

WORKING GROUP 2

Affect and mathematical thinking

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AFFECT AND MATHEMATICAL THINKING. ROLE OF BELIEFS, EMOTIONS, AND OTHER AFFECTIVE FACTORS

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One of the goals of the working group is to enhance discussion in the CERME - conferences and research between the conferences. Working Group 2 “Affect and mathematical thinking - This includes the role of beliefs, emotions, and other affective factors“ at the CERME 4 -conference was successful in creating an atmosphere of collaboration among its 16 participants. In preparation to the conference took place a call for paper and as a consequence of a reviewing process, 11 papers were accepted for presentation at the conference. The conference program scheduled 7 sessions, each 105 minutes, for work in the group. The chair of the organizing team worked out a concept for this 7 sessions. In 6 of these 7 sessions should take place a presentation of the key ideas and results of the accepted papers followed by a general discussion to the papers. Each session was extended by further activities (small group discussions to various themes, role play, analysis of data, problem solving, etc.). The last session was used for a summary of activities during the conference and highlighting important research questions for the following years.

Session 1

The chair opened the working group and welcomed all participants. To help the members of the working group to get to know each other better in a presentation game all participants had to introduce themselves and to present their interest in the field of affect. The interests in affect of the participants were very widespread: From meta-aspects (meta-cognition and meta-affect), various aspects from the relationship of affect and learning, reasons why students reject mathematics and leave schools, motivate students to learn mathematics and create a motivating atmosphere for learning meaningful mathematics, improve creativity in mathematics classrooms, for instance by problem solving, teachers relationship to mathematics and its influence to students mathematics learning processes and different kinds of research and research methods in the field.

After this first activity the organizers started with a short report about the work of the working group on affect at the CERME 3. The main message was the list of research questions that were the result of discussions at CERME 3:

- * dimensions of affect, and measures of these: a need for multiple methodologies
- * a deeper study of the relationships between affective dimensions and mathematical outcomes, such as performance
- * the need to clarify the role of affect in problem solving episodes

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- * influences on a person's affective relationship with mathematics: e.g. early experiences with mathematics
- * exploring differences in affect over the age-range, and across social groups.
- * the possibility / difficulty / modality of changing teachers' and students' affect

After a discussion of these research questions from CERME 3 Moscucci and Piccioni presented results of a research project of an Italian group (Moscucci, Piccioni, Rinaldi, Simoni and Marchini: Mathematical discomfort and school drop-out in Italy) to the relationship of affect towards mathematics and school drop-out. The causes of school drop-out there were separated in two groups: "exogenous variables" (social and familiar background) and "endogenous variables" (causes for drop-out that are connected with school system and education process). Mathematical discomfort – a negative attitude towards mathematics – was identified as a crucial reason for school drop-out. Especially elementary algebra leads to misconceptions and furthermore to problems in the learning process. Consequences are low performance and bad results in tests. Students dislike algebraic problems and develop negative attitudes toward mathematics, followed by negative attitudes in relation to school learning in general.

Session 2

Panaoura & Philippou and Schlöglmann discussed in this session two aspects of metalevel concepts (meta-cognition and meta-affect).

The concept of meta-affect was introduced by DeBellis and Goldin and describes affect about affect, affect about and within cognition and monitoring of affect, in a short form the notation encapsulates the ability of humans to handle affective situations. To get more insight in the complex process of formation and effect of meta-affect Schlöglmann applied Ciompi's concept of "affect logic". Affect logic postulates that thinking and acting of an individual is a consequence of his or her affective-cognitive schemata required in assimilatory and accomodatory processes. That means that learning processes have as a result not only cognitive knowledge, they lead also to knowledge about affective circumstances in connection with the cognitive context. Repeated learning processes to a certain content results in a meta-affect. This meta-affect controls following learning processes and the development of learning strategies.

Panaoura & Philippou (The measurement of young pupils' metacognitive ability in mathematics: the case of self-representation and self-evaluation) gave in their presentation an insight in the complexity of the concept of metacognition and the difficulties to measure metacognitive ability of young pupils. The authors use two dimensions of metacognition, self-representation of one's mechanisms about her/his knowing and self-regulation of cognition and investigate their interrelation to mathematical performance. As instrument for the investigation a questionnaire with 30 Likert type items is used to get an image of pupils' self-representation and three pairs of problems for evaluating their difficulty and the degree of similarity should

give insight in pupils self-evaluation. Mathematical performance was measured through numerical, analogical and verbal tasks and matrices. Statistical analysis shows that the constructed instrument is suitable for measurement of young pupils metacognition. Furthermore the results show that pupils with more precise relation between their performance and self-representation are able to classify the problems in a more precise way. As an important consequence it seems that low achieving pupils are often unaware of their cognitive processes and abilities although this awareness is a necessary prerequisite for an improvement of performance.

The session was finished with a discussion of affect in nonroutine problem solving processes. Schlöglmann presented Goldins' (Goldin, 2000) description of affective pathways in a problem solving process and a model of Hannula (1998) to describe the influence of affect to cognitive processes. The group agree with the observation by Liljedahl that even a successful problem solving needs to include struggle in order to be emotionally rewarding. During the discussion of meta-affect and meta-cognition Hannula presented a division that is based on his earlier work (Hannula, 2001) (Figure 2).

Figure 2. The four aspects of the meta-level of mind (Hannula, 2001).

Metacognition (cognitions about cognitions)	Emotional cognition (cognitions about emotions)
Cognitive emotions (emotions about cognitions)	Meta-emotions (emotions about emotions)

Session 3

Mellone (Di Martino and Mellone: Trying to change attitude towards maths: A one year experimentation) presented results of an experiment in a grade 12 classroom to change students' attitudes towards mathematics. The project started with a questionnaire with open-ended questions to explore students' attitudes toward mathematics. The answers also gave information for the teaching experiment. Many students answered they would like to have more connection to everyday problems. To fulfil this demand, a course in trigonometry started with an experimental situation in the context of measurement. After this experimental phase followed a reflective phase with a systematisation of the results from the experiment, then utilization and on the end recapitulation. As a result of this project we have to take into consideration that changing the attitudes of students need a new situation for students as well in the teaching method as in the organization of the learning process.

Session was continued by an input from Kaasila. All participants got the following task: Write down an experience, a situation that you remember from your school time. This experience should have some significance for you. To discuss the written document the panel was divided in groups and the document interchanges between

the group members. The main points of these remembrances lead to a very intensive discussion within the groups.

The session was finished with a homework for all participants. Liljedahl presented two nonroutine problems that should be solved by the working group members. An intensive problem solving process in the evening was the consequence.

Session 4

Eaton & Kidd (Self-conceptualised perceptions of attitude and ability among student teacher) presented results of a study about self-conceptions of students who started an education for primary school teacher. In Northern Ireland there are two pathways to become primary school teacher, a four-year undergraduate course to require a Bachelor of Education (BEd) and a one-year postgraduate education to require a Postgraduate Certificate in Education (PGCE). The last one is open for all those who have a degree in a subject that is related to subjects in primary school. The investigation used questionnaires and interviews to collect data to students' general attitudes, personal attitudes and abilities, their feelings when thinking about mathematics and their views on teaching mathematics. As a first result, the majority of students see themselves as possessing average or above average ability in mathematics but they are lacking in confidence in mathematics. They think that teachers have a strong influence on pupils' attitudes and ability, even as society influences both. Comparing the two groups in teacher education there is a relevant statistical difference in the level of competence in mathematics – the postgraduate students feel more competent and more confident.

Polo & Zan (Teachers' use of the construct "attitude"- preliminary research findings) investigated teachers' use of the attitude concept in their practical work at school. In research two concepts for attitudes are used: A "simple" definition of attitude, that describes attitude as a positive or negative degree of affect associated with a certain subject and a "multidimensional" definition, that includes three components in attitude – an emotional response, the beliefs regarding the subject and the behaviour toward the subject. The aim of the study is to see whether teachers in practice use the construct "attitude" and if yes what kind of definition is used. A further goal is the development of a diagnosis instrument for the practice in school. After a pilot study a questionnaire with multiple choice and open-ended questions was used to explore the situation. A first analysis of data shows that teachers mostly use a multidimensional idea of attitude but there exists a lack of a clear distinction between the definition of attitude and the identification of indicators. This is the reason because the definition is not really operative. Furthermore in describing causes for negative attitudes of students, teachers use characteristics and behaviours that hide their own responsibility for these attitudes. The diagnosis that a student has a negative attitude is more the result of a process to interpret students' failure and not the starting point for a remedial action.

Both presentations lead to an intensive discussion about the relationship of behaviour, observation of behaviour through researchers by using an observation concept and the interpretation of this observation.

Session 5

In session 5 two papers of Finish team were presented. Both papers belong to an ongoing project about primary teacher students' affect.

Hannula (Hannula, Kaasila, Laine and Pehkonen: The structure of student teachers' view of mathematics at the beginning of their studies) presented a statistic analysis of a survey study to explore the structure of student teachers' view of mathematics. Especially for elementary school teachers their view of mathematics is seen as an important factor that influence the way of teaching and has a crucial effect to young pupils belief in a very formative stage of their mathematical development. Analysis of a questionnaire investigation showed 10 components that identify students' view of mathematics. Two components grasping students past experience (encouragement by the family and estimation of the own mathematics teacher), three students beliefs (own talent in mathematics, estimation of their own diligence and difficulty of mathematics as a field), one the emotional relationship to mathematics and two to persons' expectation about further success in mathematics learning and as a mathematics teacher. Using correlation between the components the authors identified three components as closely related and forming a core of a person' view of mathematics – the own talent in mathematics, the estimation of the difficulty of mathematics and liking mathematics. Students with a positive view in the core components are also more confident in being a good teacher. Furthermore the background variables gender, course selection and grade are related to many of the variables and are explaining a fair amount of variation.

Kaasila (Hannula, Kaasila, Laine and Pehkonen: Autobiographical narratives, identity and view of mathematics) presented the second study of the Finish group. While the aim of the first paper was to analyse the structure of students' view of mathematics the second study used the results of the measures on self-confidence (measured by 10 items from the Fennema-Sherman attitude scale) and performance (measured by an mathematical skills test). 21 students were chosen for an interview (6 with positive self-confidence from the top 30 percent of the mathematical skills test, 8 with low self-confidence and weakest 30 percent in the skill test and 7 presenting the neutral level). For this presentation the focus was on 7 of these students who had advanced studies in mathematics in upper secondary school to answer the research question to the impact of own school experience to the view of mathematics and the construction of their views by using autobiographical narratives. Their stories lead to a division in three groups – success stories, victory through hardship stories, leaving eventually to a positive view with a negative dimension too and regression stories, leading to a negative view. To explain the mathematical identity of a student as a product of an

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education process their socio-emotional orientation (task orientation, socially orientation and ego-defensive orientation) and their coping strategies were important.

Session 6

This session started with Morsellis' presentation of a case study to creativity (Furinghetti and Morselli: Reflections on creativity: The case of a good problem solver). Basis of the study was a protocol of a students' proof to a number theoretical problem. The protocol contained not only the proof but also a drawing that the student used in the problem solving process. Especially this drawing lead the thinking process as a metaphor and later the construction of a formal proof. Problem solving was not only based on cognition but it was also influenced by affective factors. Following DeBellis' and Goldin's affective category values, ethics and morals, the authors see aesthetic values strongly linked to creativity and creativity as an expression of personality. To be a good problem solver a student needs in addition to specific mathematical knowledge also flexibility, fluency and originality in thinking, openness to new experiences, motivation to search for novelty, concentration and persistency. Especially these non-specific mathematics-related characteristics are an expression of personality adept for good mathematical problem solving. An important sign of a successful problem solving is also the use of metaphors that help to express thoughts and lead the thinking process.

The last presentation during working group sessions was given by Liljedahl (Liljedahl: Sustained engagement: Preservice teachers' experience with a chain of discovery). Students' engagement in mathematical activities is an important aim in mathematical classroom, but usually it is very difficult to sustain students' engagement to the same task for a longer period. The author presented the concept of "chain of discovery" that facilitates a state of sustained engagement. Using the "Pentominoe Problem" he created an opportunity for a series of discoveries for students. Because each student had success, the solution initiated new questions and in the following new discoveries. This process sustained students' interest in the task and strengthened their self-confidence. Successful experiences with respect to mathematical problem solving was also suitable to change individuals' beliefs and attitudes about mathematics and about their own ability as a mathematical problem solver. Many studies about beliefs and attitudes showed that a change of beliefs, attitudes and self-concepts is an important prerequisite for a more successful learning process.

Session 7

The last session was dedicated for a discussion about the implications of the working group presentations and discussions. The following conclusions were presented at the closing session of CERME4:

1) Teacher education and school practices:

When we pay attention to affect, we can influence affect through interventions.

We need to train teacher students to pay attention to affect.

An open question remains whether change of affect will lead to a change in practice. How stable will the change be?

2) Improvement in research:

Recent research has added more clarity in terminology

Research questions, theory and methodology have been linked.

More refined methods have been used.

Research has built links between theory and practice.

3) Implications to educational policy:

Decline in affect precedes decline in performance; failure in mathematics is a major cause for school drop-out.

Positive affect towards mathematics would allow more students to choose mathematically oriented lines of education. A great need for this exists.

Summary and reflections

The specific of CERME is to stimulate research in a field through the concept of the conference with working groups as the kernel of activities. This concept opens the opportunity to more discussions in small groups. In continuation to the discussion the contribution to CERME 4 should be considered in relation to the research question at CERME 3.

Dimensions of affect, and measures of these: a need for multiple methodologies implications to teacher education and school practices.

The discussion to the dimensions of affect are extended. On the one side the category values, ethics and morals is seen as necessary supplement to the categories beliefs, attitudes and emotions. On the other hand meta-level concepts are used to explain phenomena. But to measure such meta-level aspect is a very complicated task as we have seen in the case of metacognition and need deep methods. The concept meta-affect is more a description than an operative definition and is not suitable for

measuring. In other concepts as attitudes we have much deeper analysis especially by using more statistical methods. A further step is to explore teachers' use of affective categories and the consequences of this observation in classroom practice.

A deeper study of the relationships between affective dimensions and mathematical outcomes, such as performance.

The relationship between affect and mathematical outcomes is in the focus since the beginning of research in affect. But now we have more results that affect is strongly interwoven with self-concept and this self-concept influences learning as well as decision to leave school. Furthermore, this self-concept is also expression of metacognition as well as of affect. To get more insight in the complexity of self-concept, research has also to use narratives as a source of information.

The need to clarify the role of affect in problem solving episodes.

Problem solving is on the one side strongly related the personality - openness of thinking, motivation to look for new challenges - but also to have cognitive means like metaphoric thinking and cognitive and metacognitive strategies. Problem solving needs also adequate classroom situations. It is necessary to give students opportunity to discover new things. Only successful problem solving processes develop motivation and self-confidence. Only if students have positive attitude to problem solving, success is possible.

Influences on a person's affective relationship with mathematics: e.g. early experiences with mathematics.

Nearly all presentations at CERME 4 give hints to the importance of earlier experiences with mathematics to the affective relationship to mathematics. Mathematical discomfort, attitudes, self-conception, identity, metacognition and meta-affect refer to a developmental perspective. In many papers one of the aims is a change of affect, which is only possible if the property is a consequence of a learning process and not an innate characteristic. The presentations give many new results that affect towards mathematics is acquired through experiences in school and sometimes outside of school, is influenced by teachers and teaching methods but also by society.

Exploring differences in affect over the age-range, and across social groups.

The possibility / difficulty / modality of changing teachers' and students' affect towards mathematics.

Many of studies deal with teacher and student beliefs. Teacher beliefs are of interest, because these are seen as an influential factor for student beliefs. Differences in affect over the age-range need longitudinal investigations. Some presentations at the conference refer to differences across social groups especially the gender aspect is considered. We have also hints to the possibility to change affect towards mathematics. It is important that teachers reflect the affective situation of their students and accept that they have a responsibility not only for the cognitive but also

for the affective situation of their students. To do this it is necessary to develop diagnostic instruments that can be used in classroom situations.

Also CERME 4 concluded with new questions that were discussed during the working group sessions and should be a guideline for research in the following years. These questions can also be seen as continuation of the discussion process started in earlier conferences. CERME 5 will show what we can say to these new challenges.

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SELF-CONCEPTUALISED PERCEPTIONS OF ATTITUDE AND ABILITY AMONG STUDENT TEACHERS

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Abstract. *This paper reports the preliminary results of a survey examining students' attitudes to mathematics at the beginning of their initial teacher training programmes. It compares the attitudes of those students in Northern Ireland taking a postgraduate course with those undertaking an undergraduate degree and particularly focuses on their views of their own competence and confidence in mathematics, and their perceptions of how those views have been informed. It also analyses the emotional response of the students to mathematics.*

INTRODUCTION

This paper reports the initial findings of a survey of attitudes to mathematics and the teaching of mathematics among students in Northern Ireland training to teach in primary schools (pupils aged 4 to 11 years). In Northern Ireland there are two pathways to teacher education, one being a four-year undergraduate Bachelor of Education (BEd) degree course and the other, a one-year Postgraduate Certificate in Education (PGCE), offered to those already possessing a degree closely related to a subject taught in the primary curriculum. In Northern Ireland primary schools, teachers deliver all subjects in the curriculum with Mathematics and English playing central roles and the structure of teacher education programmes reflects this. Two out of the three institutions in Northern Ireland involved in providing training to such students were involved in this wide-ranging study. This paper reports on one aspect of the study, namely that part concerning personal ability, emotional response and attitudes.

THEORETICAL FRAMEWORK

As the international view of teaching has shifted from didactic to constructivist with its image of learner as participatory, so research on teacher education has moved from a focus on the transfer of a body of knowledge to a more dynamic view of the classroom, with teachers being facilitators of learners' knowledge construction. In this view of teaching, teacher beliefs and attitudes play an important role in shaping classroom practice (Bolhuis and Voeten 2004) and there is a substantial body of evidence examining this supposed link between teachers' attitudes to and beliefs about mathematics and teaching, and classroom practice (Ernest 1988, Bishop and Nickson 1983, Fang 1996, Macnab and Payne 2003).

Whilst there is a growing body of research literature concerning teacher beliefs and attitudes there appears to be no consistent definition for either of these terms despite some recent attempts to clarify thinking in this area (Di Martino and Zan, 2001). Alternatives include a single dimensional definition of attitude as emotional disposition (McLeod, 1992) while other definitions are more complex taking into account, emotions, beliefs and behaviour (Hart, 1989). Ernest (1988) also argues that attitude is multi-dimensional and distinguishes between a number of components including liking and enjoyment, difficulty, confidence and anxiety. In this paper the term belief is taken to refer to the personal constructs that influence a teacher's practice (Nespor, 1987) while the multi-dimensional definition of attitude is used.

Often, when teacher education courses are designed, little consideration is given to the set of beliefs which students carry and it is perhaps for this reason that student teachers are more likely to teach mathematics in ways in which they were taught (Ball 1988, Meredith 1993). In particular, the experiences that a student has during their own formative years in the classroom as a pupil have been shown to have a major impact on their behaviour as a teacher (Ernest 1989, Ball 1988, Hill 2000, Cooney et al.1998). It seems to be the case that student teachers revert to models of teaching that they themselves have experienced rather than try the often new and unfamiliar models that they study during their teacher training programmes (Borko et al. 1992).

This would not be an issue if teaching styles had remained unchanged in the last twenty years or so, the time during which most of these student teachers have experienced mathematics classrooms, but new paradigms have come to light and in order to move student teachers to constructivist or even socio-constructivist approaches, where cognisance is taken of the complex interplay among all participants in the learning process – pupil, teacher and wider society – an analysis of beliefs must take place with a view to appreciating the importance of often latent ideas concerning the teaching process.

The first section of this survey asks student teachers to explore their own beliefs and attitudes about mathematics with the two-fold role of initiating the self-reflection necessary as a precursor to change (Korthangen and Kessels 1999) and of informing the construction of teacher education courses designed to be responsive to current student thinking.

The second section however looks at students' perceptions of who has influenced those beliefs and who has influenced their ability in mathematics. Much has been made in the literature of examining what attitudes teachers hold but little on asking teachers why they hold the views that they do and how much influence others have on their belief systems. The aim is to encourage student teachers to think about the complex factors impacting on the learning experience and provide a starting point for analysing such views in subsequent courses. It could be argued for example, that a

teacher who does not believe that their own views have been affected by their teachers will not take account of how their own outlook will impact on their pupils. Likewise a teacher who believes their ability in mathematics is not fixed but has been influenced by the teaching they received is perhaps more likely to encourage their own students to improve in mathematics and see the centrality of the role of the teacher in improving pupil performance. This survey provides a snapshot of current views of these student teachers and it is hoped to follow up this work with an analysis of just how this interface between beliefs and practice affects interactions in the classroom.

METHODOLOGY

A main objective in designing the research was to generate data from as many perspectives as possible using both qualitative and quantitative methods. The quantitative instrument used was a questionnaire survey. Qualitative data was generated from one-to-one interviews and focus groups with students from both institutions.

The questionnaire was designed to elicit information about student teachers' general attitudes, personal attitudes and ability, their feelings when thinking about mathematics and their views on teaching the subject. This paper will focus on the student teachers' personal ability and attitudes.

The questionnaire consisted of a list of statements and emotions and a five-point Likert scale was used throughout the questionnaire to facilitate the efficient collection of standardised data from a large proportion of the target population. To obtain a sample of student teachers taking undergraduate and postgraduate courses, data was collected from two of the three institutions offering primary teacher education courses in Northern Ireland. To maximise the number of responses, lecturers from the two institutions were asked to administer the questionnaires after their classes and to collect the completed questionnaires. In the event, 130 out of 156 BEd (83%) and 69 out of 70 PGCE (99%) students returned completed questionnaires.

The BEd sample consisted of 130 students enrolled in the first year of a four year primary teaching course. The majority of the sample, 83%, was female. Approximately 7% of the sample were mature students i.e. 21 years and over. Of the 69 PGCE students, 11 were male (16%) and 43 were mature students (63%). In this institution a student is defined to be mature if they have not come directly from tertiary education.

The questionnaire was designed by the authors and took into account the work of Macnab and Payne (2003). A small representative sample of students (7 in total: 3 BEd, 4 PGCE) from both institutions were interviewed in small groups (maximum 3) or individually. The semi-structured interviews, consisting of questions designed to draw out responses to the original questionnaire, were recorded and transcribed and

the data categorised in relation to the key questions in the questionnaire. These interviews took place later in the course and at this stage the students had some experience of teaching in primary schools. Whilst it is realised that this is a very small sample and cannot be taken to represent the whole sample under study it was felt that the additional insight it provided was very valuable. This project is ongoing and only data from the 'Personal Ability and Attitudes' section will be discussed in this paper.

RESULTS

Students were asked to state their perceptions of their own ability in mathematics and the results can be found in Table 1. Responses show that the majority of both cohorts rated their own ability as 'average' with very few indicating 'below average' or 'well below average' (5.7% 'below average' or 'well below average' for PGCE and 10% for BEd).

	PGCE &BEd	PGCE	BEd
Well below average	2.5	1.4	3.1
Below average	6.0	4.3	6.9
Average	68.4	72.5	66.2
Above average	21.0	20.3	21.5
Well above average	0.5	1.4	0

Table 1: Percentage rating of mathematical ability

Furthermore, over one fifth of PGCE and BEd students did report their mathematics ability to be above average. It is hoped to explore in further detail, as more focus groups are carried out, what measures students use to rate their own ability.

Student teachers were then asked to respond to ten statements regarding their personal ability and attitudes to mathematics and Table 2 summarises the responses.

There was a notable difference between the two cohorts' answers with regard to competence in mathematics. It would be interesting to explore in more detail the definition of competent used by the students when responding to this particular question but given that the purpose of this survey was to ascertain views of students at the very beginning of their training it was felt that giving them a more precise definition would be meaningless as they had not yet the experience to interpret possible definitions. While almost 67% of PGCE students 'agreed' or 'strongly agreed' that they felt competent in mathematics less than 45% of BEd students felt the same way and further analysis using t-tests revealed that this was a statistically significant difference ($t(197) = 2.969, p < 0.01$). 17.3% of PGCE and 30% of BEd students did not feel competent in the subject. Both cohorts were even less positive about their confidence in the subject with less than 50% of PGCE and 30% of BEd students feeling confident about their own mathematical ability. It should be noted

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that this survey occurred at the very beginning of the students initial teacher education programme and it would be interesting to note any change in these results on completion of their programme.

	'agree' or 'strongly agree'			'disagree' or 'strongly disagree'		
	PGCE & BEd	PGCE	BEd	PGCE & BEd	PGCE	BEd
I feel competent in mathematics*	52.2	66.7	44.6	25.6	17.3	30.0
I feel confident in mathematics	36.7	47.8	30.7	34.6	27.5	38.4
My mathematical ability is influenced by my parents/family	36.7	44.9	32.3	43.7	40.5	45.4
My mathematical ability is influenced by my teachers	77.4	78.2	77.0	9.0	7.2	10.7
My mathematical ability is influenced by my peers	22.1	23.1	21.6	52.2	50.7	53.1
My mathematical ability is influenced by society*	23.1	36.2	16.2	39.7	29.0	45.4
My attitude to mathematics is influenced by the attitudes of my parents family	34.2	40.6	30.8	51.7	47.8	53.9
My attitude to mathematics is influenced by the attitudes of my teachers	68.4	71.0	66.9	15.1	13.0	16.2
My attitude to mathematics is influenced by the attitudes of my peers	27.1	28.9	26.1	48.7	44.9	50.8
My attitude to mathematics is influenced by the attitudes of society*	24.1	34.7	18.5	45.2	39.1	48.5

**Items where a statistically significant difference exists between the two independent variables using t-tests.*

Table 2: Personal ability and attitudes: percentage 'agree' or 'strongly agree' and 'disagree' or 'strongly disagree' combined for each statement by course type.

The next four statements related to factors influencing students' own ability. Both cohorts felt that their teacher had the most influence on their mathematical ability (77.4% PGCE and 78.2% BEd 'agreeing' or 'strongly agreeing'), with the

'parents/family' category coming second. For both groups, peers had a smaller amount of influence on their ability with less than 25% 'agreeing' or 'strongly agreeing'. In contrast, 20% more PGCE students (36.2%) felt that society influenced their mathematical ability than their BEd equivalents and further analysis using t-tests showed that this was a significant difference ($t(197) = 3.125, p < 0.01$). The following comments made by students during interview support the findings. (Institutions are identified anonymously by capital letters.)

Teachers have a very powerful influence on their pupils' ability. I remember way back in P2(second year of primary education) being stood out at the front of the class while the teacher told everyone that I did not know the answer to a particular sum. I will never forget that experience. I can still remember the embarrassment and shame and would say that it convinced me of my lack of ability in maths. (PGCE student 3 Institution A)

If you (teacher) are enthusiastic about the subject and you want to encourage the children to become better at it (mathematics) then if you get them at a young enough age, you might be able to help them rather than (have them) get problems later on. (BEd student 2 Institution B)

A similar pattern of results is revealed from the statements regarding third party attitudes influence over students' own attitude to the subject. Both contingents felt that their teachers' attitude to mathematics influenced their own attitude most (71.0% PGCE and 66.9% BEd 'agreeing' or 'strongly agreeing'), with the attitudes of parents/families coming second. Those students interviewed reinforced this point.

I think a good teacher can help to motivate children. I wasn't very good at maths at the beginning but I had good teachers and stuck at it because they helped. They motivated me and helped me want to be good at maths. (BEd student 1 Institution B)

I feel my attitude to maths was influenced by both my teachers and my parents. Until this point I wasn't particularly inspired by anyone who taught me maths and found the subject difficult. But it was OK at home because my mum isn't good at maths and she understood. I remember her telling me not to worry when I couldn't learn my (multiplication) tables because she couldn't remember hers either. I have a son and I don't want him being influenced by my negative experiences. (PGCE student 2 Institution A)

For both groups the attitudes of their peers had less influence on their own attitude to the subject with less than 30% 'agreeing' or 'strongly agreeing'. However, PGCE students (34.7%) felt that the attitude of society to mathematics influenced their own attitude to the subject more than BEd students (18.5%). Further analysis using independent sample t-tests ($t(197) = 3.125, p < 0.01$) indicated that this difference was significant.

Student teachers were asked to respond to ten emotions they might feel towards mathematics using a five point scale from ‘not at all’ to ‘very strongly’. The results are displayed in Table 3. Overall both cohorts were not very positive in their own feelings towards mathematics, although PGCE students were more positive than BEd students in all ten emotions. While more than 40% of PGCE students rated their interest, satisfaction and motivation in the subject ‘moderately’ or higher, 50% of BEd students responded ‘little’ or ‘not at all’ to the same emotion, with the difference being statistically significant for interest and satisfaction. Furthermore, while 20% and 16.9% of BEd students felt strong or very strong fear and anxiety respectively only 8.7% of PGCE students felt ‘strongly’ or ‘very strongly’ about either of these emotions, with the difference in anxiety levels being statistically significant. Out of all the emotions specified both cohorts indicated they felt anger least. However, while BEd students did feel less enthusiasm and more bored and frustrated towards mathematics, over 70% of both groups indicated that they felt little or no excitement about the subject. These findings are supported by the following comments.

I’m not sure this is an emotion but the word I would associate with maths is fascination. It is much more complex than I had initially thought and I am fascinated by children’s ideas and thought processes. (Student 4 PGCE Institution A)

I feel satisfaction. I love the logic and sequence in teaching maths and you have a feeling of satisfaction seeing children make progress. I think it is easier to track (children’s progress) in maths than in say something like English which is so broad. (Student 1 PGCE Institution A)

	‘not at all’		‘a little’		‘moderately’		‘strongly’		‘Very strongly’	
	PGCE	BEd	PGCE	BEd	PGCE	BEd	PGCE	BEd	PGCE	BEd
Interest*	10.1	23.1	29.0	30.0	40.6	32.3	17.4	12.3	2.9	2.3
Fear	24.6	19.2	49.3	44.6	17.4	16.2	2.9	15.4	5.8	4.6
Anger	81.2	72.3	8.7	15.4	8.7	10.0	0.0	1.5	0.0	0.8
Satisfaction*	10.1	19.2	21.7	30.0	46.4	32.3	14.5	14.6	7.2	3.8
Boredom	36.2	14.6	39.1	53.1	17.4	19.2	7.2	6.9	0.0	6.2
Frustration	29.0	10.0	34.8	53.8	21.7	19.2	10.1	11.5	4.3	5.4
Excitement	50.7	60.0	20.3	24.6	23.2	9.2	4.3	3.1	1.4	2.3
Anxiety*	36.2	26.9	47.8	37.7	7.2	18.5	2.9	13.1	5.8	3.8
Enthusiasm	20.3	30.8	30.4	31.5	37.7	27.7	10.1	6.9	1.4	3.1
Motivation	20.3	21.5	23.2	36.2	44.9	27.7	8.7	10.8	2.9	3.1

**Items where a statistically significant difference exists between the two independent variables using t-tests*

Table 3: Percentage response of both PGCE & BEd students for each emotion

Comparing this table with Table 1 reveals that while 5.7% of PGCE students stated that their ability was below or well below average, 8.7% felt strong or very strong anxiety. Among BEd students, 10% rated their ability below or well below average with 16.9% feeling anxiety strongly or very strongly. This suggests that even those students who regard themselves as having at least average ability in mathematics also experience strong or very strong feelings of anxiety.

CONCLUSION

The findings above provide us with an image of student teachers, undergraduate and postgraduate, the majority of whom see themselves as possessing average or above average ability in mathematics. They are lacking in confidence in mathematics and generally do not feel very competent. It is however not surprising that those who have chosen teaching as a career feel that teachers have a very strong influence on the attitudes and ability of pupils. The generally younger BEd students do not feel as strongly as the postgraduates that society influences the attitude and ability of pupils but this may be due in some part to the postgraduate's greater engagement with the wider world. The students were generally negative in their emotional response to mathematics with however some differences coming to the fore between the contingents.

One area in which there were statistically significant differences between the views of undergraduates and postgraduates was in their level of competence in mathematics. Considering that the questionnaire was completed at the beginning of both courses one must look perhaps to the experience of the postgraduate students to account for their greater competence. This may also be a reason why the postgraduates felt more confident in mathematics. It will be interesting to observe whether or not their confidence decreases or increases as their course progresses. There is an argument that at this stage they do not appreciate the subtleties of mathematics and are over-confident in their own abilities, as is perhaps the case with similar Scottish students (Macnab and Payne 2003), and as they learn how much they do not know they may actually lose confidence in their own abilities. This would be highly undesirable as already less than 40% of the combined group currently feel confident in mathematics.

Other studies (Brown et al. 1999) have discussed the inability of students to fully articulate their understanding of the subject when reflecting on their own experience and while this survey is certainly constrained by this it is hoped that by asking students to at least begin the process of self-reflection at this early stage of their training they may to some extent overcome this hurdle. It will be very interesting to follow these students as they progress through their initial teacher education courses, and hopefully beyond, to see how, and if, these attitudes change and how the process of self-reflection can be embedded in the practice of these students.

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REFLECTIONS ON CREATIVITY: THE CASE OF A GOOD PROBLEM SOLVER

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“There is no permanent place for ugly mathematics”
G.H. Hardy (*A mathematician’s apology*)

Abstract: *In this paper we report a case study of a good problem solver who faces the proof of a statement in number theory. We have assumed as a starting point the existence of a mutual influence of affective and cognitive factors and we have carried out our analysis with the aim of verifying this hypothesis. Among the elements that shape the behavior of our good problem solvers we singled out aesthetic values and feelings of freedom in facing the problem.*

Keywords: *creativity, aesthetics, affect and cognition.*

INTRODUCTION

In the years from the 1902 to 1904 the famous journal devoted to mathematics teaching *L’Enseignement Mathématique* published an extensive questionnaire addressed to professional mathematicians. The aim was investigating their way of working. The findings were published in issues of the journal from 1905 to 1908 with the comments of Henri Fehr, a mathematician editor of the journal, and two psychologists of the University of Geneva (Édouard Claparède and Théodore Flournoy). This inquiry had the merit of bringing to the fore an issue which would have had remarkable developments in the successive years, e.g. the factors influencing the mathematical activity. Poincaré (1908) referred explicitly to this inquiry in his famous paper on the mathematical invention. Hadamard acknowledged the importance of this inquiry in his famous book (1954), but raised the following criticism:

No [...] precise question was asked concerning the influence of the psychical state of the worker and especially emotions that he may be experiencing [when doing mathematics]. (p. 9)

Aspects of this criticism apply also to research in mathematics education. Traditionally, research on students’ performances (problem solving, proof,...) has concentrated primarily on cognition, less on affect¹, and still less on interactions between them. This way of looking at students’ behavior when engaged in mathematical tasks has shown its limits. Schoenfeld (1983) claims that:

“purely cognitive” behavior is extremely rare, and what is often taken for pure cognition is actually shaped - if not distorted - by a variety of factors. [...] The thesis advanced here is that the cognitive behaviors we customarily study in experimental fashion take place within, and are shaped by, a broad social-cognitive and metacognitive matrix. That is, the tangible cognitive actions produced by our experimental subjects are often the result of consciously or unconsciously held beliefs about (a) the task at hand, (b) the social environment within which

the task takes place, and (c) the individual problem solver's perception of self and his or her relation to the task and the environment. (p. 330)

It is widely recognized² the complex nature of the factors influencing students' processes of proving. Moore (1994) reviews a mass of studies which analyze the areas of potential difficulty that students encounter in learning to do proof and concludes that "These studies suggest that the ability to read abstract mathematics and do proofs depends on a complex constellation of beliefs, knowledge, and cognitive skills" (p. 250) We share this opinion and in our recent paper (Furinghetti & Morselli, 2004) we have discussed a case in which the intertwined nature of affects and cognition was evidenced through the study of the negative performance of a university student proving a statement of number theory. In this paper we consider the other side of the coin, that is the good performance of a university student facing the same exercise in the same external conditions reported in (Furinghetti & Morselli, 2004).

THEORETICAL BACKGROUND

DeBellis and Goldin (1997) studied the complexity of internal representational systems as human beings engage in mathematical problem solving. According to them, there are five kinds of internal representational systems, constructed over time, that interact continually in symbolic relationships with each other: verbal/syntactic system, imagistic system, formal notational system, system of planning and executive control, affective system. The affective system refers to changing states of feelings during problem solving (local affect) and more stable and longer-term constructs (global affect). Considering affect as a representational system, it may be said that states of feeling interact with other modes of representation, encode important information and influence problem solving performance. Attitudes and beliefs are aspects of global affect, emotions are part of local affect. According to DeBellis and Goldin (1997), emotions have not a low level of cognitive activity. They speak of "affective pathways", that are sequences of states of feelings that interact with cognitive representational configurations.

DeBellis and Goldin (1997) consider also aspects of the solver's values, morals and ethics. These are much than a belief about what mathematics is: for example, a student may feel bad when he doesn't follow the instructional procedures, because he is contravening his moral values. Another example of moral component is the mathematical self-acknowledgement, that is the student's ability to acknowledge an insufficiency of mathematical understanding. This acknowledgement may lead to surface-level adjustment or to efforts for a deeper understanding. The strongest problem solvers show a straightforward recognition of insufficient understanding and productive responses.

In our opinion the values/morals/ethics component of affect includes aesthetic values. When we use the term aesthetics we are not referring to the way it is intended by

professional mathematicians³, but to classroom aesthetics. Sinclair (2003) clarifies this distinction as follows:

A student's aesthetic capacity is not equivalent to her ability to identify formal qualities such as economy, cleverness, brevity, simplicity, structure, clarity or surprise in mathematical products. Rather, her aesthetic capacity is her ability to combine information and imagination when making purposeful decisions regarding meaning and pleasure...(p. 200)

The study reported in (Sinclair, 2003) shows that students' "aesthetic behaviors have very functional, yet pedagogical desirable, purposes: establishing personal and social value." (p. 204) A similar interpretation of aesthetic values is discussed by Featherstone and Featherstone (2002), who comment the work of David Hawkins. These authors focus on the way this philosopher "connects aesthetic experience to interest and engagement" (p. 24). In the words of Hawkins reported in the paper "[aesthetics] is a mode of behavior in which the distinction between ends and means collapses; it is in its own end and it is its own reinforcement."

According to us aesthetics is linked to creative thinking and creativity. Creativity has been described in different ways. According to (Imai, 2001) the key aspects of creativity are "the ability to overcome fixations in mathematical problem-solving and the ability for divergent production within the mathematical situation" (p. 187) and the related concept of divergent thinking is characterized in terms of the following features:

fluency, shown by the production of many ideas in a short time; flexibility, shown by the students varying the approach or suggesting a variety of methods; originality, which is the student trying novel or unusual approaches; elaboration, shown by extending or improving of methods; and sensitivity, shown by the student criticising standard methods constructively. (ibid.)

Fluency and flexibility, that is to say the abilities to overcome fixations and to produce creative thinking within mathematical situations, were already acknowledged as important features also by Haylock (1987). In the following we will summarize the features linked with fluency and flexibility with the expression of 'sense of freedom'.

There are other description of creativity. For example, Eryvnyck (1991, p. 47) gives the following tentative definition

Mathematical creativity is the ability to solve problems and/or to develop thinking in structures, taking account of the peculiar logico-deductive nature of the discipline, and of the fitness of the generated concepts to integrate into the core of what is important in mathematics.

A totally different point of view, which comes from the exterior of mathematics, is presented in (Urban, 1995). The model of this author identifies six dimensions of creativity which stress the concurrent presence of knowledge, abilities and affective factors, see Figure 1.

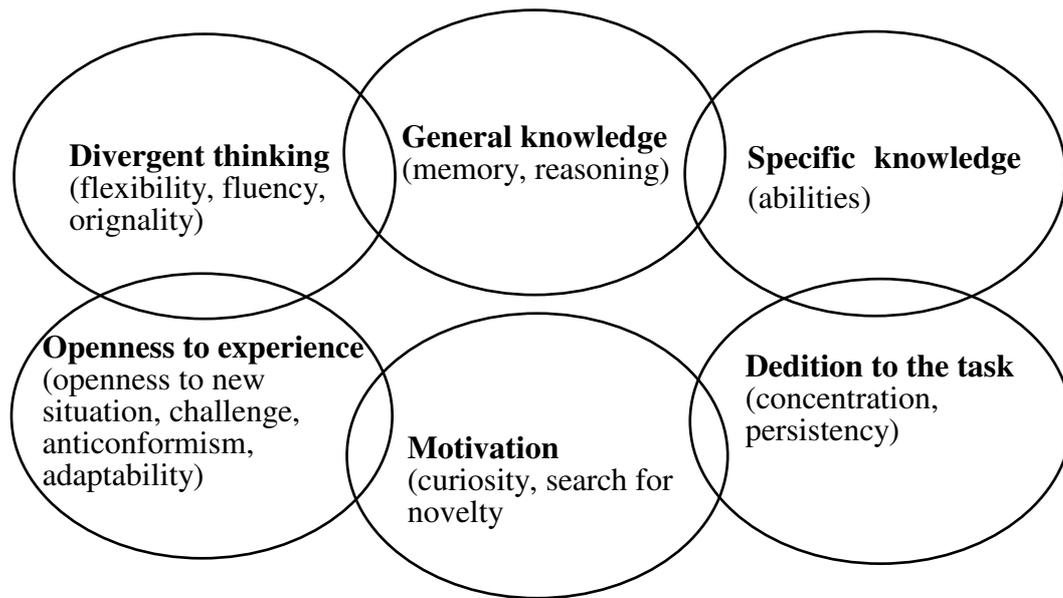


Figure 1. Urban's model for creativity

Maslow (1962) distinguishes two degrees of creativity: primary creativity is related to spontaneous behaviors and takes place when a person does not fear his own thinking; secondary creativity is related to the ability of putting order in personal or others' ideas. When both degrees are present creativity is termed by Maslow "integrated creativity". In particular, Maslow deals with *self-actualizing creativity*, which does not come from a particular talent ("genius"), but exactly from *personality*. Self-actualizing creativity is revealed by any behavior of the subject, who tends to act creatively in any situation. According to Maslow, self-actualizing people are receptive, i.e. open to exterior stimulus, spontaneous and expressive. These subjects act in a more natural way respect to ordinary people; they are less inhibited by self-criticism and by fear of judgments by other people.

METHODOLOGY

In our study we consider one student of the final year of the university course in Mathematics. He had attended all basic courses (algebra, geometry, analysis...) and advanced courses in mathematics. His curriculum encompassed one course of mathematics education, in which our experiment was carried out. In this course the students are regularly engaged in activities of proving, developed as follows⁴:

- a problem is given
- the students are aware that the problem is at their grasp
- the students are asked to write the solving process and to record in the protocol the thoughts⁵ that accompany their work
- the students work out the problems, solving them individually
- the protocols produced are analyzed by all class.

The goal of these activities is not the proof by itself, nor marks are given to the performances. Instead, the students are asked to focus on the analysis of what they think and do when proving.

The students are allowed to use pseudonyms and are allowed to work as long as they need in order to avoid the influence of time in the performance, see (Walen & Williams, 2002). The case study we refer to is set in this context. The statement to be proved was the following:

Prove that the sum of two coprime numbers is prime with each of the addends⁶
(Two natural numbers are coprime if their only common divisor is 1)⁷

We have selected our student because he was very collaborative in providing us information on his thoughts. Moreover he was one of the few cases of a student who was able to fill the task. Our study is based on the protocol and a subsequent interview with the student involved. In the following we report the translation into English of the protocol⁸. To perform our analysis we have split the protocol in numbered sentences which for us identify the component steps of the student's reasoning. We accompany these sentences with comments. In line with the ideas of Leron and Hazzan (1997) the focus of our analysis will be on the students' experience by itself rather than on the comparison of the students' production with an expert's one.

OUR ANALYSIS

Since our student used names of trees as a pseudonym for labeling his protocols we will refer to him as *Albero* (the Italian word for 'tree').

Already at the first glance the protocol of this student appears extremely well organized and clear. It contains also a drawing. We will see in the following that the protocol is sharply divided in two parts. We analyze the first part (steps 1-5) step by step; the second, which simply refers to the formal proof of the statement, will be analyzed globally, since it is less relevant to the focus of this paper.

1. First of all I want to see prime numbers, I want to grasp their secrets.

We catch *Albero's* need to get a personal sense for the problem, as a prerequisite for constructing a proof. He recalls his previous understanding of integers to his wish of *grasping the secrets of prime numbers*. He looks at the concepts and relations involved in the problem, in a particular way that becomes effective for having an insight in the problem. The use of a colloquial expression such as "to grasp their secrets" will be the leitmotiv of his way of communicating.

2. The first way in which I see them is "as jumps", I tell you in this way.

I imagine a straight line with many equidistant stops (the stops are the numbers). Two stops are coprime if⁹ - I'm roughly speaking - [in considering] the frog that jumps from stop to stop,

the frog that jumps every two stops

the frog that jumps every three stops,

There is not any frog that reaches both stops (except the frog that jumps every stop).

His first strategy is a sort of translation of the problem in terms of “*his mathematical world*”. In this world the institutional rules are completely accepted, but they are expressed in a *non-conventional way*. He translates the concepts and the relations involved in the statement of the problem (which of course, are mathematical) into *dynamical images taken from his imagery*. We stress that he refers to images that reflect his own way of conceptualizing natural numbers and turn out to be useful for reasoning.

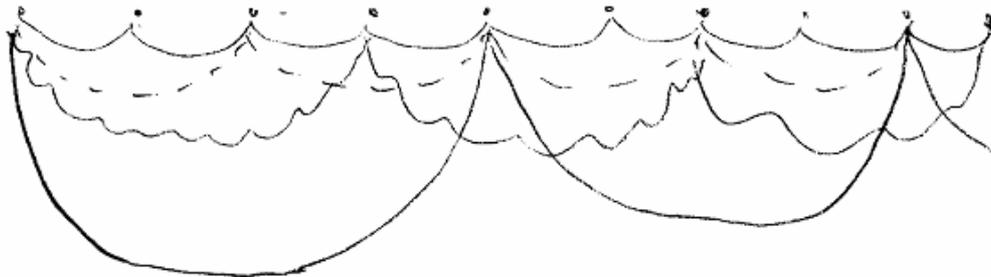


Figure 2. Albero's drawing

The second component step of the protocol is accompanied by a drawing, see Figure 2. Albero's view of the relations involved in the problem turns out to be efficient; in particular, his way of representing the relation “*being coprime*” is dynamic and allows to grasp the mathematical mechanism behind the definition of common divisor. The metaphor is accurately chosen and shows a high degree of conceptualization. This is evidenced by the fact that, as we see from an erased row, initially the chief character of the metaphor was a traveling man stopping any two stages. This metaphor, which may evoke continuity, was discarded in favor of the metaphor of the frog which is more evoking discrete aspects. What makes really efficient the metaphor is the drawing, which is a support and a suggestion for the reasoning.

3. Ooh now I see!

I consider the two stops A and B. I call $A + B$ the stop (do not ask me to be formal, otherwise I loose the good thing)...

I have to prove that, except the first frog, a frog that stops in A does not stop in $A + B$, a frog that stops in B does not stop in $A + B$.

So I would have: if a number divides A or B it does not divide $A + B$, and this is enough for the thesis, because I would have that if a number divides $A + B$ it does not divide A nor B”.

By making the drawing, Albero gets an insight in the relations involved in the problem and when he looks at the finished drawing the solution comes out immediately (“Ooh now I see!”). From now on he explains his solution to the reader keeping alive the metaphor of the frog. We stress the fact that his reasoning is sharp and formal-like, even within the ‘amusing’ metaphor. He translates hypotheses and theses into the metaphorical language, keeping the isomorphism between mathematics and the pond. He is aware that his way of reasoning is not conventional, but he does not want to get out of the metaphor so as not to break his stream of thoughts. The sentence “do not ask me to be formal, otherwise I loose the good thing” offers a clue on Albero's approach to mathematics.

4. It is easy. If a frog stops in A it can not stop in $A + B$ because going from A to $A + B$ is like going from O to B . And, for hypothesis, the frog that, by jumping from O reaches A , can not reach also B . The same argument holds for a frog that stops in B and then it does not stop in $A + B$.

We note that Albero's proof resounds Peano's axioms of arithmetic (1889). The clever idea was to set the problem in a discrete domain (the frog versus the traveler). This fosters the use of the sum and its invariance under translation ("*going from A to $A + B$ is like going from O to B* ").

The following quotation from the interview taken after the lesson adds further information on Albero's approach:

If one wonders... I have to work on multiples, then let's look the numbers in the face! [...] I spoke of frogs, but I can imagine this series of numbers, this meter, as bulbs that switch on at the same distance: and you look them, and once you have looked them.... Perhaps someone does not understand anything and imagines numbers as sacks, does not look multiples in the face [...] We are talking about numbers, what are they? Numbers are equidistant things, which count equally and never finish. Adding is like going back to zero and starting again to jump.

Albero needs to have an insight on the problem to solve it and this insight comes from the his reflection on the structure of natural numbers and their properties. The use of expressions such as "*look the numbers in the face*" recalls us Sfard's (1991) claim that being capable of somehow seeing the invisible objects of mathematics appears to be an essential component of mathematical ability. We underline Albero's awareness of having adopted a discrete approach that is functional to the problem and his skill in analyzing and explaining his way of reasoning. Furthermore we stress the accuracy in choosing the metaphors which are isomorphic to the structures he will use.

5. In this way in intuitive arithmetic and with intuitive methods I have proved (I take on the responsibility for this word, here among friends) the thesis.

We note that at this point Albero reflects on the nature of his proof. Firstly he focuses on intuitiveness: for him "*intuitive*" means being outside the formal mathematics (no formulas, no symbols, no explicit rules of inference). Actually Albero's reasoning is "isomorphic" to a correct proof of the statement, only the way of communicating is not mathematical. Furthermore, he regards his proof as a real proof ("I have proved"), and he feels completely responsible ("I take on the responsibility") of it. This behavior is not common among students: usually they are influenced by the belief that a proof is acceptable if it presented in a certain formal way.

The second part of Albero's protocol has a completely different flavor from the first one. He writes:

6. "Now, since there is time time left, we'll make it in the form of an algebraic proof."

His proof is correct, clear and concise as it could be found in a textbook, see Figure 3. It appears as an automatic product coming from the brain and not from the heart. The natural fluency of Albero's formal proof using algebraic tools confirms that the previous informal proof did not originate from a lack of knowledge or skill, but from a natural inclination to to metaphorical thinking.

Working Group 2

Siano $a, b \in \mathbb{N}_*$ primi fra loro (se $a \circ b = 1$ la tesi è banalmente, inconfutabile, facile)

Per la tesi basta provare $c|a \Rightarrow c|a+b$ con $c \in \mathbb{N}_*$
 $d|b \Rightarrow d|a+b$ con $d \in \mathbb{N}_*$

Provo la prima
 suppongo PA $c|a$ e $c|a+b$ con $c \in \mathbb{N}_*$
 $c|a \Rightarrow \exists d \in \mathbb{N}_* \text{ t.c. } a = dc$
 $c|a+b \Rightarrow c|dc+b \Rightarrow c|b$ ASSURDO

\uparrow
 $c|dc+b \Rightarrow \exists e \in \mathbb{N} \text{ t.c. } c = e(dc+b) \Rightarrow c = edc+eb \Rightarrow c - edc = eb$
 $c|dc+b \Rightarrow \exists e \in \mathbb{N} \text{ t.c. } c = dc+b \Rightarrow \exists e \in \mathbb{N} \text{ t.c. } c(e-d) = b \Rightarrow c|b$

applico tutte
 le proprietà delle operazioni che mi servono
 delle due operazioni di $(\mathbb{Z}, +, \cdot)$

la seconda è strutturalmente uguale.

Figure 3. Albero's formal proof

REFLECTIONS

Our initial assumption on the intertwined nature of cognitive and affective factors has led us to scrutinize with two different kinds of lens the student's behavior. The cognitive lens revealed that he knows mathematics and he is able to use his knowledge. He shows:

- good mastering of proving strategies
- richness of language for communicating
- good mastering of the mathematical language
- flexibility and fluency

Through the affective lens other elements emerged:

- pleasure for the challenge
- sense of freedom
- aesthetic as a value
- strong emotional involvement in doing mathematics
- mathematics as a personal business

Definitely we may say that Albero has a good relationship with mathematics. For him mathematics is not an external construction, rather it is a living part of himself. He takes the mathematical activity easy and doing mathematics is a pleasure for him. The pleasure in solving problems was indicated by Polya (1945) as one of the elements fostering the good solution. We note that all along Albero's protocol there is the distinction of the two levels of mathematics: the public (formal mathematics of the professional mathematicians) and the private (his personal construction of meaning.) For him mathematics is firstly a private business and there is strong emotional involvement: this context fosters his creativity.

Albero is a pregnant example of integrated creativity, see (Maslow, 1962): he has a powerful intuition, an easy relationship with his thoughts, and manages his ideas through a sharp reasoning. Maslow (1962) claims that creative subjects do not fear

unknown and puzzling: while ordinary people feel uncomfortable with situations of doubts and uncertainty, creative people live such situations as a pleasant challenge. In this concern, the comparison of Albero's protocol and the protocol of the unsuccessful solver analyzed in (Furinghetti & Morselli, 2004) offers interesting insights. The unsuccessful solver makes explicit her anguish about her past (lack of knowledge) and looks at the future feeling unsafe and uncomfortable. On the contrary, Albero feels good about his past (we have seen that he is able to exploit his past experience productively) and looks at the future with confidence. He throws himself into the challenge with enjoyment. He feels the pleasure linked with aesthetic values.

In reading Albero's protocol we realized how much right is Maslow when he claims that being creative means keeping something of one's childhood. If Saint-Exupéry says that all adults have been children¹⁰, we would add that all adults should remember that and should not repress this aspect.

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¹ It is well known that there is a problem of terminology, see the discussion in the book (Leder, Pehkonen & Törner, 2002). We adopt the view of McLeod (1992), who claims that "the affective domain refers to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition. H. A. Simon (1982), in discussing the terminology used to describe the affective domain, suggests that we use affect as a more general term; other terms (for example, beliefs, attitudes, and emotions) [are] used as more specific descriptors of subsets of the affective domain." (p. 576)

² A literature review about this point is in (Furinghetti & Morselli, 2004).

³ See (Hadamard, 1954; Poincaré, 1952).

⁴ For a full account of the course and the experiment see (Morselli, 2002).

⁵ Emotions and feelings were not explicitly mentioned as required information, nor they were mentioned in the theoretical lectures delivered during the course of mathematics education.

⁶ This problem is an adaptation of a part of Euclid's proposition VII, 28 "If two numbers be prime to one another, the sum will also be prime to each of them; and, if the sum of two numbers be prime to any one of them, the original numbers will also be prime to one another" (Heath, 1956, v. II, p. 329). Nowadays instead of "numbers prime to one another" we say "coprime numbers".

⁷ The definition of coprime numbers was given in the statement to prevent the difficulty of remembering it.

⁸ We have translated it as faithful as possible. We are aware of the pitfalls of translation, but this was the only way to make the original texts accessible to foreign readers.

⁹ From the sentence erased by the student, but still readable in the protocol, we know that initially the student thought to use as a chief character a traveler, but he shifted to the image of the jumping frogs that fit better to the metaphor of integer numbers seen as stops in a path.

¹⁰ "Toutes les grandes personnes ont d'abord été des enfants", *Le petit prince* by Antoine de Saint-Exupéry.

AFFECT, MATHEMATICAL THINKING AND INTERCULTURAL LEARNING

A STUDY ON EDUCATIONAL PRACTICE

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Abstract: *Studies dealing with affect and mathematical learning in multicultural contexts are rare. This paper explores the interaction between mathematical thinking and affect in these contexts. A study made in 2003, in Belgium, carried out with secondary level students of Portuguese origin is presented. The methodology and the schema of analysis involved have been previously used by this author in earlier investigations in Spain. It is a framework with parameters which disclose the origin and the evaluation of the emotional responses in the subjects.*

The reconceptualisation on affect and mathematical thinking in the present decade is marked by two essential features. One is the attempt to consolidate a satisfactory theoretical framework for its interpretation, the other is the development of measurement instruments and other methodological tools for researcher within the social context in which learning takes place (see e.g, Evans, Hannula, Philippou & Zan (2003), Hannula et.al. (2004)). In accordance with this aim our paper seeks to increase the coherence between the observing instruments and the theory itself.

In a previous published paper I presented a model for the study of the interactions between cognitions and affects in the learning of Mathematics (Gómez-Chacón, 2000a and 2000b). This model is used here for describing emotional responses, their origin as well as for surveying their evolution in the subjects under consideration. Three dimensions related to affects and cognitions are specifically focused, namely, affect itself, meta-affects and belief systems. Attention is drawn to the importance of taking into consideration these dimensions in investigations of this nature, particularly in the case of school-failing students and in multicultural contexts.

Two groups of students, both of secondary level, have been involved in the study, one from Spain the other from Belgium. The study in Spain was accomplished first. Here I present the study carried out in Belgium, from February to June 2003, with students of Portuguese origin living in Brussels, in their 7th to 12th grades of education.

1.- Theoretical Framework

Working from a holistic perspective, which requires to consider the particular situation of the individual person, forces us to approach the theoretical framework of our work from a variety of cognitive fields:

- The appropriation of insights from a socio-cultural and socio-constructive perspective;

- The affective dimension in mathematics education, applying theoretical frameworks from psychology and sociology into the context of mathematics education.
- The perspective of social and cultural identity.

Dimensions related to affects and cognitions, such as affect itself, meta-affects and belief systems have been already used and defined by other authors (Goldin, 1998). At the beginning of my research (Gomez-Chacon, 1995, 1997) I considered them as a departure point, but moved toward a more precise and operational definitions of the constructs. Since then, myself and others, have distanced ourselves from this approach in order to seek a more dynamic viewpoint, which is required for dealing with the complexity of affect-cognition interplay in social learning situations. Taking into account the embeddedness of students' knowledge as well as beliefs in the social context (see e.g. de Abreu, Bishop, and Pompeu, 1997) the interpretation and appraisal processes that ground students' emotions in the classroom are constituted by the social-historical context in which they are situated. The emotions clearly have a rationale with respect to the local social order, giving rise to a particular notion of identity, all of which play a central role in the Mathematics classrooms (Cobb & Hodge (press))

With others, I hold the general view that cognitive theories and socio-cultural theories can be brought together in an effort to create a comprehensive theory of human activity (Eisenhart, 1988:10). My specific contribution in this respect is to integrate the emotional dimension in mathematics learning into this general view. I consider that the group, with its culture, its communication system and its institutional structure (accepted to be basic social and anthropological phenomena in mathematical education), is as important as the personal dimension, with its intra-individual aspects of cognition and of psychological relationships. I claim to have verified that a critical integration of these perspectives assists our understanding of the complex interaction of the affective, cognitive and cultural factors which play a role in learning mathematics.

In this perspective emotions are not a mere result of automatic responses or consequences of physiological impulses but rather are a complex result of learning, of social influences and of interpretation. In social interaction, emotions play a basic role in establishing relationships of social belonging and of social status. Social identity is considered to be the central organising principle which mobilises the totality of emotional responses of each individual towards mathematics and its learning. The central tenet of this perspective is that the individual's social identity shapes the social and global structure of affectivity in mathematics.

The adoption of this holistic perspective requires of us to look for an understanding of the research questions at three levels: the individual level; the micro level of classroom and workshop interaction; and the level of the social and cultural contexts.

As I have already indicated, emotional responses show their qualitative character if they are placed in the social context which originates them. Consequently the analysis of emotion has not been limited to simple laboratory scenarios (specific phases of problem-solving, mistakes, etc.) but I include more complex scenarios¹ which regard the students in their social context and take account of students' self-concept as learners of mathematics, and the values and feelings that the students have as members of a social group.

2.- Instruments for identifying (diagnosing) the cognition-affect interaction

The investigation is qualitative in character, combining methods proper to ethnography and case-studies. (Clearly, a form of research involving a multi-methodological dimension cannot easily be described in detail in the limited space at our disposal in a paper. Here the essentials are presented and we direct the interested reader to consult Gomez-Chacon, 1997 and 2000).

The aims of the study were:

- To establish and describe significant relationships between cognition and affect (local and global affect, two constructs defined in the study);
- To trace the origin of the affective responses and identify the undergone evolutions in the subjects (differences, changes, etc.) after their participation in a programme of active learning which integrates the affective dimension (meta-affect);
- To analyse if it is possible to interpret the emotional responses of the young from a perspective of social identity and of cultural identity;
- To promote school instruction (teaching-and-learning) as a *process of socialization and counter-socialization* in the students beliefs, and the need to revise these methods of instruction.

Description of the study:

The study in Belgium was done with Portuguese students living in Brussels (displaced or emigrants) (Figueiral and Gómez-Chacón, 2003) of grades 7th to 12th, between Feb. and June 2003. During a five-month period, forty pupils of grades 10th, 11th and 12th were closely monitored². Two of the groups were followed in their classrooms, in the Belgium schools, and three groups in classes of Portuguese

¹ The terms *complex scenarios* are used here with the sociological meaning of *scenario*. Therefore to speak of scenario is to speak of what makes a scene to be organised and how it is organised. It is, particularly, to speak of what is being acted in a concrete space and time, with specific resources. Whenever this scenario is acted in similar circumstances, the persons participating will behave, more or less, in the same way because their individual and social learning predisposes them to act so. For readers interested in this aspect see Gómez Chacón (2002). Here we have exemplified some of the various *scenarios* in which students behaviours originate and are recognised by their teachers and by a number of other persons. By means of this typology we have aimed at showing the influence of various processes, (cognitive, meta-cognitive, social, cultural), making explicit the causes and consequences of emotional interaction in learning.

² It is relevant to note that 64% of the students in the group researched were born in Portugal, and more than half of these, 56%, arrived in Belgium when they were over 5 years old.

language and culture (each group followed once a week). The researcher was as an action researcher who collaborated as teacher with their official teacher. The students were integrated in multicultural classes. There were students from Portugal, Belgium, Cameroon, Turkey, Albania, Italy, Morocco and children of mix-marriages, Belgium-Italian, Turkish- Albanian, Italian- American and so on. The common characteristic of the groups was that the largest number were Portuguese.

In the study we³ explored and described the possible “tensions, conflicts and resistances” that could occur in learning mathematics in multicultural contexts, and that could be related to the stance the students take in relation to how their social and cultural group is represented.

The data was collected through field notes, interviews, questionnaires and other biographical and family data of aspects related to their experiences and learning situations as a “displaced group”. The research sought to find out their experience in the school in Belgium, and their academic development in both countries, how competent they were in the languages (French / Portuguese), their identity and their expectations for the future. Data was also gathered on their affects, --attitudes, beliefs and emotions-- in relation to Mathematics.

Summary of data collection tools:

1. Interviews about situations. A tool to collect data about the belief systems and values associated with mathematical knowledge at school level and about applied knowledge, in an educationally disadvantaged context (level 1 and 2, global emotion, complex scenario).
2. Mood map of the problems. A tool to diagnose emotional and self-regulatory responses (level 2 and 3, local emotion, simple scenario)
3. Questionnaire on beliefs about mathematics and learning mathematics (levels 2 and 3, local emotion, simple and complex scenarios)ç
4. Field notes. Field notes written by the (author as) researcher, either during the class or afterwards. This exercise completed the transcription made in classroom session. The researcher took the role of teacher by developing learning modules, designed for classroom practice. Also the researcher registered the notes to describe the conduct of students and teacher outside the classroom (level 1, 2 and 3, local and global emotion, and simple and complex scenarios).

3.- Analysis and results

The different categories of beliefs about mathematics learning and problem solving is not only determined by the mental context of the student and the classroom. Other influential factors are, the way classes are presented, the activities in which the student takes part, the family culture, parents beliefs about mathematics, the social

³ I use “we” because several researchers are collaborating in this project.

ideas about mathematics, etc. The stimulation to learning, beliefs, interpretations, instant emotions, and the construction of meanings they all take place in this interface experience (Volet, 2001).

All these aspects are not always easy to analyse or interpret. We found difficulty in analysing and interpreting the intersubjective perceptions when there was a cultural displacement, as was the case with the study done in Belgium.

The results of the investigation on the emotional dimension of the students regarding mathematics showed that of all the levels mentioned above the one most clearly visible was the level of appraisal of the experience which includes the immediate classroom context.

The classes of mathematics, in spite of the different and diverse styles of teachers, were all based on photocopied cards with a summary of the topic and mechanical and routine exercises to do or solve by the students. Therefore, the same methods were repeated: to do the card work, to correct it by the students or by the teacher on the blackboard. The method followed conditioned their emotional reaction: routine exercises without emotion.

The interviews with the teachers showed that, among the teachers themselves, there was very little acknowledgement of the process of change and development of a cultural identity that these students were experiencing as a result of their moving between European areas with different cultures. The adaptation of these students to school is seen mainly in cognitive and linguistic terms, no other dimensions were considered, nor were there any questions or informed debate as to the role the schools may play in forming their cultural identities.

To the question: *Have you observed any difference in the behaviour of X and in the rapport of his/her family with the school when you compare him/her to students of other countries?* The answers were: *No. I don't believe that the differences are traceable to the country of origin.*

And in relation to the question, *Have you had contacts with the parents of X?* The answers indicated that belonging to a particular cultural group was not the cause of their behaviour: “yes”, “no”, “*the same as for other students*”. However, it was considered obvious and normal, in Belgium, that these students will be one year behind because of their language, but they do not recognise that it is in fact a difficulty⁴.

In relation to the teaching and learning of mathematics and its relationship to the students' cultural and social identity we observed that:

- None of the activities of solving problems or exercises made any reference to the history of mathematics nor to mathematics as a constructed and cultural body of knowledge.

⁴ This scenario is not unique to the schools that received Portuguese immigrants in Belgium (Cline Abreu, Gray, Lambert and Neale, 2002).

- There were no activities for investigation and solution of problems.
- There were no applications or didactic or dynamic resources.

This type of activities would have facilitated their integration and would have allowed them to tackle mathematics from a more cultural perspective and as part of social and cultural identity.

The students outlined as difficulties some of the elements of what we have called “interface” experience. Such experience has to do with the climate generated in the classroom, with those invisible elements that give quality and stimulate learning and that, from our point of view, are linked to a characteristic of Portuguese culture and their way of interacting.

They missed a warmer kind of communication on the part of the Belgians. The students valued⁵ positively the multicultural society in Belgium, because of the co-existence of many displaced groups from different countries. They considered negative the intrinsic double linguistic culture of Belgium. We also observed a tendency to attribute positive aspects to non-Belgians and negative ones to Belgians : “they are cold”. However, it is interesting to point out how these students saw themselves. Although they considered themselves Portuguese their place of belonging was not so clear and at times this realisation became painful.

In describing the most difficult part of their experiences a student wrote: “to have your arse between two stools” and another expressed it with no room for doubt: “having the label of foreigner in Belgium and in Portugal”.

In view of all these data we claim that teaching-learning is fundamentally a communication process based on social interaction.

For the learner to take part in this process a double task is called for: The first is centred in the conceptual content to be learned, and the other in the form of interaction-communication in which the content is inserted.

When we compared these data with other studies done in Spain, a new example emerges of what we called *complex scenario and expression of the global affect of the person*. In our investigation, the data specifically showed that the reaction to mathematics in the classroom of the first group of students was manifested basically in scenarios of re-adjustment, self-righteousness, demand for interdependence and answers to messages or distinctions (resistances). Here, in the investigation with Portuguese students in Belgium, it appears again the demand for interdependence – obviously with different nuances-- and a new scenario that may be called *scenario of interaction-communication* that refers to the characteristics of the interchanges between student and teacher.

⁵ Questions such as the following were asked in the interview: What do you like best of Belgium society? What do you like least? What do you value most of what you consider to belong to your Portuguese culture? What do you appreciate least of your Portuguese culture? In relation to your experience of coexistence with different cultural realities: what do you personally consider most positive?

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In the data collected about this scenario of interaction-communication, we noticed that interaction-communication between student and teacher was not frequent and did not go beyond questions and answers about the exercises.

In the data emerging from the students about the question of ‘what they would like in the classes of mathematics’ we gathered three types of suggestions:

- *Ones related to resources and methodology*, for example: “ I would like to be able to use a computer to see the drawings and graphs”; “to have less theory and make the classes more lively”, “to give more practical examples”
- *Ones that considered that more time given to explaining the concepts would help the understanding*: “more time to get the sense of it, that the teachers give more time to these subjects”, “there should be the minimum number of students possible, so that the teacher could explain better to each one”, “to have the classes in the afternoon to have more time for mathematics”, “the teachers know a lot, but they should have a lighter programme to give them more time to explain it to us”,
- *Ones related to the mathematics motivation that the students think that the teachers should foster in the classroom*: “I would like to change the teacher because mine explains things very badly”, “I think that the teachers should explain things for all the pupils and not only for the good ones”, “the teacher should explain things better for all and not think that it is simple and that all should follow his ideas”, “ that they don’t start from the premise that is all logical”, “I would advise that the teachers stimulate love for mathematics because often they are the cause of us not liking it”, “a teacher should be near the students and be patient”, “ I believe that it is necessary to have a teacher that likes teaching to awaken the interest of the students”, “ I believe that they explain things well but they are too serious”.

In relation to this scenario of interaction-communication we recorded the following dialogue in an interview of group debate about particular “cases”. It shows clearly the deficit in learning on the part of the students.

- Researcher: “and the classes of mathematics?”
- Diana: “ The teacher is a pain. All teachers are a pain but this one is the worst”
- Researcher: “The class is a bit unruly and you, although not indisciplined, don’t do much work. You begin work late, don’t have the materials... calculators...”
- Bruno: “today I have everything, but we are tired”
- Diana: “With her (the teacher) you cannot rest. She makes us work too much. Always go, go; that is why, she sometimes gets on our nerves”
- Bruno: “ we lack motivation”
- Diana: It is not the study, it is understanding and if I miss a class it becomes much more difficult”
- Researcher: “You can take advantage of the extra recuperation classes”

- They react with laughter: “ that’s just what we need. With all the hours of mathematics that we have now one more...” (Extract from an Interview of a Group Debate School Ma Ch.) (there is a contradiction between what they say here and what they asked for before, that more time and less pupils would help their understanding).

The same *protocol of Interviews about situations* was used in Belgium and in Spain (Gómez –Chacón, 2000 or 1998). The objective was to compare the beliefs of both populations (beliefs and social context).

As already indicated we constructed an in-depth semi-structured interview on situations. The situations were presented through photographs. For each photograph a number of questions were prepared in relation (a) to the use and success of school mathematics, (b) to the beliefs about mathematics as an object of knowledge. The questions directed the interviewee to describe what he/she saw and to explain –from his own point of view—what he/she believed about the manner in which the action was taking place.

As the interview progressed the beliefs and responses concerning mathematics were ascertained. They spontaneously emerged in the conversation with the student, or they were provoked by the interviewer’s deliberate intervention at a given moment, thus manifesting the local and global affect of the subject.

The protocol of this interview had questions which could not be answered except as responses about the true beliefs the subject had. For example, the students were asked, when the photographs were shown, to suggest possible scenarios while thinking about their friends, or to give advice relevant to situations in which other young people, like themselves, are participants. The student projected himself in the choices he makes.

Ultimately what we sought to know was not merely what these young people thought and how they lived, but rather how they manifested their beliefs and emotional responses concerning their educational and cultural world.

In the results gathered from the interviews concerning beliefs about mathematics we ascertained a clear tendency to identify mathematics with three large areas: school mathematics; mathematics as a body of knowledge needed for some professions (mainly the more intellectual type; for the others only some “basic mathematics” were needed), and mathematics as a means to develop certain types of reasoning (logic and strategy). Some of the answers present a diffuse idea of mathematics as something that is present everywhere and explains the world in a complicated manner.

Levels of inclination and tendencies were more difficult to ascertain in this study due to the individual personal expressions of the students in mathematics classes. However, the global data collected, clearly manifested that the adaptation of

Portuguese students to Belgium made a significant impact in their sense of self and demanded from them to reconsider their cultural identity.

The four main characteristics in the way they related their new identity were:

- Commitment to family customs and the historical patrimony of their country.
- A reinforced feeling of wanting to perfect cultural instruments associated to their Portuguese heritage (for example: to become fluent in Portuguese) and to speak French very well to be well integrated.
- A reinforced feeling (because Belgium is itself a multicultural society) of wanting to co-exist with many displaced groups, although they denounce the tensions generated by biculturalism and by their own bicultural situation.
- A feeble feeling when the differences associated to their Portuguese origin were not accepted / recognised or taken as lack of competence.

Conclusion

In summary this study illustrates the significance of the multidimensional and dynamic aspects of learning and identity in context. The studies done in different contexts, and in the interface experience, have shown that what constitutes the adaptative and appropriation elements of learning, is not only a subjective perception, but it is also objectively situated in communities of practices.

This is of extreme importance for educational practice in intercultural contexts. The contexts should not be considered as static but rather as dynamic units.

There is a tendency --identified in our investigation--to conceptualise the cultural contexts and the culture of origin as something fixed, even in those high schools that mention the intercultural dimension in their school plan. They manifest a static and respectful vision towards the other but without interaction or estimation of the value of what is proper of each culture.

From our own perspective, we claim that in order to build a good framework and to interpret the emotional dimension of the person in their context, it is necessary to conceptualise the macro-micro relations and the role that communication and face-to-face encounters play in the building of these relations. This is a *complex matter*, and many authors have written extensively about it. They denounce that face-to-face meetings are for many researchers, an element (and a minor one at that) among many others in society. However, for the micro-ethnograph authors, these encounters are the central constitutive element of the social world.

These issues raise new questions for investigation: Can the identity, cognition, and emotion be conceptualised and *treated empirically* at three or more levels of specification? What complex emotional scenarios can be typified in multicultural contexts? What beliefs are stable and which ones are linked to context? What are the differences inter and intra individuals in the different levels of the context?

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THE STRUCTURE OF STUDENT TEACHERS' VIEW OF MATHEMATICS AT THE BEGINNING OF THEIR STUDIES

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Abstract: *The elementary school teachers' view of mathematics is important because it will influence the way they will teach mathematics. In the present article we explore the structure of student teachers view of mathematics. Based on a survey study in three universities we found eight factors for the view. In the core of this view there are belief of one's own talent, belief of the difficulty of mathematics and one's liking of mathematics. Gender, grade, and mathematics course selection at high school each contribute to the variation in these factors.*

Introduction

Elementary school teachers have a profound significance in regard to competences in and beliefs about mathematics, because they are the first ones to teach mathematics to children. Furthermore, they teach mathematics during the age when children's beliefs are still at their formative stages (Hannula, Maijala & Pehkonen, 2004).

In Finland, roughly quarter of elementary school teacher students have negative view of mathematics at the beginning of their studies (Pietilä, 2002; Kaasila, 2000), and similar problems have been identified in other countries as well (e.g. Uusimäki & Kidman, 2004; Liljedahl, 2004). Such negative view can seriously interfere their becoming good mathematics teachers, unless they can either overcome their anxiety or find constructive coping strategies. There are a disturbing number of elementary teachers on the field who have not been able to do so. However, efforts to promote positive view of mathematics during teacher education have often been successful (e.g. Kaasila, 2000; Liljedahl, 2004; Pietilä, 2002; Uusimäki, 2004).

In our research project "Elementary teachers' mathematics", our aim is to explore the structure of teacher students' view of mathematics and how it develops in three universities that use different approaches to promote positive affect among students. This report will focus on revealing the structure at the beginning of studies.

Theory

The three basic elements of the human mind are emotion, cognition and motivation. Hence, we need to pay attention to the students' feelings, beliefs, and wants as the elements of their view of mathematics. The feeling aspect of the view of mathematics consists of the emotions one experiences while doing mathematics. However, in any survey study it is only possible to find about expectancies or memories of these emotions. (Hannula, 2004).

For the term “belief”, there is no single, exact definition. Furinghetti & Pehkonen (2002) have tried to clarify the problems of the concept belief, and they conclude that it seems to be impossible to find a universally accepted characterization for beliefs. Here, we define beliefs as purely cognitive statements to which the holder attributes truth or applicability. According to this view, beliefs do not include an emotional component, although a belief can be associated with an emotion. (Hannula, 2004)

The aspect of motivation, or wanting, relates to the goals and desires one has. These wants are based on human needs that include need for competence and autonomy. We can distinguish between intrinsic (the act is rewarding in itself) and extrinsic (there is an external reward) motivation or between mastery (“I want to understand”), performance (“I want good grades”), and ego-defence (“I don’t want to be humiliated”) as motivational orientations (Hannula 2004).

View of mathematics has a structure. We can distinguish between one’s view of different objects, such as 1) mathematics education (mathematics as subject, mathematical learning and problem solving, mathematics teaching in general), 2) self (self-efficacy, control, task-value, goal-orientation), and 3) the social context (social and socio-mathematical norms in the class,) (Op’t Eynde, De Corte & Verschaffel 2002). With regard to the social context, Op ‘t Eynde & DeCorte (2004) found later that the role and functioning of one’s teacher are an important subcategory of it. The spectrum of an individual’s view of mathematics is very wide, and they are usually grouped into clusters that influence each other. Some views depend on other ones, for the individual more important views. When discussing beliefs Green (1971) uses the term ‘the quasi-logical structure of beliefs’ which means that the individual himself defines the ordering rules. We assume that emotions, cognitions and motivations form a system that has a quasi-logical structure. The view of mathematics also has a hard core that contains the student’s most fundamental views (cf. Green 1971: the psychological centrality of beliefs; Kaplan 1991: deep and surface beliefs). Only experiences that penetrate to the hard core can change the view of mathematics in an essential way (Pietilä, 2002).

Methods

The research draws on data collected on 269 trainee teachers at three Finnish universities (Helsinki, Turku, Rovaniemi). Two questionnaires were planned to measure students’ competences in and view of mathematics in the beginning of their studies.

The ‘view of mathematics’ indicator consisted mainly on items that were generated in a qualitative study on student teacher’s view of mathematics (Pietilä, 2002) it also included a self-confidence scale containing 10 items (B1-B10) from the Fennema-Sherman mathematics attitude scale (Fennema & Sherman 1976), four items from a ‘success orientation’ scale found in a study with pupils of comprehensive school (Nurmi, Hannula, Maijala & Pehkonen, 2003) and some background information

about earlier success in mathematics and experience as a teacher. The statements were structured around the following five topics:

- Experiences as mathematics learner (A1-A29),
- Image of oneself as a mathematics learner (B1-B16),
- View of mathematics, learning of mathematics, and teaching of mathematics (C1-C12),
- View of oneself as mathematics teacher (D1-D6), and
- Experiences as teacher of mathematics (E1-E7).

The mathematical skills test contained altogether 12 mathematical tasks related to elementary level mathematics. Four tasks measured understanding of some key concepts and eight tasks measured calculation skills. The questionnaires were administered within the first lecture in mathematics education studies in all universities in autumn 2003. Students had altogether 60 minutes time for the tests and they were not allowed to use calculators.

For a principal component analyses we chose the 63 items on topics A to D. We excluded the topic on experiences as a teacher because almost half of the respondents did not answer these questions. We chose to use Maximum likelihood method with direct oblim rotation. In the first stage we extracted factors with eigenvalues greater than 1. The items that had communalities below 0.30 were removed and then factor analysis was repeated. When an acceptable result was achieved, principal components were constructed of the items that had the highest loadings in respective factors. This solution was compared with other solutions suggested by scree-test. The criteria for comparing the solutions were the ratio chi squared/degrees of freedom (Nummenmaa, Konttinen, Kuusinen & Leskinen, 1997), Cronbach alphas of the constructed principal components and the sense each component made.

When a solution was chosen, the structure of the principal components was analyzed. We calculated the correlations between components and also the effects of main background variables: gender, mathematics course at high school, and grade.

Results

Principal component analysis

According to the scree test, solutions of 5 to 10 components were deemed reasonable. In the first extractions with the eigenvalue-greater-than-one criterion there were items with low communality. When 12 items had been removed, all items had good communality and a solution of nine components was found. We compared this solution with other solutions where the number of components varied from 5 to 10. The main difference between the different solutions was in the distinction between different aspects of self-confidence. As the values for Cronbach alphas were highest for the ten-component solution and the extracted factors also made sense theoretically we chose it for further analysis (Table 1). The ratio chi squared/degrees of freedom is regarded good, when it is less than 2.5 (Nummenmaa & al., 1997). In all models the

ratio was good, and it got better together with the number of factors to be extracted. In the ten-factor model it was 1.31. As two of the constructed components included only two items each we did not take these to further analyses. However, it should be noted that the goodness-of-fit for all models were poor. This indicates that the principal component solutions tap only a small fraction of the variation between the respondents' answers.

Two of the principal components relate primarily to the student teachers' past experiences (*My family encouraged me, I had a poor teacher*), three to the personal beliefs (*I am not talented in mathematics, I am hard-working and conscientious, and Mathematics is difficult*), one to emotions (*I like mathematics*), and two to the person's expectations about future success (*I am insecure as a mathematics teacher, I can do well in mathematics*).

Table 1. The ten-component solution of teacher students' view of mathematics.

Principal components and statements	Loading
F1 I am not talented in mathematics (alpha = .91)	
B9 I am sure I could do advanced work in math.	-.573
B8 I'm no good in math	.554
B6 I'm not the type to do well in math	.522
B7 I am sure of myself when I do math	-.460
B4 Math is hard for me	.438
B3 Math has been my worst subject	.431
A15 I made it well in mathematics	-.418
A9 Being compared with others made me anxious	.326
F2 I am hard-working and conscientious (alpha = .81)	
B13 I am hard-working by nature	.812
B12 I have not worked hard enough	-.761
B15 I always prepare myself carefully for exams	.690
A4 I worked hard to learn mathematics	.588
B11 My attitude is wrong	-.477
F3 My family encouraged me (alpha=.83)	
A23 My family encouraged me to study mathematics	.870
A17 The importance of competence in mathematics was emphasized at my home	.839
A18 The example of my parent(s) had a positive influence on my motivation	.678
F4 I had a poor teacher in mathematics (alpha = .84)	
A21 My teacher did not inspire to study mathematics	.754
A26 My teacher was a positive example	-.727
A27 I would have needed a better teacher	.697
A3 The teacher could not explain the things we were studying	.636
A2 The teacher created appropriately challenging learning situations	-.598
A6 The teacher did not explain what for we needed the things we	.530

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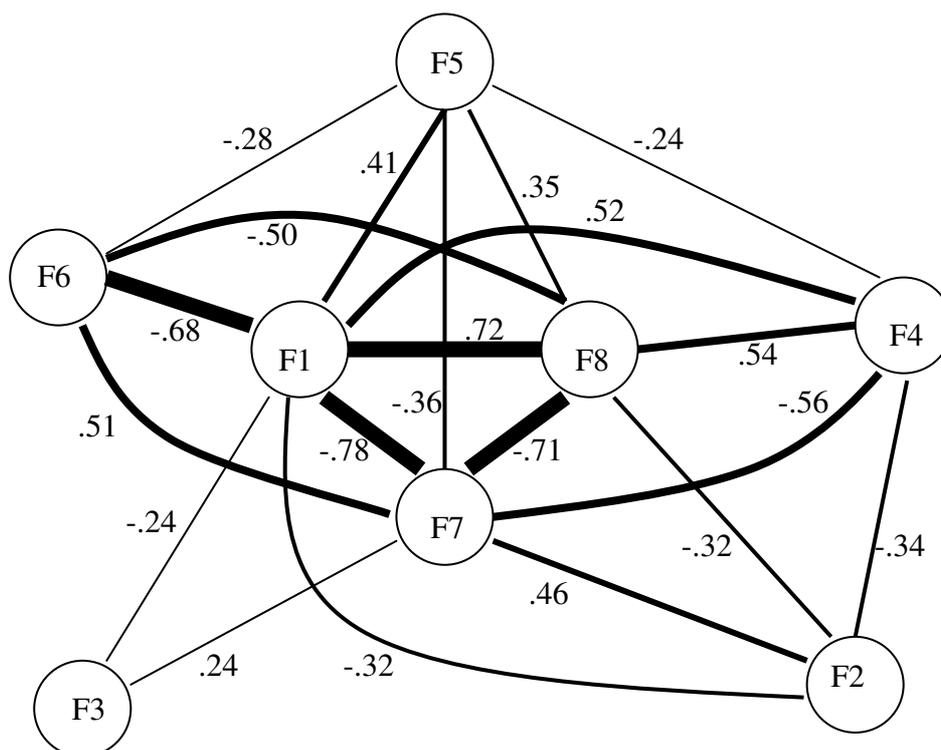
were learning	
A20 During the mathematics lessons we did only tasks from our mathematics book	.490
A28 During lessons I usually did not want to ask for advice, in order not to be labeled stupid by classmates	.264
F5 I am insecure as a mathematics teacher (alpha = .74)	
D4 I am insecure (as a mathematics teacher)	.767
D2 I am inexperienced in teaching mathematics	.690
D1 I am not able to give pupils clear enough explanations	.657
D5 My level of competence in mathematics causes me problems	.419
F6 I can do well in mathematics (alpha = .80)	
B1 I am sure that I can learn math	.916
B2 I can get good grades in math	.711
B10 I know I can do well in math	.516
B5 I think I could handle more difficult math	.354
F7 I like mathematics (alpha = .91)	
A8 Doing exercises was pleasant	.728
A7 It was boring to study mathematics	-.653
C1 Mathematics is a mechanical and boring subject	-.590
A12 Mathematics was my favorite subject	.524
A13 To study mathematics was something of a chore	-.483
A22 Mathematics was the most unpleasant part of studying	-.468
A25 I enjoyed pondering mathematical exercises	.420
D6 My attitude towards mathematics helps me as a teacher	.301
F8 Mathematics is difficult (alpha = .78)	
A5 I did not understand teacher's explanations	.556
A11 Mathematics was difficult in high school	.412
A24 The teacher hurried ahead	.380
C4 Mathematics is difficult	.369
C2 Learning mathematics requires a lot of effort	.364
A19 Mathematics was a clear and precise subject to study	-.207
F9 Mathematics is calculations (alpha NA)	
C5 Mathematics is numbers and calculations	.634
C6 One learns mathematics through doing exercises	.526
F10 I am motivated (alpha NA)	
B16 It is important to me to get a good grade in mathematics	.574
B14 For me the most important thing in learning mathematics is to understand	.308

Structure of student teachers' view of mathematics

When we look at the correlations between the constructed principal components (Figure 1), we see that three of the components are closely related and form a core of

the person's view of mathematics. This core consists of three aspects of person's general view of mathematics. The first aspect (*I am not talented in mathematics*) focuses on beliefs about self, the second aspect (*Mathematics is difficult*) on beliefs about mathematics, and the third aspect (*I like mathematics*) on the person's emotional relationship with mathematics. Around this core there are five factors, each of which relate primarily to the core and some secondarily also to each other. The encouraging family (F 3) had only a minor effect on the core view, whereas experiences of poor teaching (F 4) related more closely to the core view and also to diligence (F 2) and insecurity as a teacher (F 5). The core had also a strong connection with personal expectations to do well (F 6), which probably differs from factor 1 in that the element of effort has a greater role in it.

Figure 1. Structure of student teacher's view of mathematics. The connection weights are Pearson correlations. The components are: *I am not talented in mathematics* (F1), *I am hard-working and conscientious* (F2), *My family encouraged me* (F3), *I had a poor teacher* (F4), *I am insecure as a mathematics teacher* (F5), *I can do well in mathematics* (F6), *I like mathematics* (F7) and *Mathematics is difficult* (F8).



Effects of gender, course selection and grade

Most means of the responses were rather near to the center of the scale. Hence, the student teachers' views in general were neither positive nor negative. We see significant gender differences in most of the variables examined. The largest difference was in that female students felt that they were more hard-working and diligent (F2). Male students had higher self-confidence regarding their talent in mathematics (F1). However, there was no gender difference in students' liking of mathematics (F7) or perceiving mathematics as difficult (F8). Female students had

more critical image of their mathematics teachers (F4). According to a regression analyses, gender accounted for 20 % of the variation in the view of mathematics.

As assumed, course selection had affected students' view of mathematics (Table 3). Those who had studied the more advanced mathematics course in high school had significantly higher self-confidence regarding their talent (F1). They also liked mathematics more (F7), but both groups perceived the subject equally difficult (F8). Those who had studied the more advanced track were less critical about their teachers (F4) and they had also received less encouragement from their families (F3). Surprisingly, the track had no effect on view of oneself as hard-working (F2), although the more advanced course is generally regarded to require a lot more work.

Table 2. Gender differences in student teachers' views of mathematics.

	F1	F2	F3	F4	F5	F6	F7	F8
Mean (3 is middle)	2.6	3.1	3.0	2.6	3.1	3.9	3.4	3.0
Standard deviation	0.9	0.9	1.0	0.7	0.8	0.7	0.9	0.7
Mean, male (N=61)	2.3	2.7	2.8	2.4	3.0	4.1	3.3	2.9
Mean, female (N=206)	2.7	3.2	3.1	2.6	3.2	3.8	3.4	3.0
Equal variance assumed	No	No	Yes	Yes	No	No	Yes	Yes
t	-2.7	-4.5	-1.9	-2.8	-1.5	2.3	-0.5	-1.1
Sig.	0.008	0.000	0.059	0.006	0.131	0.024	0.642	0.274

Table 3. Differences in view of mathematics between students who took advanced or less advanced mathematics in high school.

	F1	F2	F3	F4	F5	F6	F7	F8
Mean	2.6	3.1	3.0	2.6	3.1	3.9	3.4	3.0
Standard deviation	0.9	0.9	1.0	0.7	0.8	0.7	0.9	0.7
Mean less advanced (N=174)	2.7	3.1	2.9	2.7	3.2	3.8	3.3	3.0
Mean advanced (N=93)	2.3	3.0	3.3	2.4	3.0	4.0	3.6	3.0
Equal variance assumed	No	Yes	No	No	No	No	No	No
t	3.4	0.8	-3.5	2.8	1.7	-2.2	-2.9	0.0
Sig.	0.001	0.398	0.001	0.006	0.097	0.028	0.004	0.982

According to a regression analyses, course selection accounted for 15 % of the variation in the view of mathematics. As there is usually a clear gender difference in course selection, we checked also for interaction effect between gender and course, but found this to be non-significant.

Those who had made it well in mathematics held more positive views about themselves and mathematics (Table 4). Grade had a significant correlation with view of oneself as talented (F1), as hard-working (F2), liking of mathematics (F7), and view of mathematics as a difficult subject (F8).

All the above effects of grade were more pronounced among female subjects and those who have studied the more advanced mathematics course in high school. In fact, none of the correlations were significant among male subjects. However, among those 26 male students who had studied the more advanced mathematics, there was a significant correlation (0.520, sig. = 0.006) between grade and trust in chances to do well (F6). According to a regression analyses, course grade accounted for 12 % of the variation in the view of mathematics.

Table 4. Correlations between student teachers' grades and their views of mathematics. Statistically significant correlations ($p < 0.01$) in bold.

	F1	F2	F3	F4	F5	F6	F7	F8
Correlation	-0.274	0.211	0.038	-0.141	-0.068	0.166	0.253	-0.268
Sig. (2-tailed)	0.000	0.001	0.539	0.021	0.271	0.029	0.000	0.000
Correlation, male	0.109	0.117	0.035	0.107	-0.121	0.059	0.086	-0.042
Sig. (2-tailed)	0.413	0.369	0.787	0.415	0.354	0.651	0.509	0.757
Correlation, female	-0.375	0.228	0.030	-0.220	-0.060	0.216	0.296	-0.319
Sig. (2-tailed)	0.000	0.001	0.664	0.002	0.390	0.002	0.000	0.000
Correlation, less advanced course	-0.219	0.177	0.030	-0.102	-0.038	0.113	0.186	-0.227
Sig. (2-tailed)	0.004	0.019	0.692	0.184	0.621	0.137	0.015	0.003
Correlation, advanced course	-0.495	0.449	0.052	-0.259	-0.153	0.376	0.517	-0.483
Sig. (2-tailed)	0.000	0.001	0.626	0.012	0.146	0.000	0.000	0.000

Conclusions

In the principal component analyses we found evidence for the distinction between emotion, cognition, and motivation. We got one emotional component ("I like mathematics") that was separate from the cognitive ones. There was also one component for motivation ("I am motivated"), but because it included only two items we did not analyse it further. In the future development of the instrument we need to add items that relate to this aspect of view of mathematics.

We also found support to the approach, where the view of mathematics is separated into different domains. We found separate components in all main categories and in most of the subcategories suggested by Op 't Eynde & DeCorte (2002):

- Beliefs about mathematics education
 - mathematics as subject ("Mathematics is calculations", only two items),
 - mathematical learning and problem solving ("Mathematics is difficult"),
 - mathematics teaching in general,
- Beliefs about self
 - self-efficacy ("I am not talented in mathematics", "I can do well in mathematics")

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- control (“I am hard-working and conscientious”)
- task-value,
- goal-orientation (“I am motivated”, only two items),
- Beliefs about the social context
 - the role and functioning of teacher (“I had a poor teacher”)
 - social norms (“My family encouraged me”)
 - socio-mathematical norms

We also found a separate component for one’s view about oneself as a teacher of mathematics (“I am insecure as a mathematics teacher”).

Regarding the structure of view of mathematics, we found it to have a core that consists of three closely related elements. A person with a positive view believes oneself to be talented in mathematics, believes mathematics to be easy, and likes mathematics. The person with a positive view is usually also confident on being able to do well in mathematics, hard-working, and satisfied with the teaching he or she had in mathematics. Such a student is more confident in being a good teacher than one with a negative view of mathematics.

The three background variables: gender, course selection and grade are related to many of the variables, explaining a fair amount of the variation. Female students perceive themselves to be more hard-working and diligent than male students. Interestingly, while we found gender differences in self-confidence, we did not find those in liking mathematics or perceiving mathematics difficult. Hence, it seems worthwhile to separate the different aspects of the core of the view of mathematics although their correlation with each other is high. Regarding the student teachers’ confidence as a teacher of mathematics, the background variables were statistically almost significant ($p < 0.03$). Hence, in future analyses there is good reason to pay attention to the effects of gender, grade, and course selection as variables that may have an effect on confidence as a teacher of mathematics.

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AUTOBIOGRAPHICAL NARRATIVES, IDENTITY AND VIEW OF MATHEMATICS

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Abstract: *The paper presents some results of a research project funded by the Academy of Finland on elementary teacher students' views of mathematics at three Finnish universities. Here, we focus on seven teacher students: they had advanced studies in mathematics in upper secondary school but their views on mathematics varied in considerable way at the beginning of teacher education. We divided the cases into three groups: Success stories, Victory through Hardship stories and Regression stories. Students' different views of mathematics may be explained by following factors: their socio-emotional orientations, coping strategies and the socio-mathematical norms of the upper secondary school class or at home.*

Mathematics experiences are of central importance in the formation and development of primary teacher students' views of mathematics. School, home, friends, myths about mathematics and temporary posts as teachers have shaped their views of mathematics before came into the teacher education. (Kaasila 2000; Pietilä 2002) About one-third of the teacher students have completed the advanced mathematics curriculum in upper secondary school. Yet it's difficulty can threaten their views on themselves as learners of mathematics (Kaasila 2000). The focus of this article is on seven elementary teacher students' views of mathematics. Because of positive experiences from elementary and secondary school they had chosen advanced studies in mathematics in upper secondary school, but their views of mathematics varied in considerable way at the beginning of teacher education. We try to find out why.

Autobiographical narratives

There are two broad ways in which people organize and manage their knowledge of the world: logical-scientific and narrative mode of thought: the first seems appropriate for treating physical 'things', the second for treating people's plights (Bruner 1986). According to Ricoeur (1992), people often develop their sense of identity by seeing themselves as protagonists in different stories. Narrating is much more than describing events or actions. It also means relating events and actions, organizing them into sequences or plots and then attaching them to a character. What creates the identity of the character is the identity of the story.

Mathematical autobiography includes the students' personal experiences in learning mathematics and ways they managed them, important persons and explanations (Kaasila 2000). Explanations are creating coherence to the story (Linde 1993).

Mathematical identity

To learn is to develop an identity through modes of participating with others in communities of practice. Identity is the who-we-are that develops in our own minds and in the minds of others as we interact. It includes our knowledge and experiences, and also our perceptions of ourselves (e.g. beliefs, values, desires and motivations), others' perceptions of us and our perceptions of others. (Wenger 1998)

Mathematical identity is a construct that describes the relationship of a person with mathematics (Bikner-Ahsbals 2003). Students' learning in the mathematics education community (for example in school class) is characterised by an actualisation of their identity through their interactions with the teacher, the books, the peers they engage with. These interactions are determined by the social context they are situated in, but on the other hand, students bring with them the experiences of numerous other practices in other communities they have participated. Students' self, their identity, is only partially transparent to them. Who they are, what they value in this context, emerges in the situation. (Op't Eynde 2004)

View of mathematics

We see that a person's view of mathematics develops through experiences connected with mathematics in the interaction of affective, cognitive and conative factors. On one hand, emotions, beliefs, conceptions and attitudes operate in the formation of view of mathematics as a regulating mechanism. Learning requires also cognitive actions, (e.g. understanding, recognizing, estimating and reasoning), and a conscious strive to act and aim at something. (Op't Eynde, De Corte & Verschaffel 2002) On the other hand, when a student's view of mathematics has developed through experiences, it will influence his understanding, solutions, affective reactions and actions in different mathematics-related learning situations (e.g. Schoenfeld 1985).

The view of mathematics is a large entity of a student's knowledge, beliefs, conceptions, attitudes and emotions. In the view of mathematics, we distinguish three components: The view on oneself as a learner and teacher of mathematics, the view on mathematics and its teaching and learning (Pietilä & Pehkonen 2003), and the view of the social context of learning and teaching mathematics. (Op't Eynde et al. 2002) Self-confidence pertains to the first component. The second component contains how instruction should be organized. The third component can be analysed for example by considering socio-mathematical norms of a school class. The norm refers to interpretations that become taken-as-shared by the community (for example a school class), and normative aspects of interactions that are specific to mathematics are called socio-mathematical norms (Yackel & Cobb 1996). An example of a socio-mathematical norm is that pupils don't laugh at mistakes made by others in a math class. The view of mathematics consists of a hard core, which contains the student's most fundamental views (cf. Green 1971: the psychological centrality of beliefs). Mathematics experiences need to penetrate to the hard core in order to change the view of mathematics in an essential way (Pietilä 2002).

Socio-emotional orientations

Emotions, goals and motivation are an important part of person's view of mathematics. Emotions have a central role in interpersonal communication. We see people around us behaving and expressing, and we learn to interpret those actions as emotions and thoughts. (Hannula 2004)

In typical (mathematical) learning situations the student is expected to cope with complex social and emotional challenges. The model of learning orientations describes how motivational and emotional dispositions develop interactively in learning situations. Socio-emotional orientations (or types of coping) can be classified into three categories: task-orientation, socially dependent orientation and ego-defensive orientation. Task orientation or task-oriented coping is dominated by an intrinsically motivated tendency to approach, explore and master the challenging aspects of the environment. The student's initial cognitive appraisal of task cues and instructions consists of recognising the task as intelligible. Emotions like curiosity, interest or enthusiasm arise. In social-dependence orientation student adaptation to the learning situations is dominated by social motives, such as seeking help and affiliation from the authority. The student is not very willing to make self-directed and independent efforts, she/he easily become helpless and seeks hints and support from others. The students' expectations of success are high and are not related to self-contained task control but instead to getting teacher's help. Positive emotions are connected with expected satisfaction of the teacher, and students are not ready to proceed independently. Ego-defensive oriented student adaptation is dominated by self-defence and self-protective motives. The student will be sensitised to task difficulty cues and signs anticipating a negative response from the teacher. He or she does not concentrate intensively the task at hand, and may try to find some compensatory tactics in order not to "lose face". The student's expectations of success are low. (Lehtinen et al. 1995)

Method

Behind this paper, there is a research project "Elementary teachers' mathematics", financed by the Academy of Finland. The project draws on data collected on 269 trainee teachers at three Finnish universities (Helsinki, Turku, Rovaniemi). Two questionnaires were planned to measure students' situation in the beginning of the studies. The aim of the questionnaires was to measure students' experiences connected to mathematics, their views of mathematics and their mathematical skills.

One part of 'view of mathematics' indicator was a self-confidence scale containing 10 items from the Fennema-Sherman (1976) attitude scale. The mathematical skills test contained altogether 12 mathematical tasks. Four tasks measured mathematical understanding and eight tasks measured calculation skills. The questionnaires were administered within the first lecture in mathematics education studies in all universities in autumn 2003. Students had altogether 60 minutes time for the tests and they were not allowed to use calculators. Additional results of this project are described e.g. in Kaasila et al. (2004) and Hannula et al. (2004).

We chose 21 students and carried out interviews with them during September and October 2003. Six students presented positive views of mathematics. Their self-confidence, measured by the Fennema-Sherman attitude scale, registered with the top 15 percent and success in the mathematical skills test the top 30 percent. Eight students presented negative views of mathematics. Their self-confidence registered with the weakest 15 percent and success in the test the weakest 30 percent. Seven students presented neutral views of mathematics. Here we will answer the following research questions: What impact does elementary teacher students' experiences from their own school time have in the formation of their views of mathematics? How do elementary teacher students construct their views of mathematics by using autobiographical narratives? Our focus is on seven students who had advanced studies in mathematics in upper secondary school. Five of them had positive or mainly positive and two had negative views of mathematics.

During the interview, every student told her/his mathematical autobiography. In the narrative analysis, we as researchers attempted to recognize the parts of the data that appeared to be significant to the student's view of mathematics and mathematical identity. The plot serves to recognize the contribution certain events make to the development and outcome of the story (Polkinghorne 1995). We also paid attention to the language, including the use of narration and vocabulary. The result of this process is called the mathematical biography. It revealed how the person had constructed her/his mathematical past, present and future. (Kaasila 2000) For adding validity, we compared the data we got through the interviews with the data of questionnaires, especially the dimensions of the view of mathematics we got by using factor analysis. The phases of the factor analysis are reported in the other article (Hannula et al. 2004). Finally we compared systematically students' mathematical biographies.

Results

At the beginning of their elementary teacher studies Anna, Kati, Leo, Sini, and Mari had a positive, while Heli and Pia had a negative view of mathematics. They had very little teaching experience on mathematics, but Mari and Pia had worked as pupil mentors for approximately one year. Everyone had positive learning experiences in elementary school: They noticed that they were faster than average in mathematics, and the teacher gave them extra exercises. Kati summed up her positive mathematical identity as follows: "*Tests were a means to show what one was capable of.*" Competitiveness emerged.

Most of them had a positive view of mathematics after secondary school as well. Mari's grade in mathematics was 10 (the best possible) all through the secondary school "*although it felt like I'd done nothing.*" Heli's grade was 9, and she was "*really interested in it.*" Heli told she succeeded well in "*mechanical exercises,*" but the word problems created some difficulty. Pia had problems because "*during elementary school the ostentatious expectations changed.*" For all, the secondary school's teacher had served as a positive role model. In upper secondary school the

majority chose the advanced studies in mathematics as a self-evident procedure, also Pia as well, although her secondary school's teacher did not recommend it.

In spite of similar experiences in elementary and secondary schools, the advanced studies in mathematics in upper secondary school had a quite varying effect on their view on mathematics. According to the interviews, we divided the cases into three groups: 1) Success stories, allowing an easy explanation to a positive view on mathematics. 2) Victory through Hardship stories, leading eventually to a positive view on mathematics. However, the view is not coherent because it also includes a negative dimension. 3) Regression stories, leading to a negative view on mathematics. Anna and Kati belong to group 1, Leo, Sini, and Mari to group 2, and Heli and Pia to group 3. Information received through factoring supports this grouping as well (see Table 1).

	Anna	Kati	Leo	Sini	Mari	Heli	Pia
My family encouraged	1.67	3.00	2.67	4.00	4.00	4.33	2.00
I had a poor teacher	2.89	1.56	2.56	2.67	3.56	2.78	3.67
I like math	4.75	4.50	3.25	4.75	4.00	2.13	1.75
I am hard-working	4.60	4.40	3.20	3.80	1.40	3.60	1.20
Math is difficult	2.17	2.00	3.67	2.33	2.83	4.50	3.83
I am not talented	1.25	1.50	2.13	-	1.88	4.75	4.25
Insecure as a math teacher	2.50	4.00	3.50	3.00	1.75	4.75	3.50
I can do well in math	5.00	4.50	4.25	4.75	5.00	1.75	2.50
Marks of math on ME ¹	e	m	e	m	c	b	a
View of mathematics	+++	+++	++-	++-	++-	--	--

Table 1: The dimensions of teacher students' views of mathematics, scale is from 1 to 5, except for the two last rows.

In the table 1 two of the factors relate primarily to the student teachers' past experiences (My family encouraged me, I had a poor teacher), four to the personal beliefs (I like mathematics, I am hard-working and conscientious, Mathematics is difficult and I am not talented in mathematics), and two to the person's expectations about future success (I am insecure as a mathematics teacher, I can do well in mathematics). (see Hannula, Kaasila, Laine & Pehkonen 2004.)

¹ The scale for marks on the Finnish Matriculation Examination (ME) is, from the highest to the lowest: laudatur (l), eximia (e), magna (m), cum laude (c), lubenter (b), approbatur (a), improbatur (i).

Success stories: Anna's view of mathematics was very positive already when she started the upper secondary school. She enjoyed math as well as the fact that teaching went *'behind the formulas'*. Her teacher was inspiring as a positive role model. The home did not play a significant role: Anna told that she had been *"very autonomous ever since an early age"*. Her grade was *eximia* on the ME's mathematics exam. Anna had a positive view on herself as a mathematics teacher, and she said to have influenced her little sister to become inspired by mathematics.

Kati had (partly by accident) been placed in secondary school in a lower-level mathematics group, and she had become frustrated because the exercises were too simple. Because of the group level, Kati felt like a *'pioneer'* having chosen the advanced studies in upper secondary school, but, nevertheless, it did not hinder her success. Kati liked very much her teacher. Her grade was *magna* on the ME. She recalled that a friend she taught in secondary school succeeded well in the exam. However, the negative experiences of teaching subjects during Kati's first period of teaching practice scarred her identity as a teacher. In summary, Anna and Kati feel talented and diligent in mathematics (Table 1). They emphasized understanding of mathematics, and regarded themselves as *'insiders'* in relation to math.

Victory through Hardship stories: Sini's view of herself as a mathematics learner changed by upper secondary school: *"I think nowadays I'm a lot worse in math than I was in elementary and secondary schools. The basic stuff stays there, but the more difficult and applied things, they are gone. Sure, they could easily come back I guess."* Her view of mathematics has deteriorated but she points out that she masters the basic issues and that through practice she could further improve the situation.

Sini's experiences of the upper secondary school teachers varied. Her father (an engineer) and older siblings, who had succeeded in mathematics, played a significant role in the formation of her mathematical identity: *"I'd seen the older ones do math, so I think I learned the basic math stuff with'em."* On the ME Sini's mark was *magna*, but she was disappointed at the result, which did not *"meet her expectations"*. She explained this by emphasizing the right to fail: *"I flunked, and it was a disappointment, but it didn't really change my life afterwards any more than that. As a teacher I should now also be able to convey to the pupils that failures happen as well."* A positive view on oneself as a teacher of mathematics reflected in Sini's speech: *"I'm sure about it, whatever I do, I won't teach wrong."*

Also Leo's mathematical identity changed in upper secondary school: *"math was done late at night"* and *"it was pretty much like forcing oneself with the math exercises."* For him the contents were too abstract: *"If anything, I wanted to understand mathematics."* This supports Leo's view of mathematics as a difficult and not-so-likable subject (Table 1). The teacher was a negative role model: *"To him, everything was as clear as a day. Sometimes I felt so dumb 'cause I just don't get it."* Leo's uncle, who worked as a secondary school teacher, had a big impact on his view: *"Whenever I myself had a feeling that it (math) makes no sense, then he'd always say that it helps you to use your brain."* Yet, hard working brought results:

On the ME his mark was e. Leo tried to make his sister understand mathematics, but *“many things that were crystal clear to me, they weren’t clear to her at all.”*

Mari had experienced a shock at the beginning of upper secondary school. She did not pass a mathematics exam: *“Because I had kinda gotten used to it that (on the secondary level) everything went always just like that.”* The upper secondary school teacher was a negative role model. Mari sometimes received help from her mathematics-oriented father or her older sister. Like Sini and Leo, at the beginning of upper secondary school, Mari had to re-evaluate the view of herself as a learner of mathematics, and began to work much harder. Her mark on the ME was cum laude. Mari felt herself very secure as a teacher of mathematics (Table 1).

Regression stories: Pia’s mathematical identity changed clearly in the upper secondary school. Her teacher was a very negative role model who favored the best students. Pia said that she should have raised her hand and *“shout out loud that the last one was total gibberish to me.”* Her siblings or parents were of no assistance and Pia had a good friend who exhibited a negative role model: *“We just sat there through classes and thought how stupid this is.”* Pia evaluated herself as follows: *“I don’t get it right away, so, I kind a panic, like, this is not gonna work at all.”* It would have been useful to *“just move over to the general math studies.”* At the beginning of the interview Pia wanted to know why she was chosen for the interview, and she answered her own question right away: *“I think it was the attitude, bad attitude.”* Her mark on the ME was approbatur. Pia felt herself quite insecure as a math teacher. There were two types of discourse in her speech: on the basis of her substitute experiences she thinks teaching mathematics is fun because *“the pupils are enthusiastic.”* On the other hand, she *“cannot structure things in a clear way.”*

Also for Heli, the upper secondary school teacher was a very negative role model. Like Pia, Heli had a good friend who exhibited a negative role model: They talked as follows: *“This is so stupid and there’s no way I can understand mathematics, I never have and I never will.”* Here the negative view of mathematics is crystallized, joining the past, present and the future. Already before the tests, Heli had decided she couldn’t do the maths: *“Thinking that you don’t know something to begin with, has a pretty big effect.”* Also, a demanding sports hobby took its time: *“I just kept crying in the evenings when I tried to do math exercises and go for a jog as well.”* When needed, Heli’s big brother and father helped her with her homework. She noted about her mother and herself: *“Me and my mom are kinda like overdoing it.”* Heli thinks, *“teaching mechanical exercises in elementary school is easy, but verbal ones may create difficulties.”* Like Pia, Heli does not consider herself as talented in mathematics (Table 1). Heli’s mark on the ME was lubenter.

The factors behind the different views: The students’ different views of mathematics may be explained by various factors. The most essential one is that their socio-emotional orientations and coping strategies differ from each other: Those who belong to the first two groups seem to be task oriented: They emphasized the importance of understanding mathematics. Heli and Pia with their Regression stories

seem to have socially dependent orientation, which is manifested as the girls being dependent on their upper secondary school teacher, who, however, was a negative role model. This orientation becomes even more apparent when Heli and Pia talk about their good friends, with whom it was natural to talk about mathematics as an appalling subject.

The students of groups 2 (Sini, Mari, and Leo) seemed to tolerate uncertainty better than Heli and Pia. The former seem to have come up with functional coping strategies in order to overcome the turning point caused by upper secondary school mathematics. Sini made it by not trying to be perfect, giving herself (at a later point) permission for a 'failure' in the ME and Mari by referring to the insufficient work amount in secondary school. Leo was balancing between two discourses: "*does this make any sense anymore*" and "*mathematics teaches you to use your brain*". Sini, Mari and Leo reached good results on the ME by working hard.

As regards Sini, Mari, and Heli the socio-mathematical norms at home seem beneficial, Pia's situation being quite the opposite (see also Table 1). Sini and Mari regarded their parents and older siblings as positive role models and identified with them, but Heli seemed to identify more with her mother who was not '*mathematically oriented*'. Maybe Heli has adopted a belief according to which men are better in math than women. For Leo, the conversations with his uncle were crucial to motivation.

Based on the interviews and Table 1, Mari, Anna, and Sini considered themselves the most secure teachers of mathematics. The most insecure ones were Heli and Kati. Kati's negative experiences from their first year's teaching practice are likely to be reflected on her view as teacher of mathematics. Everyone criticized the teaching methods of their elementary school teachers. This may be due to the fact that the students regarded the issue from their future career's viewpoint.

Discussion

The majority of the students had positive experiences from mathematics in elementary and secondary school. Yet, Pia's starting point in upper secondary school seemed weaker than that of the others: the teacher had not recommended advanced studies to her. According to the experiences from upper secondary school, we divided the cases into three groups: 1) Success stories, allowing an easy explanation to a positive view on mathematics. 2) Victory through Hardship stories, leading eventually to a positive view of mathematics. However, the view also includes a negative dimension. 3) Regression stories, leading to a negative view of mathematics.

Students' different views of mathematics may be mainly explained by their socio-emotional orientations and coping strategies. The task (or problem solving) orientation was the best explainer for the grades of mathematics in the ME (see also Yrjönsuuri 1989), and for the positive view of mathematics at the beginning of teacher education. Yet, it is difficult to say whether students, already when entering upper secondary school, had different orientations that led to different coping

strategies. Additionally, the socio-mathematical norms of the upper secondary school class did not seem to have supported Heli and Pia with their Regression stories.

If a student is task oriented (like Mari and Leo), even a ‘bad’ teacher does not as such affect the student’s view of mathematics in a negative way. If a student has a socially dependent orientation and has experienced the upper secondary school teacher’s activities in a negative manner (like Pia and Heli), other positive role models do not necessarily suffice to maintain a positive view of mathematics.

Only Heli and Pia did not consider themselves as mathematically talented (Table 1). It is a question of a permanent and uncontrollable reason that is mostly internal (cf. Weiner 1986). So the beliefs of the social context of learning and teaching mathematics (Op’t Eynde et al. 2002) form an important part of Heli’s and Pia’s view of mathematics, and also influence their goal-settings and expectations.

Although Heli and Pia emphasize their willingness to forget the unpleasant experiences, they surface especially in scary situations (e.g. mathematical test). Their automatic emotional reactions may prevent changes in the goal-setting even when they consciously try to change their way of thinking (Hannula 2004). According to the psychological principle of centrality it seems that the beliefs related to their self are difficult to change. A negative view of the self seems to be embedded in the hard core of their view on mathematics. (cf. Pietilä 2002)

In mathematical autobiographies – as in autobiographies in general – defending one’s self through explanations is important (Linde 1993). The largest number of explanations is found in Regression stories: Heli and Pia added coherence to their identity crises by criticizing their upper secondary school teachers, the excessive amount of work, and the theoretical contents of their studies.

It is important in teacher education to take care of students who had lost their self-confidence during advanced courses of mathematics in upper secondary school. The studies of Pietilä (2002) and Kaasila (2000) show that elementary student teachers’ self-confidence can be improved by creating a challenging but safe atmosphere in the studies and by offering students opportunities to elaborate their negative experiences. It is important that students can experiment similar things that they will teach in the future to their own pupils (Pietilä 2002). We shall report later in which way these teacher students’ views of mathematics changed during teacher education.

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SUSTAINED ENGAGEMENT: PRESERVICE TEACHERS' EXPERIENCE WITH A CHAIN OF DISCOVERY

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Abstract: *Engagement and sustained engagement can be two quite different phenomena in a mathematics classroom. In this paper I put forth an argument for this aforementioned claim as well as present a brief summary of a study with a group of preservice elementary school teachers in which a form of mathematical discovery called a 'chain of discovery' is used to facilitate a state of sustained engagement. The results indicate that not only is sustained engagement reliant on a person's beliefs and attitudes, but that a 'chain of discovery' can help to change even very negative beliefs and attitudes.*

Much has been written about engaging the minds of mathematics students, most of which can be summarized as an appeal to a student's emotional dimension. Even strategies that purport to create engagement through the facilitation of cognitive challenges can be distilled down to the fact that what is being created is an 'enjoyable experience' (Csikszentmihalyi, 1996). This not only speaks to the complex and interwoven relationship between the affective and cognitive domains (Di Martino & Zan, 2001), but it also speaks to the powerful contribution that enjoyment makes on learning (Williams, 2001). But is it enough? If the goal is engagement in mathematics, it seems to be. However, if the goal is *sustained engagement* then it may not be. In this paper I argue that in order to create a situation of sustained engagement an appeal to, and eventual transformation of, a person's beliefs and attitudes towards mathematics is needed. I further argue that one mechanism for changing these aspects of the affective domain is through mathematical discovery. These arguments are then combined to put forth the idea that a *chain of discovery* can facilitate the sustaining of engagement in a mathematical problem. Finally, I present a brief summary of one study in which a *chain of discovery* helped to sustain the engagement, as well as to change the beliefs and attitudes, of 184 preservice elementary school teachers.

A CHAIN OF DISCOVERY AND SUSTAINED ENGAGEMENT

Simply put, a *chain of discovery* is an experience in which a person, in the process of solving a mathematical problem, makes a series of mathematical discoveries. However, there are subtleties about such an experience that are lost through such a simple description. As such, a more detailed and descriptive explanation is necessary. In what follows the idea of a *chain of discovery* is more thoroughly developed through the presentation of a progressively narrowing focus on specific types of mathematical problem solving. I begin with a cursory discussion of mathematical problem solving, and then examine three specific types of problem solving, of which *chain of discovery* is the third.

Mathematical Problem Solving

Much has been written about mathematical problem solving and to try to summarize it in a concise fashion is difficult; difficult primarily because opinions vary as to what constitutes a mathematical problem solving experience, what constitutes mathematical problem solving, and even, what constitutes a mathematical problem. As such, rather than embark on either a synthesizing or a differentiating analysis of these varying understandings of mathematical problem solving I present a brief working definition that will allow me to discuss the subsequent three specific cases of problem solving more effectively. First of all, mathematical problem solving can be thought of as being divided into two distinct, but related, processes – the logical and the extra-logical; the logical processes will be dealt with in this section, the extra-logical processes will be treated in the next section.

The primary logical process of problem solving is alternatively known as *problem solving by design* (Rusbult, 2000). The process begins with a clearly defined goal or objective after which there is a great reliance on relevant past experience, referred to as *repertoire* (Bruner, 1964; Schön, 1987), to produce possible options that will lead towards a solution of the problem (Poincaré, 1952). These options are then examined through a process of conscious evaluations (Dewey, 1933) to determine their suitability for advancing the problem towards the final goal. In very simple terms, problem solving by design is the process of deducing the solution from that which is already known (Pólya, 1957).

Mathematical Discovery

Perkins (2000) makes a clear distinction between problems that a person cannot solve and problems that a person has not yet solved. A problem that does not yield to a process of *design* is not necessarily unsolvable, but may merely be *problematic*. Such problems will require input from the extra-logical processes in order for a solution to be found. The extra-logical processes of problem solving are those processes that lie outside of the "theories of logical forms" (Dewey, 1938). Included in the cadre of the extra-logical are such mysterious phenomena as intuition, imagination, insight, illumination, serendipity, and aesthetics, each of which may contribute to the solving of a problem in a fashion that may defy explanation, or more appropriately, defy logic. Also included in this collection are two phenomena that received much attention within the field of mathematics over the course of the last one hundred years, creativity and discovery.

In 1908 Henri Poincaré (1854–1912) gave a presentation to the French Psychological Society in Paris entitled '*Mathematical Creation*' (Poincaré, 1952). This presentation, as well as the essay it spawned, stands to this day as one of the most insightful, and thorough treatments of the topic of mathematical invention and put forth the proposition that the unconscious mind plays an invaluable role in the creative process. Inspired by this presentation, Jacques Hadamard (1865-1963) began his own empirical investigation into mathematical invention (Hadamard, 1945). Hadamard

took the ideas that Poincaré had posed and, borrowing a conceptual framework for the characterization of the creative process in general, turned them into a stage theory. This theory still stands as a viable and reasonable description of the process of mathematical discovery. In what follows I present this theory, referenced not only to Hadamard and Poincaré, but also to some of the many researchers whose work has informed and verified different aspects of the theory.

The phenomenon of mathematical discovery, although marked by sudden illumination, consists of four separate stages stretched out over time, of which illumination is but one part. These stages are *initiation*, *incubation*, *illumination*, and *verification* (Hadamard, 1945). The first of these stages, the *initiation* phase, consists of deliberate and conscious work. This would constitute a person's voluntary, and seemingly fruitless, engagement with a problem and be characterized by an attempt to solve the problem by trolling through a repertoire of past experiences (Bruner, 1964; Schön, 1987). This is an important part of the inventive process because it creates the tension of unresolved effort that sets up the conditions necessary for the ensuing emotional release at the moment of illumination (Barnes, 2000; Hadamard, 1945; Poincaré, 1952). Following the initiation stage the solver, unable to come to a solution stops working on the problem at a conscious level (Dewey, 1933) and begins to work on it at an unconscious level (Hadamard, 1945; Poincaré, 1952). This is referred to as the *incubation* stage of the inventive process and it is inextricably linked to the conscious and intentional effort that precedes it. After the period of incubation a rapid coming to mind of a solution, referred to as *illumination*, may occur. This is accompanied by a feeling of certainty (Poincaré, 1952) and positive emotions (Barnes, 2000). With regards to the phenomenon of illumination, it is clear that this phase is the manifestation of a bridging that occurs between the unconscious mind and the conscious mind (Poincaré, 1952), a coming to (conscious) mind of an idea or solution. The correctness of this idea or solution is then evaluated. This is referred to as the *verification* stage and is the fourth and final stage of the discovery process.

Flow and Discovered Complexity

Flow (Csikszentmihalyi, 1996) is most simply described as the pleasurable state that a person may find himself or herself in when doing an activity. It is a state where one's actions are "automatic, effortless, yet it is also a highly focused state of consciousness" (p. 110), and enjoyment and engagement are at a maximum. Csikszentmihalyi identified nine key elements in people's descriptions of such states: there are clear goals every step of the way, there is immediate feedback on one's actions, there is a balance between challenges and skills, attention is focused on one's actions, distractions are excluded from consciousness, there is no worry of failure, self-consciousness disappears, the sense of time becomes distorted, and the activity becomes satisfying in its own right.

Williams (2001) has taken Csikszentmihalyi's idea of flow and applied it to a specific instance of problem solving that she refers to as *discovered complexity*. Discovered

complexity is a state that occurs when a problem solver, or a group of problem solvers, encounter complexities that were not evident at the onset of the task and are within their zone of proximal development (Vygotsky, 1978). This occurs when the solver(s) "spontaneously formulate a question (intellectual challenge) that is resolved as they work with unfamiliar mathematical ideas" (p. 378). Such an encounter will capture, and hold, the engagement of the problem solver(s) in a way that satisfies the conditions of *flow*¹. What Williams' framework describes is the deep engagement that is sometimes observed in students working on a problem solving task during a single problem solving session. What it does not describe, however, is a student's willingness to return to the same task, again and again, over several days or weeks until the problem is solved. Such *sustained engagement* requires a different theoretical framework to explain, a framework built from a *chain of discovery*.

A Chain of Discovery

Willingness to engage in a problem solving activity resides within a student's affective domain. It may reside within the student's beliefs and attitudes or it may reside within the student's emotions (McLeod, 1992). Beliefs are, just that, what students believe; what they believe to be true about mathematics and what they believe about their ability to do mathematics. Beliefs about mathematics are often based on their own experiences with mathematics and are slow to form, and slow to change. For example, beliefs that mathematics is '*difficult*', '*useless*', '*all about one answer*', or '*all about memorizing formulas*' stem from experiences that have first introduced these ideas and then reinforced them.

A qualitatively different form of belief is with regards to a person's beliefs in their ability to do mathematics, often referred to as efficacy, or self-efficacy. Self-efficacy, like the aforementioned belief structures, is a product of an individual's experiences with mathematics, and is likewise slow to form and difficult to change. Self-efficacy with regards to mathematics has most often been dealt with in the context of negative belief structures (Ponte, Matos, Guimarães, Cunha Leal, & Canavarro, 1992) such as '*I can't do math*', '*I don't have a mathematical mind*', or even '*girls aren't good at math*'.

Attitudes can be defined as "a disposition to respond favourably or unfavourably to an object, person, institution, or event" (Ajzen, 1988, p. 4) and can be thought of as the responses that students have to their belief structures. That is, attitudes are the manifestations of beliefs. For example, beliefs such as '*math is difficult*', '*math is useless*', or '*I can't do math*' may result in an attitude such as '*math sucks*'. A belief that '*math is all about formulas*' may manifest itself as an attitude of disregard for explanations in anticipation of the eventual presentation of a formula.

¹ Although flow can be used to describe the pleasurable state that anyone may find himself or herself when doing an activity, it is most often used to describe the pleasurable state that occurs when a person is doing something that they are *interested* in. That is, interested in the sense of "individual interest" (Schiefele, 1991). What Williams (2001) is describing can also be referred to as "situational interest" (Schiefele, 1991).

Emotions, on the other hand, are relatively unstable (Eynde, De Corte, & Verschaffel, 2001). They are rooted more in the immediacy of a situation or a task and as a result are often fleeting. Students with generally negative beliefs and attitudes can experience moments of positive emotions about a task at hand or, conversely, students with generally positive outlooks can experience negative emotions.

So, the willingness of a student to engage with a problem during a single problem solving session may, in fact, be due to a temporarily heightened positive emotional response to the situation. This is certainly in keeping with the foundation of '*enjoyment*' existing within the frameworks of both Csikszentmihalyi (1996) and Williams (2001). To extend this engagement across several sessions, however, beliefs and attitudes about the task must also be in a positive state. Within some student this may already be the case in that the student may have positive beliefs and attitudes about either problem solving in general, or the topic in which the problem is set in particular. Within other students, however, this may be more problematic.

For a student who has already developed negative beliefs and attitudes about mathematics and/or problem solving, engagement in a problem solving task across many sessions will be difficult. Research has shown that, although, negative beliefs and attitudes are slow to form in a learner, they are equally slow to change once formed (Eynde, De Corte, & Verschaffel, 2001). Ironically, change is most often achieved through the emotional dimension in that repeated positive experiences will eventually produce positive beliefs and attitudes. However, change is slow and will generally require a large number of successive successes before any positive change is observed. As such, the '*enjoyment*' that may be experienced in the first session of an extended series of problem solving will likely not effect enough change to encourage engagement in subsequent sessions.

However, there is one mechanism by which change in attitudes and beliefs can be quickly realized. Research has shown that the experience of discovery in the context of mathematical problem solving has an immediate and powerfully transformative effect on learners' beliefs about mathematics and their ability to do mathematics and the attitudes that govern their behaviour in the context of doing mathematics (Liljedahl, in press). These changes can be as wide sweeping as '*I now like mathematics*', but are more likely to fall into the domain of '*I can do this*'. Extrapolating this positive effect across a series of discoveries will not only magnify change in the affective domain, but will also maintain the engagement of the student in the problem solving task across several distinct sessions.

A *chain of discoveries* will facilitate such an extrapolation. It occurs when a problem solver, or a group of problem solvers, encounter successive discoveries in the course of solving a problem over an extended period of time requiring an extended number of problem solving sessions. Each new discovery provides the solver(s) with new information and new tools to aid in the advancement towards an eventual solution, as well as provides the necessary changes in the affective domain to sustain engagement across these many sessions.

In what follows I use the idea of a *chain of discovery* to explain the sustained engagement observed in a group of preservice elementary school teachers working to solve a problem.

SUSTAINED ENGAGEMENT, CHAIN OF DISCOVERY, AND THE PENTOMINOE PROBLEM

Participants in this study are preservice elementary school teachers enrolled in six different offerings of a *Designs for Learning Elementary Mathematics* course involving a total of 184 students. Each offering of the course enrolls between 28 and 35 students, the vast majority of which are extremely fearful of having to take mathematics and even more so of having to teach mathematics. This fear resides, most often, within their negative beliefs and attitudes about their ability to learn and do mathematics. As such, a large part of the focus of the course is to provide positive mathematical experiences that will allow the students to revisit, and hopefully revise, these negative beliefs and attitudes. The primary mechanism for this revision is through engaging and successful problem solving encounters. One such problem solving encounter occurs in the context of the *Pentominoe Problem*, a problem which requires a *chain of discovery* in order to solve.

A pentominoe is a shape that is created by the joining of five squares such that every square touches at least one other square along a full face. Now consider a 100's chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

If a pentominoe is placed somewhere on a 100's chart will the sum of the numbers it covers be divisible by 5? If not, what will the remainder be? Explain how you can know **quickly**!

Figure 1: The Pentominoe Assignment

The *Pentominoe Problem* (as seen in *figure 1*) is given to the participants at the beginning of the course and they are given between 10 and 13 weeks to work on it on their own and in self-determined groups of four.

Data for this study comes from a variety of different journals that each participant kept during their enrolment in the course. These journals ranged from personal

problem solving journals, to group problem solving journals, to personal reflective journals. These journals were collected and sorted according to the groups that worked together. The relevant entries from all of these journals were then coded for emergent themes and cross-checked with journals from other group members. Together these entries provide an invaluable picture of what went on while the participants struggled to solve the *Pentominoe Problem*, both on their own and in groups.

The *Pentominoe* Chain of Discovery

The *chain of discovery* that the participants experienced in working on the *Pentominoe Problem* can be summarized in the diagram presented in *figure 2*. It should be noted that although many of the participants traversed this chain exactly as presented here, not all did. Some of the participants bypassed the discoveries represented in cells two and four as shown with the arrows. It should also be noted, however, that all participants made the discoveries summarized in cells one, three, and five, and furthermore, they made the discoveries in this order.

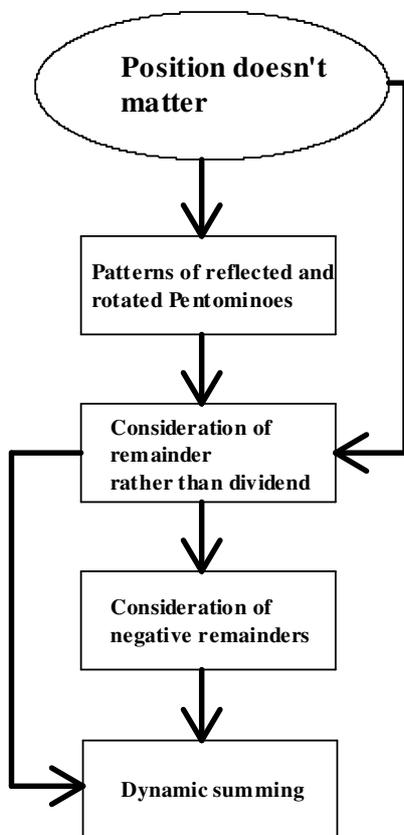


Figure 2: The *Pentominoe Problem* Chain of Discovery

The initial realization that position doesn't matter is more of an immediate observation than a discovery and hence is represented differently than any of the other discoveries in the diagram. Nonetheless, this realization leads to some number theory related concepts regarding the contribution of the ten's part of a number to the remainder.

The first real discovery that is made pertains to the patterns that are observed with respect to the remainders of reflected and rotated pentominoes. The next two discoveries that are made have to do with the realization that it is advantageous to consider the contribution that each number makes to the remainder instead of the contribution that each number makes to the dividend. This comes from the fact that the remainder of a sum will be the same as the sum of the remainders. The initial discovery brings to light this idea in the context of only positive contributions that a

number can make to the remainder and is represented in the third box of *figure 2*. The secondary discovery of this concept comes when it is realized that contributions can be both positive and negative in nature and is represented in the fourth box of the diagram. Finally, the last box represents the discovery that all discussions of remainders can be liberated from the 100's chart all together. This then leads to the dynamic construction of pentominoe shapes and tracking of the effect that each move of a block has on the final remainder.

The chain of discoveries *summarized* here is more than just a set of understandings that are achieved. Each new understanding is punctuated with a discovery. For the participants the concepts that are presented to them at each link in the *chain of discovery* arrive in a flash of illumination on the heels of much deliberate effort and periods of incubation. For example, John saw the pattern that exists between rotated pentominoe shapes '*jump out*' at him when he suddenly saw how the property of reflection also applied to rotation; Jessica was '*suddenly struck*' by the realization that individual numbers contributed to the eventual remainder and thus warranted the renumbering of the grid to reflect these contributions; Alyssa '*suddenly saw that* [her group] *was doing it wrong*' and that the pentominoes could be placed more strategically for the calculation of remainders; Dianne had an '*AHA!*' and '*knew right away*' that remainders could be seen as being both positive and negative; and Sharon '*suddenly started to see a pattern*' that the dynamic construction of pentominoes was producing.

These moments of discovery bring with them all of the positive effects that normally accompany such experiences (Liljedahl, in press). In particular, the majority of the participants demonstrate significant changes in their beliefs and attitudes about mathematics, as well as their beliefs and attitudes about their own ability to do mathematics. Most attribute this change to their work on the Pentominoe Problem. This change, as well as comments on engagement, can be seen in Melanie's comments.

Of all the problems that we worked on my favourite was definitely the pentominoe problem. We worked so hard on it, and it took forever to get the final answer. But I never felt like giving up, I always had confidence that we would get through it. Every time we got stuck we would just keep at it and suddenly one of us would make a discovery and we would be off to the races again. That's how it was the whole time – get stuck, work hard, make a discovery – over and over again. It was great. I actually began to look forward to our group sessions working on the problem. I have never felt this way about mathematics before – NEVER! I now feel like this is ok, I'm ok, I'll BE ok. I can do mathematics, and I definitely want my students to feel this way when I teach mathematics ...

What is of further interest is the discussions of the context in which many of the discoveries were made. Certainly one of the indicators of *sustained engagement* is the frequency, and the contexts within, which the participants worked on the problem. The reflective journals revealed a wealth of information on both of these factors. The participants reported working on the problem on their own for many hours at a time, coming back to the problem several times a day, and even finding themselves waking up to work on the problem. Some of these ideas are presented in Sarah's comments.

The most significant discovery that I had so far is during the Pentominoes puzzle. I was stuck on trying to figure out what the

remainder was going to be just by looking at the numbers ... I couldn't possibly imagine that you could memorize all of the possible combinations. I had been working on the problem all day, and struggling with it, and had finally given up trying. I went out for the evening and came home and sat in the hot tub for about half an hour. I started to think about the problem again and that's when it hit me ...

In discussing how hard she worked on the problem Sarah also presents the context in which she made her discovery – she was sitting in the hot tub. This, as it turns out, was not so extraordinary. The majority of the participants reported making at least one discovery on the golf course, in the car, in the kitchen, in the shower, while talking to a friend or family member, and so on. In some of these contexts, the individual was not consciously thinking about the problem at the moment of discovery. However, in many of the situations (like Sarah's) they were. That is, they were engaged enough in the problem to think about it during non-structured time, even in their 'recreational' time.

CONCLUSION

Engagement in a mathematical activity is a tenuous thing. While it is relatively easy to engage someone in a mathematical task, it is quite another thing to sustain that engagement in the same task over an extended period of time. This is because it is relatively easy to capture a person's attention by appealing to their emotions. However, the emotions are much too unstable to alone sustain the attention of the individual over the course of several problem solving experiences. For this, what are in need of capture are the attitudes and beliefs of the individual. Herein lies the key to *sustained engagement*. In this paper I have shown one possible mechanism for occasioning sustained engagement – the mechanism of *chain of discovery*, which sustains engagement by changing individual's beliefs and attitudes about mathematics as well as their beliefs and attitudes about their own ability to do mathematics.

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TRYING TO CHANGE ATTITUDE TOWARDS MATHS: A ONE-YEAR EXPERIMENTATION¹

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Abstract. *According to modern trends of research in mathematics education, attitude towards mathematics is a very important construct to interpret students' behaviour. In this paper we present a one-year experiment carried out in a 12th grade classroom in Naples (Italy) aimed at changing students' attitude towards mathematics (the classroom was characterized by low interest in mathematics). We will describe the steps of the research experiment and then describe and comment upon the obtained results.*

Introduction

The idea that the attitude toward mathematics is relevant in the teaching and learning process is shared in the mathematics education research community, but research on attitude is characterized by many problems, more or less investigated in our field. Amongst them the problem of a clear and shared terminology (Pekkonen & Furinghetti, 2002) and the problem of designing and experimenting observational tools that are consistent with theory (Di Martino & Zan, 2001a; Di Martino 2004).

Another evident aspect in research on attitude is the rarity of studies focused on teaching experiments aimed at changing attitude toward mathematics: we want to quote Philippou & Christou (1998) (describing the results of an study on pre-service teachers), Yosuf & Tall (1999) (describing the results of an innovative mathematics course in a whole classroom) and Hannula (2002) (in this case focus is specifically on a single student).

One of the difficulties of this kind of research is theoretical: if we want to deal with change in attitude toward mathematics we have to establish a *direction* of a suitable change, that is we have to clarify what we mean by negative and positive attitude (Di Martino & Zan, 2002; 2003). Moreover we need the tools to highlight this change.

In this paper we present a one-year teaching experiment in a 12th grade classroom of a school in Naples (Italy).

The particular feature of this study is that it was developed *from the bottom*, from the help request coming from a teacher.

The willingness of the teacher is fundamental to the success of the teaching experiment, in particular we worked with the classroom for the entire school-year (five hours weekly).

¹ This research was partially funded by a grant from the MIUR (Project FIRB RBAU01S427).

The teacher was initially motivated to intervene on affective factors by the fact that the class was very achieving poorly and at the same time showed a strong and widespread lack of motivation in studying mathematics in particular. So she was just persuaded that affective, cognitive and motivational factors are extrinsically linked.

So the starting point for this research study was the hypothesis, made by the teacher together with researchers involved, that attitude towards mathematics could play an important role in this situation.

Methodology

The following questionnaire, made of open-ended questions, was distributed in the class at the beginning of the school year and it was useful to decide which of the most accepted notions of attitude (simple or multi-dimensional, Di Martino & Zan, 2001b) could be the most suitable to intervene on the troubles described by the teacher.

Complete the following sentences:

I would like school more if ..

I would like school less if ..

I would like mathematics less if

I would like mathematics more if ..

Describe:

Situations in which you feel more uncomfortable during the mathematics class ...

Situations in which you feel more comfortable during the mathematics class ...

Something you did not understand in mathematics and you always felt ashamed to ask ...

Something you liked in mathematics (throughout your school years) ...

Our *conviction* that a multidimensional notion of attitude (going beyond the simple emotional disposition like/dislike) fits the description of the class situation arose from pupils' answers to this preliminary questionnaire, from which very strong beliefs and emotional reactions emerge:

Serena: *'when I cannot complete an exercise at the blackboard I feel useless in that context, it is a feeling that makes me feel bad, even if I try to pretend I don't mind: I almost feel like crying.*

Starting from this preliminary investigation, our research hypothesis was that it is possible to achieve a change in attitude (in a multidimensional sense) towards mathematics through suitable teaching practices that may favour, at the same time, the achievement of important cognitive objectives.

An evaluation of whether cognitive objectives have been reached was asked to the teacher, whereas an important theoretical effort has been made to set up suitable instruments for the observation of students' attitude (in the most complex notion) towards mathematics in different moments of the school year.

After this preliminary phase, the research study can be divided into three phases from a theoretical viewpoint:

Phase 1- Observation of the class initial situation in order to identify a *starting point* (this is clearly necessary for any kind of research aiming at making comparison like before/after).

As mentioned earlier (see note 1) this research study is part of an Italian national research program in which innovative instruments for the observation of attitude have been designed, through the identification of a number of indicators considered as particularly meaningful (see also Polo & Zan, to appear).

Three of these instruments were used in this initial observational phase:

- A questionnaire in which students must associate their emotional disposition (on a three levels scale: like, dislike, indifferent) with a list of rather frequent activities in a mathematics lesson.
- A semantic differential questionnaire to describe which adjectives or activities students view as closer to their mathematical experience.
- An innovative questionnaire (Di Martino, 2004) investigating on students' systems of beliefs as well as on the associated emotions towards mathematics.

Phase 2- Ongoing and continuous observation throughout the school year. This phase was characterised by a sequence of choices for tuning instruments to be used in a systematic monitoring of the class during the whole experimentation.

In particular we used: open questionnaires, semi-structured interviews, direct observation (weekly lessons were all audio recorded and listened to later) and lesson reports ("activity diaries" written by students were collected: these were self-reports including students' thoughts on the carried out activity).

Instruments of this type provide results that are more difficult to analyse and to cross-compare over time than results provided by standard questionnaires or Likert-type scales: however the choice of this type of instruments was consistent with the theoretical choices.

We believe that instruments that leave little space to complex and diverse answers are generally inadequate for the observation of a construct like attitude in its complex definition; in particular they seem inadequate to capture the psychologically central role played by beliefs (Green, 1970).

The complexity of the observational instruments we employed required the collaboration of other people (two trainee teachers and a final year mathematics undergraduate) not only in the analysis of results but also in collecting and transcribing data: without their collaboration this work would not have been possible.

Phase 3. Final observation. The personal semi-structured interview was chosen as instrument for the *final diagnosis*: this choice enables us to adapt the interview to any single student and therefore to take into account each one's peculiarities.

Teaching experiment

The experimentation's teaching and learning objectives were the outcome of an initial negotiation between students and teacher: this was done in order to be able to assume that choices were somehow shared, although the teacher kept the role of "guarantor" with respect to disciplinary achievements required by the curriculum.

In particular, some answers to the initially administered questionnaire served as a cue for the design of the teaching experimentation. For instance it was clear that students considered very important to see that the topics studied could be useful to model observable phenomena:

I would like mathematics more if ...*we could start from observing an everyday phenomenon and then find the means for a "mathematical" investigation.*

We worked in the context of trigonometry, a topic included in the curriculum, but usually not very appreciated by students because one of the most distant from everyday needs (hence one of the most quoted topics in justifying the question "why do we study this? What is it for?").

This choice was partially motivated by our conviction that avoiding topics that students consider *hard*, only not to attack their sense of self-efficacy, often gives implicit messages that can be detrimental (in this sense we highlight the importance of tackling problem situations, hard by their very nature, accepting the idea that it is possible to make mistakes, thus enacting a discussion on the role of errors in mathematics, see Borasi (1996). Another motivation came from the idea that it is easy to show some culturally meaningful applications of trigonometric instruments: hence we attempted to attack some commonplaces, stigmatising the tendency to make sharp judgements on unknown things.

To schematise we can divide the structure of the didactical path into four phases:

1.Exploration: the need to introduce trigonometric instruments is justified through a collective discussion around the problem of measure, proposed by means of

instrumental laboratories (with very simple tools). In this first phase two games are used to introduce problematic issues, so that the use of mathematics may be aimed to a search for a winning strategy.

The rope game

The first of these games, called ‘the rope game’ consists in dividing the students into two groups, leaving a student apart having the function of a messenger. The two groups are set in two different places and they both have a rope of the same length (chosen by the students). A member of the first group acts as pivot keeping one end of the rope; another member, keeping the other end of the rope, makes a movement as he pleases along the circumference whose radius is the length of the chosen rope. During the shifting the rope must be kept tight, the student decides how far he wants to shift, in which direction and how to communicate it to the other group. At this point the student belonging to the other group must move following the directions so that both groups must be in the same position. The game is repeated with a rope of different length.

The students may encounter many problems and there are several difficulties, first of all about ‘not shared’ measurement units.

Naval battle game

The other game, called ‘naval battle game’, runs over the idea of the previous game again.

Once again the class is divided into two groups, placed so that one group cannot see what happens to the other. Each group is given a poster on which two or more concentric circumferences have been drawn (the radiuses of the circumferences are the same on both posters). One group must communicate the positions of some points marked on their own poster to the other group who must single them out and represent them on their own poster.

Recordings and transcriptions are used also for the teaching and learning activity, in order to be able to go back to moments that students consider meaningful and to introduce elements for discussion.

2. Systematisation: the second phase begins with a shared need of teacher and students to systematise the discoveries made during the games. This need suggests the idea of dividing students into small groups of three, each keeping a **ship’s log**. This phase of “critical revisiting” of previous activities goes together with a series of activities aiming at the elaboration of knowledge through guided worksheets prepared by the teacher and finalised to calculate trigonometric functions as well as to construct the graphs of sine and cosine functions. Moreover in this phase proofs of some fundamental goniometric relationships were tackled also by means of “**Proofs without words**”².

3. Utilisation: In this phase the students were expected to design and construct a theodolite, using few simple objects. Both in designing and in utilising they

² They use figures for calculating areas: a suitable interpretation allows one to justify directly algebraic and geometric properties in unusual ways. Anyway, proofs of this type were always compared to those proposed by the textbook.

experience a problem posing activity: in actual fact classical problems such as measuring the distance between two points visible from each other's position but with not directly measurable distance (for instance two points separated by a river, or the distance between two mountain peaks) were tackled together with problems, sometimes complex, proposed by themselves.

4. Recapitulation: The initial idea was to construct a class book, not only for a further and final need of systematisation but also to leave a "historical trace" of the work done and of the process, thus using it as a metacognitive instrument.

Results

The richness of the available material on the one hand offers several points for reflection and on the other hand imposes a choice of communication: since it is not possible to deal with every aspect we decided not to focus on a single individual but rather to describe the evolution of the "class community" throughout the experimentation, by means of interventions and reactions by a number of students. This choice clearly leads to omit many interesting features and it is extremely difficult to highlight the plurality of positions emerged in the classroom throughout the experimentation. We will try to illustrate the observed differences as much as possible.

Initial observation. The analysis of initial questionnaires highlights an emotional discomfort expressed by many students:

"The situation in which I feel more uncomfortable at school is when I cannot understand something that my classmates, for instance, have understood from the very beginning. In my opinion I feel bad when I cannot cope with the situation in any occasion."

The sense of inadequacy reported by many students can explain also negative emotions (as we could notice in the questionnaires used in the first observational phase) associated with all the assessment moments in mathematics, such as oral and written tests. One might correctly object that these moments are not "mathematical" ones but the three used questionnaires show beliefs and negative emotions towards specific mathematics *objects*, such as theorems, expressions, formulae and problems.

The bulk of the class shows beliefs about mathematics that might be called, using Skemp's (1976) terminology, relation-type beliefs, described by expressions as "mathematics is ... *reasoning to understand, procedures to create*".

It is also interesting that the class is almost entirely convinced that everyone can understand mathematics and that all can manage although the most recurrent adjective to describe it is *difficult* and the emotion associated to it by most students is *fear*.

On going observation. In this case we collected a huge amount of data. We report here a transcription of an audio-recorded discussion referred to one of the very first

activities. This discussion seems to us extremely meaningful, since students discuss and understand the *sense* of some mathematical choices.

The discussion centres around the choice of anticlockwise orientation as positive direction of the axes, around the fact that the point's position depends on the distance from the centre and from the angle at the centre, all remarks ending up with a search for a conversion law – as a student named Daniele calls it- to transform Cartesian coordinates into polar coordinates.

(Serena):...*moreover starting from the abscissas semi-axis the point rotates (and indicates what she says moving her hands) ...gets far from the abscissas axis, i.e. as the angle increases abscissa decreases and ordinate increases.*

(Davide): *yes, but only in the first quadrant because in the second quadrant abscissas decrease again toward negative values and also ordinates decrease.... In the third quadrant abscissas start to increase and ordinates decrease again, whereas in the fourthlet's make a graph going up and down like an electrocardiogram*

(Teacher): *but have you not seen this graph with your Physics teacher already?*

(chorus): *...no...*

(Serena): *...I imagine a point going up and down...*

and discussion ends with

(Daniele): *oh!!! For this I invent sine and cosine!*

From the students' ship's logs we can capture their reactions to this collective discussion on some mathematical choices:

Alessia: *The lesson was no longer given from top to bottom, but we were constructing it.*

Daniele: *We played ...just like children, and this led us all to participate; also those who normally do not intervene much were protagonists, enriching the discussion with their own doubts and capacities`.*

It is interesting that for Daniele this approach is not only revolutionary for students but it also engages the teacher in the activity: *I think that the teacher was the one who went through the deepest revolution ... the woman of surprise tests, the woman of running debts converted herself to a weird type of mathematics.*

But there are some students scared by something completely new (interestingly we noticed that the best achieving students are not among those who got scared, and anyway those who were globally happy with their own performances with the "traditional" teaching method did not get scared):

Marco: *The new approach...left me with lots of doubts, since I could not understand the usefulness of that activity, the reason why we were doing all that.*

Francesco: *I was initially uncomfortable because I could not understand what the teacher expected and I could not understand how I should behave in front of those activities. Many times I withdrew into myself and didn't say anything, being afraid to say something that had nothing to do with the activity.*

Although the ship's logs show that some students were puzzled, there is a prevalence of positive comments and someone changed their minds over time. Marco who was initially lost, comments: *"It is possible to have a different didactical relationship with teachers, which is helpful in understanding better"*.

One of the central activities in the experimentation was the construction of a theodolite, for which a number of different competencies were necessary: manipulations, measurements, predictions and mathematical knowledge. This enabled each student to give his/her own precious contribution and the whole class participated enthusiastically to achieve the objective³. Someone, like Serena, shows her enthusiasm by claiming that she stopped *"playing with my mobile phone"* during mathematics classes and that the *"atmosphere was joyful and encouraging me to work hard to understand"* or someone, like Ornella who says *"It is very important to carry out activities personally, make calculations, measure, verify whether hypotheses are correct or wrong"*.

Moreover, during the actual realisation of the instrument (design, choice and collection of materials, actual construction) students discussed about many mathematical choices.

Final observation. In our opinion, the successful results of the experimentation are showed by students' decision of changing the nature of the work programmed for the end of the year. Students were not happy with having only a "historical memory" of the work done, to be used in the classroom, but they valued their experience in such a positive way that they wanted to build a web site to share the most significant moments of this experience with peers. The site starts with these words by students themselves: *"At the end of our path we decided to publish our experience, with the achieved results, on this web site, hoping that we could be an example for those who have problems in learning mathematics, attempting to provide suitable solutions"*.

The hard work necessary to build the site is one of the clearest proofs that the objective of involving the class was achieved. Students needed to enact their IT competencies as well as reconstruct the experience with communicative objectives (thus negotiating every time what could be shared with the "rest of the world"). The produced materials were continuously revised: this care for details seems to us meaningful in terms of our initial didactical objectives.

³ We must notice that the positive attitude towards the ongoing experimentation does not guarantee that attitude towards mathematics is equally positive. In our opinion this is a necessary sub-objective for the experimentation to be successful.

During this period the interviews we made showed a radical change with respect to the initial analysis: there is no longer fear for mathematics. Moreover collective discussions characterising the activities, together with the fact that often everyone played a role in the lessons construction, made many students' focus shift from the assessment of results (product) to the assessment of contributions (process). Besides students understand that also mathematically wrong interventions play an important role within a discussion about choices to be shared (there was a reevaluation of the role of errors, as suggested by Borasi).

Concluding remarks.

If also a single result is important, being aware of the initial situation, Serena's words would be enough to confirm the successful outcome of the experimentation:

"This type of completely new approach to the study of a subject that I never found particularly enjoyable, certainly helped me to forget about things like: I cannot do it, I will never be able and therefore it is useless to even try".

Also from a theoretical point of view the experimentation's results appear meaningful: in particular they show that an intervention aiming at changing attitude towards mathematics (in its complex definition) is possible also with students who might be considered as having set beliefs and emotions. Serena's words prove that it is possible to change 'fatalistic' attitudes that lead students to give up any possible approach to mathematics⁴.

Many open questions remain as a consequence of these results: first of all how stable in time this change of attitude can be. Linked to this, we may wonder whether the attitude we are evaluating is towards mathematics or rather towards a particular approach to mathematics: what happens if we go back to the traditional method? This question is justified by results achieved by Yusof and Tall (1999) who highlight how in their case going back to traditional methods corresponds to going back to the starting "affective" situation.

Once the efficacy of this type of approach has been recognised, it would be interesting to imagine this type of planning for other mathematical contents and in this sense the problem shifts to teacher training.

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MATHEMATICAL DISCOMFORT AND SCHOOL DROP-OUT IN ITALY¹

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Abstract: *The phenomenon of drop-out has important social aspects concerning the 20% of the European young people aging from 18 to 20. The Italian Education Board individuated and classified the causes of drop-out as “exogenous variables” and “endogenous variables”. This research deals with the phenomenon of drop-out and is related to the studies about affect for mathematics. We analyse the phenomenon of drop-out in high secondary school and particularly the role played by mathematics inside this phenomenon. An endogenous cause of drop-out, we call it discomfort in mathematics, arises from our surveys; its elimination might reduce more than the 50% the critical number of the drop-out variables.*

1. School drop-out in Europe and in Italy

The phenomenon of drop-out has been widely studied in the last ten years in many European countries, because of its diffusion (Iacomella *et al.*, 1997): a survey carried out in the whole European community shows that the 20% of the European young people aging from 18 to 20, are kept out of “the society of knowledge”. As far as Italy is concerned, we refer to publications of MIUR (Italian Education and Research Ministry) (MPI, 2002; PON, 2000), and particularly to a survey carried out in primary, junior and secondary schools (MIUR, 2002). The data show that in primary schools drop-out is of a “physiological” type and it has become quite relevant in junior school. To clarify better the Italian education system, primary and junior schools coincide for all pupils. The secondary education system is differentiated: there are high schools named ‘liceo’ specializing in foreign languages, in classical studies, in science, in technology, in music, and in art. Other high schools are technical schools specializing in trade, industrial technology, computer science, agriculture,

¹ Work done in the framework of activities of Local Research Units in Mathematics Education in Universities of Parma and of Pisa - Italy

chemistry, etc. Vocational schools present the same differentiation in subjects as technical schools but levels of syllabuses are lower than the other schools. At the end of any school year, a student must not have got more than four subjects with "not sufficient" mark; otherwise s/he fails and has to repeat the whole year. So there are other private junior and high schools dedicated mainly to rescue students from school failures.

In secondary schools, the drop-out phenomenon has risen a lot and the highest rate has to be found in vocational and art schools.

The data from (MIUR, 2002) show that the students not assessed at the end of the school year 2001-2002 amount to 8,9% in vocational schools and to 6,5% in art schools. Moreover (PON, 2000) shows that the percentage of "failed students" in high secondary schools in the first four years is of 13%; this means that about 130.000 students of the high secondary school suffer from a condition of serious difficulty. Then if we compare the figure 13% of failed students with 8,4% of pupils repeating a year, we obtain a difference of 4,6% students leaving school. Basing on PON, we can state that half of these young people can be recovered through alternative curricula i.e. either vocational training or 'experience courses'; the other half i.e. 2,3% is the percentage of drop-outs. The relevance of this phenomenon is easily tested: for example in the south of Italy about 450.000 young people aging from 15 to 18 do not go to school and during these three years they become drop-outs. Moreover there is a population of about 150.000 young people aging from 15 to 24 without a junior school leaving certificate, even if today's laws provide for ten compulsory years of school. Of course they are destined to be included in the number of people without a high school diploma; in the year 2004, 44,3% of Italians from 25 to 64 have not finished the high secondary school and Italy is in Europe last but two, followed by Spain with 41,6% and Portugal 20,6% (data from European Parliament):

We think that these data show such a critical situation to justify the alarm raised in our country and in Europe.

2. Exogenous and endogenous variables of drop-out

(PON, 2000) collects the results of the various researches on the problem of school drop-out in Italy; in particular, it shows a classification of causes (statistically named variables) of drop-out as "exogenous variables" and "endogenous variables".

The former are factors related to social-economic troubles, familiar indigence, and social background poverty. Owing to this factors young people loose motivation towards school (no importance to intellectual power and too much value assigned to "material richness", often in regions where the job offer is high).

The causes related to the school system itself, specifically concerning education, are defined endogenous variables.

Following (PON, 2000):

«it is not proper at all to value only exogenous variables. There are troubles, stickinesses and other hostile factors operating inside the educational system that produce the conditions for drop-out».

For example misunderstanding of logical-linguistic connectives and quantifiers is indicated as an endogenous variable by (MPI, 1994).

Results of a survey on young deviance (MPI, 2000), show that, in any case, there are more than one cause or better a combination of causes that imply this phenomenon (statistically an average of 1,95 causes).

The authors of the survey above state that:

«(...) the same principle might explain drop-out itself too».

Endogenous variables play a very important role inside the problem of school drop-out: so, it is necessary to take them into consideration, to analyze them and work on them in order to reduce to the minimum the risk of school drop-out itself.

The strategy to prevent school abandon drawn up by PON does not take into consideration a specific element, expressly the failure in mathematics. In this paper we state that this neglected aspect is an endogenous variable.

3. Theoretical framework

3.1. Pupils' difficulties with mathematics

The problem of difficulties in mathematics has been analysed under many aspects by the National Education Boards and by the researchers in Sciences of Education and Mathematics Education all over the world. It is already proved that in the teaching of Mathematics the affective links among teacher, student and classmates play a fundamental role in mathematical results (success or failure) (Prawat & Anderson, 1994).

To overcome the problem it is necessary to relate failure with affect.

According with McLeod's (1992) framework we assume a multidimensional description of the affective domain, consisting of interdependent constructs, such as emotions, attitudes, beliefs and values (Evans *et al.*, 2004).

In our study, we consider *discomfort* in mathematics as an attitude with three components: emotional response, beliefs on the subject, and behaviour towards the subject (Di Martino & Zan, 2001 and 2003). Discomfort is revealed through uneasiness, annoyance, inquietude, pain or fear. Pupil's 'hostile' attitude towards mathematics (lectures, homeworks, school tests) in our opinion could be ground on discomfort. We can consider it either a symptom of the process towards failure in mathematics or an aetiology of the process itself. If pupil does not overcome the causes of discomfort, s/he risks to become alienated from mathematics and then to fail in it and to abandon school.

3.2. *Teacher's practice*

Teachers should take care of relationship with students to improve learning:

«learning is personal, but it takes place in the social context of interpersonal relationships»

and

«the facilitation of learning depends on the quality of contact in the interpersonal relationship that emerges from the communication between the participants» (Alró & Skovsmose, 2002).

Moreover, the professional competence of the teacher had better take into consideration the results of the research concerning Educational Psychology and particularly meaningful learning and meta-cognition. All this leads to go beyond the transmissive teaching that prevents from an adequate conceptualization, because it does not take into consideration the ways for constructing concepts, on the whole, and the mathematical objects, in the specific instance (Sfard, 1991). Besides we must consider the advantage that originates from the epistemological analysis of the conceptual crux of the subject (Schoenfeld, 1985).

3.3. *Arithmetics, algebra and drop-out.*

Difficulties in arithmetic that originate and develop in the primary school, if they are not diagnosed and cured in the junior school, in the high secondary school they become obstacles to the access to communication through the mathematical language and therefore they become more and more difficult to eliminate (Kieran, 1990 and 1992). The lack of the semantic control (Arcavi, 1994), due to the increase in formal complexity, is also responsible for the arising of discomfort in mathematics (Silver & Kenney, 1997).

Too often, teachers do not give importance to the difficulties met by students in the use of the algebraic language and with some of its functional aspects such as the use of words as: “variable, unknown, undetermined, parameter” (Marchini, 2002), (Marchini & Kaslova, 2003). In an epistemologic perspective, such difficulties reflect the fact that in the history of mathematical thinking, these ideas have been focused along many centuries. Just from the abstract nature of the symbolic language and its consequent applicability to a multiplicity of different contexts, originates one of the greatest difficulties of the semantic treatment so that

«the student can think that this language applicable to all contexts, does not belong to any indeed» (Arzarello *et al.*, 1994).

4. The research

This research relates the phenomenon of drop-out to the studies about affect towards mathematics. We analyse the implications of mathematical teaching inside this

phenomenon focusing on secondary school, especially on the “critical” bracket of the first two years.

In a previous survey (about 800 junior and high secondary school students tested through questionnaires and interviews about their attitudes towards mathematics and schooling) we proved that when school discomfort is only due to endogenous variables, then mathematical discomfort is always present.

The present research shows that **when a student fails at school (i.e. s/he is drop-out) then, in the same school year, s/he failed in mathematics** .

The obtained results are fully consistent with the following meaningful event: in the bracket of the first two years of high school, disaffection to mathematics arises and embodies, with the highest frequency compared to the other brackets (Silver & Kenney, 1997; Busk, 1977).

Now, mathematical failure always follows mathematical discomfort. Then **we can classify discomfort in mathematics as an endogenous variable of school drop-out**, moreover considerable **a constant**, as our survey shows.

We have just seen that, on the average, 1,95 causes are necessary in order to determine drop-out: as the discomfort in mathematics represents “1” cause, its elimination means that more than the 50% of the critical number of the drop-out variables is reduced.

In other words, getting over the discomfort in mathematics becomes a decisive factor in order to avoid drop-out.

The research is based on three different surveys.

4.1 *The written - tests survey*

This survey has been carried out among students attending the first two years of high secondary school; drop-out almost always occurs during the first school period or soon after, just when the algebraic language is being used in mathematics and requires increased abstraction processes. We have analysed, vertically, the classtests made by 38 students of high secondary schools (grammar, scientific and educational psychology schools, technical high school) who haven't enrolled the following class in the same school. The tests examined are relative to the written tests from October to March, month, this one, when students usually withdraw (i.e. stop going to school officially) (under the present rules that allow to repeat, one year later, enrolment to the same class left) reference year school 2002-03.

All the examined tests consist of a sequence of exercises classifiable exclusively as computation exercises, in aiming to checking the ability to use the algebraic symbolism. In both 1st class and 2nd class, the exercises face the ordinary themes developed in the syllabuses and can be divided in the following types:

1st class

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- Computation in the set of rational numbers \mathbf{Q} , with attention to application of the exponentiation laws
- Standard algebraic computation; operations between monomials and polynomials, reduction of algebraic expressions, recognition of products and decomposition in factors
- Solution of numerical integral and rational 1st degree algebraic equations
- Solution of literal algebraic equations
- Solution of algebraic equations systems

2nd class

- Computation with literal expressions, and arithmetical radicals
- Solutions of 2nd degree equations

We clearly notice that the tests analysed do not include exercises concerning inequalities and algebraic equation systems with degree greater than 1, topics treated in the last part of the school year, when drop-out had already occurred.

The mistakes highlight, in the 100% of cases, inability to recognize the properties of the arithmetic operations and the algebraic terms involved. They can be divided into the following categories with different characters of generality and conceptual relevance:

- mistakes in the treatment of powers (resp. of the basis and of the exponent);
- difficulty in the calculus with operations, in particular division with powers having the same basis;
- incorrect use of conventional rules in the treatment of algebraic expressions;
- incorrect use of the sign “-” in the polynomial ring;
- algorithms wrongly applied for operation in the polynomial ring;
- incorrect use of the letters as unknowns, constants, parameters;
- difficulty in performing substitutions of values for letters;
- problem about the presence of a null product;
- insufficient recognition of the type of a letteral equation with respect to parameters values;
- inadequate knowledge of the interplay of operations and their "inverse".

The nature of the tests shows that the evaluation made by the teachers consists only in checking the ability of manipulation of symbols. What is certified as “insufficiency in mathematics” is, at the most, “insufficiency in the manipulation ability”. It is not correct to express a general judgement on the absence of a mathematical mind in the examined students or inability to construct one, since, perhaps, educational work to this purpose had been missing. The mathematical knowledge owned by the students

at most seems to be nothing but rules memorised and not supported by adequate mental images. On the other hand, the corrections made by the teachers suggest that these ones ignore the conceptual genesis of found mistakes.

This analysis of the mistakes points out heavy and primary gaps in the mathematical learning:

- inadequate construction of the sense of symbols
- incomprehension of the terms usually used in the subject
- lack of any process of justification of the computation rules
- inability to semantic treatment and control.

From this we realize that the students do not master algebraic language. Moreover we notice that the structure of the written tests is inadequate i.e. they should have been prepared in such a way to establish if students have got specific capacity, ability and competence. At the same time the tests show that teachers can not interpret properly the genesis i.e. the locus of the students' difficulties.

4.2 *The interviews survey*

In this survey we have interviewed 179 students enrolled in 4 "Rescue" Schools.

We asked everyone just this question:

"Did you have a sufficient mark in maths when you leaved your school?"

In 99% of the cases, the answer was that **they had a bad mark in mathematics.**

In the same survey we asked the analogous question about the other two main subjects (literature and foreign language): in both cases the average does not overcome the level of 70% of the interviewed students; moreover, these subjects are not always simultaneously present.

4.3 *The vocational/artistic schools data survey*

The choice to check mathematical failure in vocational/artistic schools is due to the data (MIUR, 2002) which refer to an average of 30% of drop-out in such schools.

We have asked data from 5 vocational and 1 artistic schools relatively to the following items:

- Number of enrolled students
- Number of successful students
- Number of failed students
- Number of failed students with bad mark in maths
- Number of failed students not evaluated in maths

The data presented in Table 1, referring to the first classes of the year school 2003-04 of the tested schools, allow to estimate the proportion between the number of students with negative assessment in mathematics and students who failed. The following scheme summarizes the data and shows that failure in mathematics is always present whenever there is a failure at school.

Tab. 1

school	enroled	successful	failed	failed with bad mark in maths	failed not evaluated in maths
vocational	302	67	43	20	23
“	215	151	49	29	19
“	485	128	147	126	40
“	273	69	62	51	11
“	295	70	28	16	11
artistic	329	94	79	58	17

4.4 General comments on the failure

Many crucial passages for the development of the mathematical thinking seem not to be supported either by the action of teachers who, in most cases, involve students only in practising calculation, or by the textbooks which, almost always, reflect this way of teaching. In particular, the “great ideas” on which mathematics is based, aren’t mentioned. This lack causes a fragmentary knowledge not controllable by the students, effectively not usable, far from the students’ interests and needs and therefore completely useless for the educational process. The importance to provide the students with the motivations of learning is not taken into consideration at all.

5. Conclusions

The analysis on the drop-out requires a careful consideration on the following problems of mathematical education. It is necessary to eliminate the separation between research and teaching. In this connection we point out something wrong with communication: the results of the researches in the teaching of mathematics are spread to the researchers of the sector and only to very few motivated and committed teachers. As a consequence there is a substantial lack in the “in itinere” training of the teachers in mathematical education. The messages not transmitted concern basically two components:

- the role of affect towards the processes of learning mathematics
- the nature of the subjects/objects proposed in the teaching routes.

A proposal that aims at raising the quality of learning mathematics must go beyond transmissive method, adopting, instead, a socio-constructivistic conception of education, that can fit the class and the single student.

To evaluate as positive the learning of a student who shows a certain ability to calculation and who can handle formulas, without caring about his real ability to semantic control, is very dangerous for the quality of mathematical learning, diminishing the interest in the subject, jeopardizing the possibility of forming mental schemes useful for the development of concepts and mathematical reasonings. It is necessary to eliminate the parameters of evaluation that are not adequate, and aim at recovering the value of the study of mathematics as an instrument of developing and bettering individual potentials. The achievement of this target enables to avoid “some students’ failure in reaching a basic qualified education” (PON, 2000). On the other hand it is in conformity with the reforms that put educational success before school success, by a wider perspective in which “education is a permanent and essential source for individual growth” (MPI, 2000).

Also the history of the mathematical ideas can contribute to make mathematics “more human”, presenting ideas and concepts not as a “simple creation of a genius”, but as a result of a long and hard conceptual work.

In the end, we want to draw the mathematical community attention towards a considerable responsibility, should other analogous surveys in Europe demonstrate the nature of *endogenous variable* of mathematical discomfort and point out relationship between the data got in this study and the final results of VII Commissione C.S.I. (MPI, 2000). In the final report of this commission, in fact, we read:

“The phenomena of abandon, failure, drop-out can be faced *inside* and *through* school. There is no direct implication between social isolation and school failure. The conditions must be considered as negative variables to compare with, not an excuse for high rates of drop-out”.

In conclusion the ordinary method of teaching in contrast to what has been checked by the research, not only prevents students from constructing a mathematical thinking, but it may be a fundamental concomitant cause of their drop-out with the consequent impoverishment of their global *intellectual richness*. On the other hand, it can make sense to check whether the interest in mathematics and therefore success in the subject, may represent motivating elements for the study in general, as instruments to make self-esteem raise. In fact, in our culture, success in mathematics is associated with “great intellectual endowments”, socially appreciated. Finally, success in mathematics could be an instrument to fight drop-out.

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THE MEASUREMENT OF YOUNG PUPILS' METACOGNITIVE ABILITY IN MATHEMATICS: THE CASE OF SELF-REPRESENTATION AND SELF-EVALUATION

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Abstract: *The main difficulty of the study on the field of metacognition concerns the development and testing of valid techniques that measure young pupils' metacognitive ability. Two important dimensions of metacognition in mathematics are pupils' self-representation as an "inner part of metacognitive knowledge" and self-evaluation as the starting point of self-regulation. In this study we propose and test some means for the measurement of the above dimensions of metacognition and investigate the interrelations between them. We found that the self-evaluation of the difficulty and the similarity of mathematical tasks were distinct procedures, which entail different cognitive processes; these constructs were independent from general self-representation. The pupils' evaluation of the difficulty of mathematical tasks were found to be overoptimistic, indicating that at the age of 8 –11 years old pupils have only a vague self-image.*

THE CONCEPT OF METACOGNITION IN MATHEMATICS

Recent investigations have established the importance of metacognition in the acquisition and application of learning skills in diverse domains of inquiry (Alexander, Fabricius, Fleming, Zwahr & Brown, 2003). Although the concept "metacognition" has been defined in numerous ways Sperling, Howard and Staley (2004) suggest a focus on its component parts, which are knowledge about cognition and regulation of cognition (Boekaerts, 1997; Fernandez – Duque, Baird & Posner, 2000). Knowledge about cognition refers to the level of the learner's understanding of his/her own memories, cognitive system, and the way he/she learns; regulation of cognition refers to how well the learner can regulate his/her own learning system, i.e., goal setting, choosing and applying strategies, and monitoring his/her actions. Though metacognitive knowledge and metacognitive regulation are interdependent variables, they are mutually interdependent so that more knowledge leads to better control and advanced control leads to construction of new metacognitive knowledge.

Historically the concept metacognition was introduced by John Flavell (1976) based on the concept of metamemory. During the past thirty years several authors have provided variable definitions, portraying different emphases on mechanisms and processes associated with metacognition (Georgiades, 2004). In this study, we consider metacognition as the awareness and monitoring of one's own cognitive system and it's functioning. Although metacognition is a multidimensional construct, we focus on two principal dimensions, knowledge of cognition or self-representation of one's mechanisms about his/her knowing, and self-regulation of cognition.

According to Demetriou and Kazi (2001) “self-representation refers to how the individual perceives himself/herself in regard to a given disposition, style, type of activity or dimension of ability” (p.33). We consider self-representation to be a wider term encompassing meanings that are normally included in related terms such as self-consciousness, self-image and self-evaluation. All these terms are important parts of the general metacognitive ability. The main function of self-consciousness is to provide integrated internal representations of the world based on experiences, perceptions and memories. As Lerch (2004) suggests the metacognitive aspects of problem solving need to be expanded to include the problems-solver’s self-image as a mathematical being. Self-evaluation refers to the subjects’ appraisal of the difficulty of the various tasks and the adequacy or success of the solutions they give to the tasks. We consider self-evaluation as the first step in the process of self-regulation and self-representation as a primary part of metacognitive knowledge.

Problem solving ability in mathematics education is recognized as a complicate interplay between cognition and metacognition. O’ Neil and Abedi (1996) view metacognition as consisting of awareness of planning, applying, and monitoring cognitive strategies. For the successful solution of any complex problem a variety of metacognitive processes is necessary. Successful problem solvers realize that they can guide their own attempt by searching for and recognizing previously overlooked ways of combining information and connections between prior knowledge and the problem situation. The less experienced solvers cannot monitor the solution process as effectively and they may continue with unsuccessful strategies (Lerch, 2004).

THE MEASUREMENT OF METACOGNITIVE ABILITY

The study of metacognition is heavily dependent on the development of valid measuring instruments and specifically appropriate tasks to measure metacognitive ability. The complexity of this task arises from two main sources, first the lack of a generally accepted conceptualisation of what really the construct means and second the fact that metacognition is an inner awareness or process rather than an overt behaviour and consequently individuals themselves are often not conscious of these processes (Georgiades, 2004). Despite Flavell’s expectation (1987) that methods for measuring and assessing metacognitive experience would soon be developed, each of the proposed so far methods has different strengths and weaknesses. For example, even though interviewing is one of the most popular methods in measuring metacognition, research has convincingly shown that verbal reports of all types are subject to many constraints and limitations (Miles, Blum, Staats & Dean, 2003). Asking children, particularly young children, about their cognitive processes involves some special problems. For instance, children’s answers may reflect not what they know or what they believe, but rather what they can or cannot tell to the interviewer.

Another widely used methodology in metacognition is the think-aloud protocol analysis. In this technique a subject is asked to vocalize his or her thinking processes

while working on a problem. The data as a protocol are then coded according to a specified model for psychological analysis, which provides insights into elements, patterns and the sequencing of underlying thought processes. At the same time report writing has been viewed as “thinking aloud on paper”. Pugalee (2004) used verbal and written protocols as a tool to compare high-school students’ mathematical problem solving processes. According to her results the strategies used by students did not vary greatly between those who provided written or verbal descriptions of the problem solving processes. Nevertheless, students who wrote descriptions of their processes produced significantly more orientation and execution statements than students who verbalized their responses. The author suggests that writing can be a tool for supporting a metacognitive framework and that this process is more effective than the use of think-aloud processes.

A demanding view is that “talking about”, as another thought process, should entail more than a simple description of previous thoughts or actions. It could be a metacognitive reflection that involves critical revisiting of the learning processes, in the sense of noting important points of the procedures followed, acknowledging mistakes made on the way, identifying relationships and tracing connections between initial understanding and learning outcomes (Georgiades, 2004). Applying this procedure with young children a number of problems might be encountered including the possible lack of verbal fluency, difficulty in discussing general cognitive events and tendency for describing specific just-experienced events.

Self-report inventories as measures of metacognitive ability are perhaps, in some ways, the least problematic technique (Sperling, Howard, Miller & Murphy, 2002), especially for very young children who are not able to express in details their thoughts. In the present study we present few of the inventories that have been used for the measurement of metacognition. Fortunato, Hecht, Tittle and Alvarez (1991) asked seventh-grade students to work on a non-routine problem and then respond to twenty-one statements reflecting their thinking while solving the problem, in order to measure their metacognitive ability in relation to their performances in mathematical problem solving.

Schraw and Sperling - Denisson (1994) developed a 52-item Likert scale self-report inventory for adults (MAI), which measured both knowledge of cognition and regulation of cognition. They set out to confirm the existence of eight factors, from which three related to knowledge of cognition and five related to regulation of cognition. The knowledge of cognition sub-scale measures an awareness of one’s strengths and weakness, knowledge about strategies and why and when to use those strategies. A specimen item on knowledge of cognition is “I learn best when I know something about the topic”. The regulation of cognition sub-scale, measures knowledge about planning, implementing, monitoring and evaluating strategy use (i.e. “I ask myself if I am meeting my goals”). Factor analysis resulted in a two-factor structure, the two factors being the knowledge of cognition and the regulation of cognition. Sperling et al. (2002) took the idea of the MAI inventory one step further;

they developed two analogous inventories, the Jr MAI version A and version B scales, appropriate for measuring younger learners' metacognitive ability along those two main factors.

THE PRESENT STUDY

The main purpose of the present study, which is a part of a bigger research on the development of metacognitive abilities in mathematics, was to investigate the interrelations between young pupils' self-representation and self-evaluation in relation to their mathematical performance. A necessary part of this goal was to develop the means for the efficient measurement of metacognitive abilities in mathematics and especially their self-representations and self-evaluations while solving mathematical problems. The measurement of mathematical performance was another necessary step towards the investigation of the interrelations between the cognitive and metacognitive performance.

The sample: Data were collected from 126 children (about 8 to 11 years old) in grades three through five (37 were 3rd graders, 40 were 4th graders and 49 were 5th graders).

Procedure: A questionnaire was initially developed measuring pupils' metacognitive ability. This instrument consisted of two main parts: The first part (Appendix A) was consisted of 30 Likert type items, of five points (1=never, 2=seldom, 3=sometimes, 4=often, 5=always) reflecting pupils perceived behaviour during in-class problem solving activity. The responses to this questionnaire constituted an image of pupils' self-representation referring to how they perceived themselves in regard to a given mathematical problem. The second part (Appendix B) was consisted of three pairs of problems for which pupils had to evaluate the difficulty of the tasks and the degree of their similarity. The first pair of tasks consisted of two quantitative problems, the second one consisted of a quantitative and a spatial problem, and the third pair consisted of two spatial problems. This part aimed to measure the subjects' self-evaluation with regard to the tasks, i.e., their judgments about aspects of their subjective experience generated at particular tasks. Mathematical performance was measured through four numerical tasks, four analogical, four verbal and four matrices. Working memory and information processing were measured as well, but those data were not used for the results of the present paper. A series of three repeated waves of measurements were taken with a break of 3-4 months between successive measurements.

RESULTS

Exploratory factor analysis was first used in order to examine whether the factors that guided the construction of the first part of the questionnaire were presented in the participants' responses. All 30 items of the questionnaire were subjected to a common factor analysis, which resulted in 10 factors with eigenvalues greater than 1, explaining 64.74% of the total variance. After content analyzing those

factors, in the light of the results of the exploratory factor analysis, we grouped them into the following four groups of factors: general self-image (two factors), strategies (four factors), motivation (two factors), and self-monitoring (two factors). The means of those four groups of factors were used subsequent analysis, to avoid a big number of variables at the structural equation modeling and the growth modeling.

A series of models about the metacognitive system were tested for each wave of measurements with the aim to specify the best fitting model. It was important to investigate the degree of the similarity of the models, which could be constructed for the repeated measurements. Structural equation modeling was used to test the hypothesis on the existence of eight first order factors, and two second-order factors in all cases, which would be consistent with the theory. The eight first order factors tested were: The difficulty of quantitative problems, the difficulty of a quantitative and a spatial problem, the difficulty of spatial problems, the evaluation of the similarity of the three pairs of problems, the self-image, the self-monitoring, the strategies and the motivations. And the on two second-order factors were the evaluation of the difficulty of mathematical tasks and the general self-representation

The analysis was conducted using the EQS program (Bentler, 1995) and maximum likelihood estimation procedures. Multiple criteria were used in the assessment of the model fit ($CFI > 0.9$, $X^2/df < 2$, $RMSEA < 0.05$).

The first of the models, which were tested, involved only uncorrelated first order factors. The fit of this model was very poor in all three waves of measurements. The fit of the hypothesized model with all the correlations among the variables, explained above, improved further after allowing a few error variances to correlate:

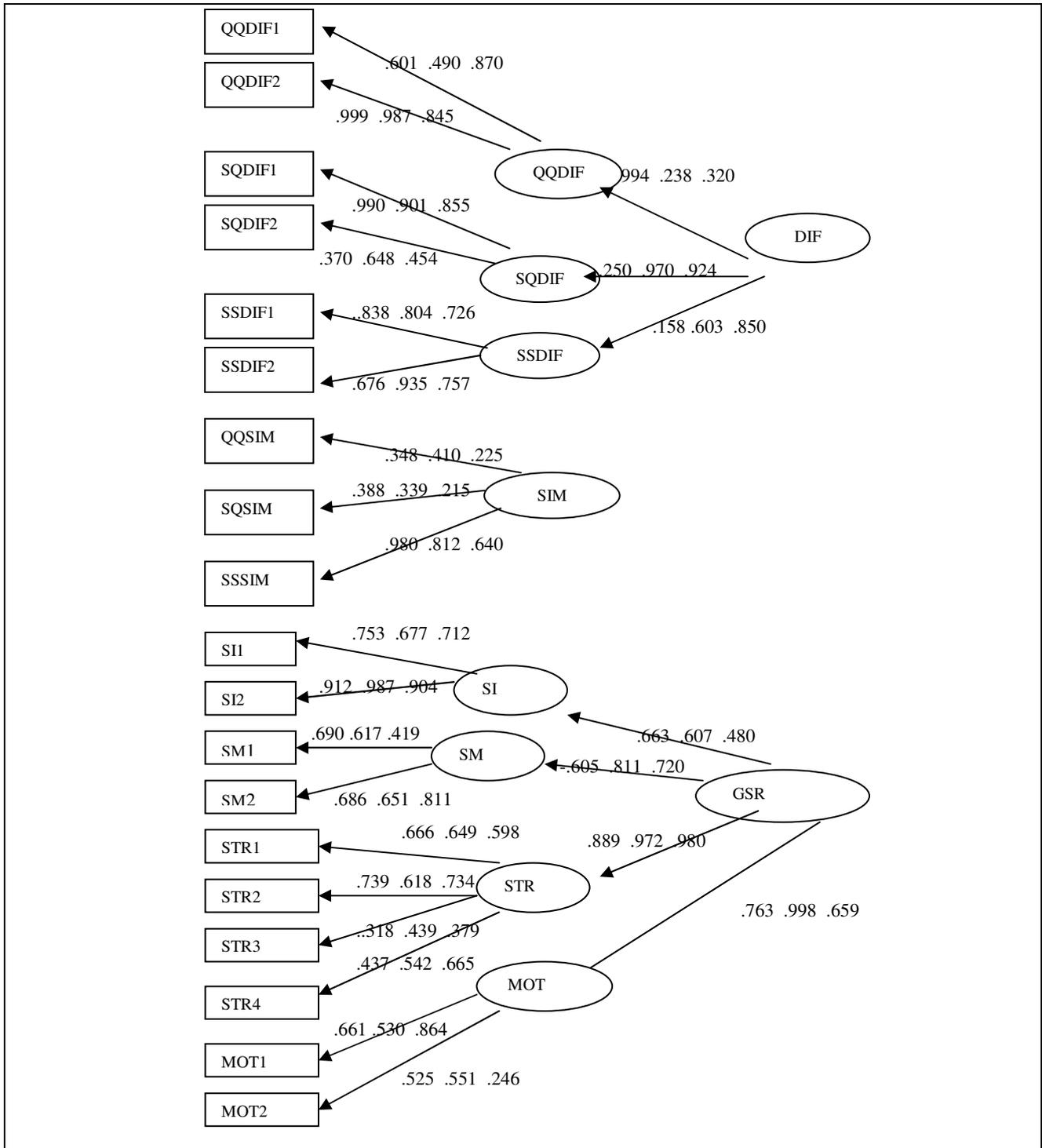
1st testing: $X^2=161.400$, $df=136$, $X^2/df=$, $p=0.06$, $CFI=.944$, $RMSEA=.043$

2nd testing: $X^2=175.172$, $df=141$, $X^2/df=$, $p=0.02$, $CFI=.939$, $RMSEA=.046$

3rd testing: $X^2=174.149$, $df=122$, $X^2/df=$, $p=0.001$, $CFI=.920$, $RMSEA=.049$

The fit of the final model was very good and the values of the estimates were high in all cases (Figure 1). It is clear therefore that the two-level architecture accurately captures the data; this two-level model is consistent with the theory. It involved two types of factors. The eight factors were regressed on the two second order factors as hypothesized. The most important result was that it was impossible for the first and second order factors, which concerned the difficulty and the similarity of the problem tasks to be regressed on a higher order factor or on the factor of general self-representation. This was an indication that those two factors entailed different processes, which were independent from the other dimensions of the system at the specific child age.

Figure 1: The model of the pupils' evaluation on the difficulty and similarity of mathematical tasks in relation to their general self-representation



QQDIF= Difficulty of quantitative problems, QSDIF= Difficulty of a quantitative and a spatial problem, SSDIF= Difficulty of spatial problems, DIF= Pupils' evaluation of the difficulty of the problems
 QQSIM= Similarity of quantitative problems, QSSIM= Similarity of a quantitative and a spatial problem, SSSIM= Similarity of spatial problems, SIM= Pupils' evaluation of the similarity of the problems
 SI= Self-Image, SM= Self-Monitoring, STR= Strategies, MOT= Motivation, GSR= General Self-Representation

After obtaining this result it was important to further investigate pupils' self-representation and self-evaluation in relation to their cognitive performance. Pupils evaluated as more similar the pair of quantitative problems and the pair of spatial problems. Pupils' evaluations of the similarity of the spatial problems improved at the second and third measurement, while the degree of similarity between the quantitative and spatial problem decreased. Specifically, the means responses about the similarity of the problems in each of the three measurements were:

The pair of quantitative problems: 1st measurement: 4.18, 2nd: 3.96, 3rd: 4.30

The quantitative with the spatial problem: 1st measurement: 2.38, 2nd: 2.29, 3rd: 2.02

The pair of spatial problems: 1st measurement: 2.67, 2nd: 2.87, 3rd: 2.89

Using the MPLUS (Muthen & Muthen, 2001) the pupils were classified into three groups, according to their cognitive performance and their self-representation (self-image and self-monitoring). The three group classification was found to have better indices, namely: AIC=2200.235 and Entropy=0.887, against AIC= 2242.973, Entropy= 0.843 for the two categories, and AIC=2225.110, Entropy= 0.862, for the four group classification, respectively. The pupils of the first group (N=51) were found to have high cognitive performance and high self-representation, those of the second category (N=10) exhibited low cognitive performance and low self-representation, while the third group (N=65) showed low cognitive performance and high self-representation. This category indicated that many pupils overestimated their cognitive abilities. Table 1 summarizes the mean responses of each group by measurement. Cross tabs analysis indicated that 8 pupils of the second group were at the third grade and 2 pupils at the fourth grade. The small number of pupils at this group permitted us to exclude it from further analyses.

Table 1: Mean responses in cognitive and metacognitive abilities by pupil group

Group	Measurement	Cognitive performance	Self-image	Self monitoring
Group 1 N=51	1 st	0.804	4.098	3.513
	2 nd	0.835	4.027	3.492
	3 rd	0.817	4.025	3.404
Group 2 N=10	1 st	0.556	3.515	3.401
	2 nd	0.524	3.751	3.173
	3 rd	0.700	3.518	2.987
Group 3 N=65	1 st	0.592	3.569	3.589
	2 nd	0.642	3.569	3.250
	3 rd	0.652	3.771	3.530

Group 1: pupils with high self-representation and high performance
 Group 2: pupils with low self-representation and low performance
 Group 3: pupils with high self-representation and low performance

To examine the development of pupils' evaluations of mathematical tasks, a 2x3x3 repeated measures analysis was applied (2-the first and the third of the above groups, 3-pairs of problems, 3-the three testing waves). According to the results, the most interesting effect was the effect of the group in which individuals belong

($F_{\text{Pillais}}(1,114)=1007.676, p<0.001$) indicating that the subjects with high performance and high self-representation evaluated as more similar the problems than the subjects with low performance and high self-representation. This finding suggests clearly that pupils with a more precise relation between their performance and self-representation had classified the problems in a more precise way, according to their characteristics and the strategies they could use for their solutions.

DISCUSSION

Most researchers agree that metacognition is an important construct to study, but difficult to measure (Schraw, 2000). Central to the problems relating to metacognition is finding ways to recording and making available to others one's metacognitive thoughts. Both identifying and measuring metacognition currently rely heavily on researchers' subjective interpretation in assessing what is cognitive and what is metacognitive. Undoubtedly it is not easy for young pupils to express their thoughts about their cognitive system and their cognitive abilities. We support the view that in our attempt to measure metacognitive abilities it is possible to interfere to pupils' metacognitive processes while they are occupied in thinking aloud about their own cognitive system. Those thoughts are metacognitive by their own. In this sense, we suggest that the inventory we have constructed can be used for the measurement of young pupils' metacognition, especially the dimensions of self-representation and self-evaluation.

According to the present results two factors (the evaluation of the difficulty and the similarity of mathematical tasks) failed to be regressed on a higher order factor, mainly because the evaluation of the difficulty concerns pupils' beliefs about their abilities in relation to specific tasks, while the evaluation of the similarity concerns their beliefs about the operations that different tasks entail. It is intuitively appealing to accept the view that people think by analogy (comparing problems with similar structures, but not necessarily the same features or story line). A key obstacle to this process is the failure of subjects to abstract the relevant principles from the problem at hand (Staats & Blum, 1999).

The results of the present study have indicated that low achieving pupils' evaluations of the difficulty and the similarity of mathematical tasks were optimistic rather than realistic; they appeared unaware of the ineffectiveness of any strategy they may use. Undoubtedly if an individual is unaware of his/her cognitive processes and abilities, we can't improve his/her performance. Learners who are skilled in metacognitive self-assessment and, therefore, aware of their abilities are more strategically thinking perform better than those who are unaware of working of their own mental system (Schraw & Sperling-Dennison, 1994).

Promoting metacognition begins with building an awareness among learners that metacognition exists, differs from cognition and affects academic success (Schraw, 1998). The first step to attaining insight into ones own mental models is

simply getting individuals to become aware of their own processes. Self-representation and self-evaluation are important dimensions of metacognition. Alternative approaches for the measurement of cognitive and metacognitive abilities have to be proposed, especially for high sample of very young pupils.

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Working Group 2

APPENDIX A

1. I know how well I have understood a subject I have studied.
2. My performance depends on my will and my effort.
3. I try to use ways of studying that had been proved to be successful.
4. I can learn more about a subject on which I have previous knowledge.
5. I can learn more about a subject on which I have special interest.
6. I understand something better if I use pictures or diagrams.
7. I define specific goals before my attempt to learn something.
8. I examine my own performance while I am studying a new subject.
9. After I finish my work I wonder whether I have learned new important things.
10. After I finish my work I wonder whether there was an easier way to do it.
11. After I finish my work I repeat the most important points in order to be sure I have learned them.
12. I use different ways to learn something according to the subject.
13. When I do not understand something I ask for the help of others.
14. For the better understanding of a subject I use my own examples.
15. I know ways to remember knowledge I have learned in Mathematics.
16. When I read a problem I know whether I can solve it.
17. I concentrate my attention on the data of a problem.
18. I understand a problem better if I write down its data.
19. In order to solve a problem I try to remember the solution of similar problems.
20. While I am solving a problem I try to realize which are its aspects that I cannot understand.
21. When I try to solve a problem I pose questions to myself in order to concentrate my attention on it.
22. When I encounter a difficulty on problem solving I reread the problem.
23. When I encounter a difficulty that confuses me in my attempt to solve a problem I try to resolve it.
24. While I am solving a problem I wonder whether I answer its major question.
25. Before I present the final solution of a problem I try to find some other solutions as well.
26. After I finish my work I know how well I performed on it.
27. I believe that some mathematical concepts are more difficult than others.
28. When I cannot solve a problem, I know the factors of the difficulty.
29. I believe that some problem solving strategies are easier than others.
30. When I encounter a difficulty in problem solving I am looking for teacher's help.

APPENDIX B

The Pair of quantitative problems

1st problem: The boys of an elementary school are 105 and the girls are 125. How many are the pupils of the school?

2nd problem: The men-teachers of a school are 18 and the women-teachers are 15. How many teachers teach at the school?

The Pair of a quantitative and a spatial problem

1st problem: Marios has four books on his desk. The Maths textbook is under the science textbook. The history textbook is between two other books. There is no book under the literature book. Which is the arrangement of the books on the desk?

2nd problem: The number of Michael's house is an even number, smaller than 20 and bigger than 10, multiple of 3 and multiple of 4. Which is this number?

The Pair of spatial problems

1st problem: Maria has a photo with her family. His father is at the upper left place of the photo. Maria is between his parents. Her sister is at the down right place of the photo and his brother is at the right hand of her sister. Draw the photo of Maria.

2nd problem: Michael has four books on his night table. The story of Peter Pan is down a story of July Vern. The book of Mythology is between two other books. There is no other book down the book of comics. Which is the arrangement of the books on the desk?

TEACHERS' USE OF THE CONSTRUCT 'ATTITUDE' PRELIMINARY RESEARCH FINDINGS¹

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Abstract: *The paper illustrates the preliminary findings of an Italian Project about attitude. The Project, aimed at investigating the phenomenon of negative attitude toward mathematics, entails various activities, focussed on purely theoretical aspects, such as the very definition of the construct, or on investigations involving students, teachers, mathematicians. Here the findings of a study are presented, which investigates teachers' use of the attitude construct in their practice. These findings suggest that most teachers refer to a multidimensional idea of attitude, not reducing attitude to a simple general emotional disposition toward mathematics, and that the teacher's diagnosis of a student's negative attitude is the final step of a process through which the teacher acknowledges the student's failure, rather than the starting point for a focussed didactical action.*

1. Introduction

Research on affect in mathematics education developed significantly in the last few years. In order to respond to criticism moved by many researchers in the '90s (see Hart, 1989; McLeod, 1992; Pajares, 1992), traditional studies limited to the analysis of specific aspects were complemented by other studies, aimed at clarifying the very nature of the involved constructs, highlighting relationships among the various constructs, and more globally at trying to construct a theoretical framework for affect in mathematics education.

According to McLeod's classification (1992), to which most researchers make reference, attitude is one of the three constructs that compose the affective domain, together with beliefs and emotions².

In actual fact studies on attitude toward mathematics evolved in a way that reflects the evolution of research on affect: from the first studies focusing on possible relationships between positive attitude and achievement (Neale, 1969), to studies highlighting several problems linked to measuring attitude (Kulm, 1980), a meta-analysis (Ma & Kishor, 1997), to end up with recent studies which question the very nature of attitude (Ruffell et al., 1998), or search for 'good' definitions (Daskalogianni & Simpson, 2000; Di Martino & Zan, 2001, 2002), or explore observation instruments very different from those traditionally used, such as questionnaires (Hannula, 2002).

The need for a theory for affect has been given several kinds of answers, differing for both the particular construct explicitly or implicitly chosen as 'starting' point (for example emotions or beliefs), and the different focus. An example of this variety of answers has been given at the Research Forum about 'Affect in mathematics education: exploring theoretical frameworks', at PME 2004 (Hannula et al., 2004): four different theoretical frameworks were presented, according to which affect 1. is a representational system, 2. is one regulator of the dynamic self, 3. is seen in a socio-

¹ This research was funded by a MIUR grant (Progetto FIRB RBAU01S427).

² More recently De Bellis & Goldin (1999) propose 'values' as a fourth construct.

constructivist framework, 4. or is seen as embodied.

As theory in a certain field of mathematics education grows, also grows the tension between practice and theory in that field. Vinner's reaction at the RF on affect quoted above is quite critical, highlighting the risk of developing theories that are not useful for practice: '(...) if a theory (...) does not explain or predict more than what we know intuitively, then it is quite superfluous' ((Hannula et al., 2004, p. 127).

More generally Burkhardt and Schoenfeld (2003), exploring the reasons why 'educational research is not very influential, useful, or well funded' (p.3), conclude their analysis saying that:

'Attending to theory in the proper ways will enhance both our work and the reputation of the field. But theory qua theory will take us only so far (and not far enough). Positioning ourselves so that we can make progress on fundamental problems of practice will make the big difference' (p. 13).

2. An Italian Project about attitude

The points made above about the need for a theoretical framework for affect, together with the importance of linking theory and practice, are fundamental points of an Italian Project about attitude.

The Project, named 'Negative attitude towards mathematics: analysis of an alarming phenomenon for culture in the new millennium', sees the participation of several researchers³.

The project's objective is to investigate the phenomenon of negative attitude towards mathematics, which has much further reaching consequences than the simple learning of the discipline, that affect various aspects of the social context: the refusal of many students to enrol in scientific degree courses due to the presence of mathematics exams, a worrying mathematical illiteracy, and an explicit and generalized refusal to apply rationality characterized by scientific thinking, or, vice versa, to uncritically accept models that are only apparently rational.

More precisely, the project's objective is to investigate both origins of negative attitude towards mathematics and factors that influence its development, through: -A longitudinal investigation on a sample of students, covering the three year duration of the project;

- Collateral investigations performed on other subjects including teachers, family members of the students, adults in general, and professional mathematicians.

These investigations develop throughout six different activities:

1. Investigation of the influence of attitude towards mathematics on the choice of university studies
2. Investigation of the teachers' use of the construct 'negative attitude towards mathematics'
3. Investigation of the attitude towards mathematics of a sample of adults

³ Besides the authors, the researchers participating in the Project are Pietro Di Martino, Pier Luigi Ferrari, Fulvia Furinghetti, Donatella Iannece, Paolo Lorenzi, Nicolina Malara, Maria Mellone, Francesca Morselli, Roberto Tortora.

4. Investigation of the attitude towards mathematics of a sample of mathematics teachers

5. Investigation of the attitude towards mathematics of a sample of mathematics professionals

6. Monitoring over three years of a consistent sample of students.

The methodology entails an integrated method approach⁴:

- the use of questionnaires, diaries and interviews for the observation of teachers and adults;
- class observation, questionnaires, structured and semi-structured interviews, conversations, essays, etc. for monitoring students over the three years.

Both planning and implementation of planned investigations will require a preliminary theoretical reflection regarding the very construct of ‘attitude’, and the setting up of observation instruments that will be continuously developed and tuned during the project, thanks to the information supplied by the experimental data made available.

Starting points for this theoretical reflection are studies carried out by Di Martino & Zan (2001, 2003), that highlight the variety of (explicit or implicit) definitions of attitude present in research, and identify in this variety two important typologies:

- a) A ‘simple’ definition of attitude, that describes it as the positive or negative degree of affect associated with a certain subject. According to this point of view the attitude toward mathematics is just a positive or negative emotional disposition toward mathematics (McLeod, 1992; Haladyna, Shaughnessy J. & Shaughnessy M., 1983). Accepting this definition, it is quite clear that ‘positive attitude’ means ‘positive’ emotional disposition, and ‘negative attitude’ means ‘negative’ emotional disposition.
- b) A ‘multidimensional’ definition, that recognizes three components in the attitude: an emotional response, the beliefs regarding the subject, the behavior toward the subject. From this point of view an individual’s attitude toward mathematics is defined in a more articulated way by the emotions that he/she associates to mathematics (which, however, have a positive or negative value), by the beliefs that the individual has regarding mathematics, and by how he/she behaves (Hart, 1989). In this case what a ‘negative’ or ‘positive’ attitude should mean is not clear: but referring only to the emotional dimension seems lessening. If we choose this point of view, a negative attitude is not only an attitude characterized by a negative emotional disposition (‘I don’t like mathematics’), but also an attitude characterized by an epistemologically incorrect view of the discipline, (i.e. a vision of the discipline that is not shared among experts). Therefore we can define as ‘negative’ the attitude of a student who likes mathematics, if this positive emotion is associated with a vision of mathematics as a set of rules to be memorized.

⁴ The apparatus used for observation will require a careful preparation. It could make use of pre-existing instruments (that have been prepared by some of the participating research units) but will also require the preparation of new instruments: in any case, the long time period available will also permit a continual experimentation and set up of these instruments.

3. Investigation of the teachers' use of the 'negative attitude towards mathematics' construct.

Within the Project, the second activity was conceived to favour a link between theory and practice through reaching two objectives:

- 1) to see whether in practice teachers use the construct of negative attitude when they diagnose difficulty
- 2) if this is the case, see how they use it, investigating:
 - what type of definition they make reference to (in particular, whether they use the 'simple' definition which sees attitude simply as an emotional disposition towards mathematics);
 - if and how the diagnosis of negative attitude constitutes an instrument for intervening in a more targeted way on recognised difficulties.

Method:

These aspects were initially investigated by performing a pilot study, which involved 12 teachers, 2 from middle school and 10 from high school. The teachers were administered a specifically constructed questionnaire to find out their beliefs towards the negative attitude pupils can have towards mathematics. The sample was too small to permit general conclusions to be drawn, and it was also unrepresentative in that all the teachers involved were taking part in a voluntary training course: both the particular context and the interest and motivation that led these teachers to take part suggested that these were all highly motivated and involved teachers.

However, it was exactly for these two reasons that the analysis performed on the data collected for the pilot study suggested some hypotheses to be further studied through a wider investigation.

In fact, the replies highlight that all teachers recognize that they refer to the attitude construct, particularly when 'explaining' the difficulties of a student with mathematics. This construct does not however seem to have the characteristics of a theoretical instrument capable of directing teachers' work (particularly in that of recovering from difficulties): it seems more a recognition of a situation that is difficult to manage and modify.

The pilot study therefore led to a wider investigation. The questionnaire used for this second investigation (see fig. 1) contains 6 multiple choice questions and 6 open ended questions. The multiple choice questions are aimed at discovering whether and how frequently teachers use the 'attitude' construct in the diagnosis of difficulty, and if they consider changing a negative attitude at the end of high school a possible thing. The open ended questions are intended to investigate what idea teachers have of negative attitude, and what indicators they use as reference⁵.

⁵ The insertion of the distinction between negative attitude of a single student and that of an entire class constitutes the most significant change made to the questionnaire with respect to the version used for the pilot study.

Sample:

The final version of the questionnaire was administered at the beginning of 2004 to 146 teachers from various school levels: 29 primary school teachers, 50 from middle school, and 67 from high school.

Fig. 1: The questionnaire

School: _____

M F Age: _____ Date: _____

1. Do you ever find yourself attributing a pupil's difficulties with mathematics to his/her *attitude* towards the subject?
 Yes No
2. If yes, is this a frequent diagnosis or have you only made it a few times?
 practically never rarely sometimes often nearly always
3. What do you mean by *negative attitude* towards mathematics?
4. What demonstrates to you that a student has a negative attitude towards mathematics?
5. Do you think it is possible to modify the attitude of a pupil at the end of high school?
 yes only to a certain extent maybe no don't know
6. If yes, *how*? If no, *why*?
7. Have you ever set yourself the specific objective of changing the attitude of one of your pupils?
 Yes No
8. If yes, how did you attempt to achieve this? What were the results?
9. Up to now we have only referred to a single student. Have you ever seen a negative attitude towards mathematics in a whole class?
 Yes No
10. How did you recognise this negative attitude?
11. If you answered yes to question 9, in this case did you explicitly set yourself the objective of changing the attitude of the class?
 Yes No
12. If yes, how did you try to reach this objective? What was the result?

4. Results

We will only present some preliminary research findings in this paper. The results related to the multiple choice questions refer to questions 1 and 2, which aim to recognise the frequency of the use of the term ‘negative attitude’ referred to a single pupil during teaching practice. The results related to the open ended questions refer to questions 3 and 4, which are aimed at identifying the common characteristics used in the definition of a pupil’s negative attitude and the indicators most widely used by teachers to identify this difficulty⁶.

4.1 The answers to the multiple choice questions

The data relating to questions 1 and 2 are reported respectively in Tables 1 and 2.

Table 1: *Do you ever find yourself attributing a pupil’s difficulties with mathematics to his/her attitude towards the subject?*

↓ Type of school	Yes	No	No answer	TOT
Elementary	24	4	1	29
Middle	47	3	0	50
High	54	12	1	67
TOTAL	125	19	2	146

Table 2: *If yes, is this a frequent diagnosis or have you only seen it a few times?*

↓ Type of school	practically never	rarely	sometimes	often	nearly always	n.a.	TOT
Elementary	4	9	5	3	0	3	24
Middle	0	12	18	10	0	7	47
High	0	11	24	8	2	9	54
TOTAL	4	32	47	21	2	19	125

The first table confirms the wide use of the term attitude in relation to the diagnosis of a pupil’s difficulty in mathematics, already found in the pilot study: 85,6% of the sample (125 out of 146) in fact gave a positive answer to the first question. A comparison of the answers given by teachers from the various school levels highlights a peak in the positive answers at middle school level (47 teachers out of 50, equal to 94%), with respect to the homogenous answers provided by the elementary and high school teachers (24 out of 29, equal to 82,8%, and 54 out of 67, equal to 80,6% respectively).

The second table confirms the importance of frequency in the use of the construct: 70 teachers out of 125 (equal to 56%) answer that they attribute a pupil’s difficulties

⁶ Therefore no consideration is given to the distinction between negative attitude of a single student and a negative attitude of the whole class.

with mathematics to his/her attitude toward the subject sometimes / often / nearly always. A comparison between the various school levels highlights that the number of responses of that kind (sometimes/ often/ nearly always) increases with schooling level: 8 questionnaires out of 21 (equal to 38,1%) at elementary school level, 28 out of 40 (70%) at middle school level, and 34 out of 45 at high school level (75,6%).

4.2 The answers to the open ended questions

The analysis of the answers to the open ended questions was performed according to an interpretative approach (see Bruner, 1990), aimed at identifying teachers' beliefs, opinions, and values, in order to understand their intentional actions, rather than at explaining their behaviour in terms of cause / effect. Final outcome of this analytical process is the construction of a set of categories, properties, relationships: a *grounded theory* (Glaser e Strauss, 1967), i.e. a theory based on collected data, the construction of which requires a continuous back and forth between the different research phases. In our case answers were read in the light of both pre-existing categories (for instance the distinction between simple and multidimensional notion of attitude) and in a free way, trying to identify meaningful categories *a posteriori*.

The reading of the answers to question 3 ('What do you mean by *negative attitude* towards mathematics?') suggests two dimensions for the analysis, both significant for the aims of the study:

1. The first dimension is the type of the given definition. According to this dimension three significant types of answers can be identified:

1.1 The answers which seem to define only the adjective 'negative', assuming the term 'attitude' as known. Sometimes this kind of answer is recognized because of the repetition of the term 'attitude' in the answer itself ('What do you mean by negative attitude towards mathematics?' 'A *refusal attitude*'). In these, very frequent, cases the most used words are 'refusal', 'opposition', 'lack of will'.

1.2 The answers which seem to define the whole expression 'negative attitude'. This type is characterized by the use of words which refer to:

- a student's beliefs about mathematics (the belief that being quick in mathematics is very important, that mathematics is made of mechanistic rules, that mathematics is useless, difficult, ...)
- a student's beliefs about his/her own capacities (he/she believes to be inadequate, to be destined to failure, not to be able to understand,...)
- a student's emotions (boredom, aversion to mathematics, anxiety, fear,...)
- a student's behaviour (mechanical application of rules, use of an inadequate method for studying, absent-mindedness, homework not always completed...)
- a student's characteristics (lack of will and intuition, ...).

1.3 The answers that do not seem to be linkable to definitions. For instance: ['What do you mean by *negative attitude* towards mathematics?'] "The sentences: 'I am not inclined to mathematics' 'Since primary school I always failed in mathematics', 'I

don't understand mathematics” [Q125,H⁷]. The example shows that some of these answers that cannot be linked to definitions attempt to provide an answer to the next question of the questionnaire, i.e. ‘What demonstrates to you that a student has a negative attitude towards mathematics?’

These three types are often simultaneously recognizable in the same answer, and this highlights the complexity of giving a definition of ‘negative attitude’:

‘The very word ‘mathematics’ terrifies. A priori refusal of every proposal of activity, also without knowing anything about it. Often also proposals of aid for a possible recovery are refused. Often it is about a negative mental attitude, not a constructive one.’ [Q 129, S]

2. A second independent dimension is that related to time. This dimension leads to the distinction between:

2.1 The answers which explicitly refer to time. This reference – very rare – is marked by linguistic indicators such as ‘always’, ‘often’, ‘never’:

‘When the student is always unfocused’ [Q109,M].

2.2 The answers, which constitute the most part of the sample, without this kind of reference.

Comparing the answers given by a same teacher to Question 3 and Question 4, confusion between the two, that we pointed out above⁸, can be highlighted together with the complexity of a distinction between ‘defining a negative attitude’ and ‘identifying observable behaviour as indicators of a negative attitude’.

Also in the case of question 4, the reading of the questionnaires suggests two dimensions for the analysis, both significant for the aims of the study.

3. The first dimension is the reference to indicators of a negative attitude, that leads to distinguish between:

3.1 Answers which refer to students’ observable behaviours.

3.2 Answers which do not refer to observable behaviours. In this second type, that is the most frequent, most answers refer to the results of the teacher’s interpretative processes: *‘he/she refuses to tackle new exercises’, ‘he/she cannot understand’, ‘he/she gives up immediately’, ‘he/she is afraid of a negative judgment’, ‘he/she is not capable of taking advantage from lessons’.*

In this case again the second dimension refers to time, and leads to a distinction between:

⁷ The last letter indicates the type of school: E (Elementary School), M (Middle School), H (High School).

⁸ For instance [Q91,M]: ‘What do you mean by negative attitude towards mathematics?’ *‘I cannot do it», «This is not for me», «I will never be able to do this»*. ‘What demonstrates to you that a student has a negative attitude towards mathematics?’ *‘He works unwillingly, he always loses confidence’.*

4.1 The (few) answers which explicitly refer to time:

'He very often declares to have no gift for mathematics' [Q18,S]

4.2 The answers, the most, which do not explicitly refer to time.

Summarizing, qualitative analysis of open ended questions suggests that:

- If present, the definition of 'negative attitude' is not operative, since it does not favour the identification of observable facts and behaviour.
- Very often the teacher's observation process is strongly influenced by his/her interpretative process.
- Reference to time is almost completely lacking.

As regards the characterization of a negative attitude toward mathematics, some recurrent aspects emerge:

- Lack of self-esteem, lack of self-efficacy in the context of mathematics;
- Lack of interest, scarce curiosity, lack of motivation;
- A view of mathematics as a schematic, non creative discipline.

In other words, answers somehow refer to categories that characterize the multidimensional definition of 'attitude' (Daskalogianni & Simpson, 2000; Di Martino & Zan, 2001, 2003), although in a fragmentary way: beliefs about self, beliefs about mathematics, and emotional factors.

5. Conclusions

As we have already said, the results here presented are only preliminary findings, that will be completed with both the analysis of the other questions in the questionnaire, and with cross-cutting analysis of different answers (by the same teacher), and with a quantitative analysis of open ended answers, aimed at highlighting the importance of the qualitative analysis performed.

The present study highlights some crucial points of the problem we tackled:

- Most teachers refer, even implicitly, to the multidimensional idea of attitude, rather than to the 'simple' one.
- The teacher's apparently 'natural' use of the 'negative attitude' multidimensional construct goes together with a lack of a clear distinction between the definition of attitude and the identification of indicators, thus making the definition itself not operative.
- Ascribing the causes of a negative attitude to students' characteristics and behaviours hides the teacher's responsibility in building a view of mathematics that elicits refusal, in the lack of interest and effort by students, in the image of the self that students construct.
- The diagnosis of 'negative attitude', referred to a single student, seems to be the final result of the teacher's interpretative process of the student's failure, rather than the starting point of a remedial action.

Further investigation is required to confirm these provisional hypotheses, for example performing individual interviews, capable to give deeper insight on the points made above.

Yet the presented results, although incomplete, suggest the importance for teachers' practice of the 'negative attitude' construct. But they also suggest the importance of making this construct become a theoretical tool capable of directing observation, interpretation, remedial actions. Therefore the theoretical studies about attitude, planned in the Project, are not 'theory qua theory', but can foster progress on an important problem of practice.

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META-AFFECT AND STRATEGIES IN MATHEMATICS LEARNING

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Abstract: *The concept of meta-affect introduced by DeBellis and Goldin describes complex effects of affect on our actions and thoughts. Ciompi's concept of affect logic together with neuroscientific research helps us explain the emergence of meta-affect as a consequence of assimilation and accommodation processes. The explanations are used to understand learning strategies of students who have problems learning mathematics, particularly "learning without understanding".*

1. Meta-affect

The concept of meta-affect was introduced by DeBellis and Goldin (1997). Just as meta-cognition describes how cognitive mental systems are handled, meta-affect describes how the mind handles affect. Goldin describes meta-affect in the following way: "An idea that has assumed a central role in our thinking is *meta-affect*, referring to affect about affect, affect about and within cognition that may again be about affect, monitoring of affect both through cognition and affect. Our hypothesis is that meta-affect is *the most important aspect of affect*" (Goldin, 2004; 113).

Meta-affect is a very complex concept – affect about affect, affect about and within cognition and monitoring of affect: in a nutshell, the notion encapsulates the ability of humans to handle affective situations (see also the description of aspects of meta-affect in Goldin (2002)). To gain more insight into this complex concept, we use the concept of "affect logic" developed by the Swiss psychiatrist Ciompi (1982, 1988, 1991, 1999).

Ciompi combines Freud's psychoanalysis and Piaget's genetic epistemology, which was based on system theory. The following is a brief summary of the most important aspects of the affect logic:

- * The psyche is seen as a unit. This means affect and cognition, feeling and thinking are inseparably combined, though dissimilar in nature.

- * The psyche is understood as a complex hierarchical structure consisting of affective-cognitive schemata. These affective-cognitive schemata are the result of maturation and learning processes based on assimilatory/accommodatory interactions with reality.

- * The affective-cognitive schemata are condensed to affective-cognitive reference systems that form an individual's "world view" and control action and thought.

- * The affective-cognitive reference system is structured by both the affective and the cognitive components. Therefore, access is possible through both components, but the access is often not complete because certain parts of the system are unconscious.

(Schlöglmann, 2000, 2004)

The term ‘affective scheme’ is also used by Hannula (1998, 2002): “Affective schemas are schemas that direct the formation of emotions. Some affective schemas are innate while some are learned automatisations of repeatedly generated emotions”. In the same sense as Ciompi, Hannula also describes the effect of an affective scheme, but Ciompi – using the term “affective-cognitive” – emphasises more strongly its interrelation with cognition, and furthermore, basing his argument on Piaget’s theory, refers to mechanisms of the development of such a scheme.

2. Affective-cognitive schemata

2.1. A short description of Piaget’s concept

For Piaget, affect is only an energy supplier: “Therefore one could say that the energetic of behaviour arises from the affectivity, whereas the structure comes from the cognitive functions.” (Piaget, 1995; 25; Translation W.S.) Ciompi’s concept extended the influence of affects to general effects on thinking, as well as specific effects comprising the basic feelings (interest, anger, fear, sadness and joy (Ciompi, 1999)). In general, affects influence cognition like operators. They provide the energy through which cognitive processes are motivated or hindered. They control attention and memory processes, and influence the hierarchy of cognitive schemata. On the other hand, Ciompi speaks of special “logics” of the basic feelings (for instance, “fear logic” or “logic of joy or anger”). The term “logic” expresses the view that the kind of thinking is different if a special feeling dominates.

To study the influence of the affective-cognitive system, we go back to Piaget’s concept of equilibrium, assimilation and accommodation that he developed for cognition. The central idea is that on the one hand, all systems make an effort to attain and remain in equilibrium with their environment; while on the other hand, systems must interact with their environment and these interactions can disturb the equilibrium. To restore the equilibrium, system-specific reactions are required. These system-specific reactions are “assimilation” and “accommodation”.

Piaget distinguishes two forms of assimilation. The simplest one needs only a simple application of a scheme in the cognitive system. Each new successful application of a scheme extends the application field of the scheme and leads, therefore, to a generalisation. But in this simple case, no change of the scheme is required in return. The second form of assimilation is the so-called “reciprocal assimilation”. This kind of assimilation leads to co-ordination of subsystems or to the integration of a subsystem into a more general system. According to Piaget (1976, 14), every assimilation scheme has the tendency to grow; i.e., to assimilate elements that are compatible with its nature. That means that for a subject, activity is necessary, too.

Accommodation is the second system-specific type of reaction of a system to the demands of the environment. An accommodation process is necessary if a problem is not solvable by assimilation and requires alteration of the system. Piaget summarises the relationship between assimilation and accommodation by stating that every

assimilation scheme is forced to accommodate the specificity of various elements. Consequently, these elements become assimilated. This implies the necessity of an equilibrium between accommodation and assimilation, which in turn implies that the goal of all assimilation/accommodation processes is a new, more stable equilibrium, since in such an equilibrium, one is able to solve more problems.

2.2. Development of affective-cognitive schemata

The concept of affect logic transfers Piaget's system-specific reactions (assimilation and accommodation) from cognition to the development of the affective part of an affective-cognitive scheme. If activity is necessary for each human, activity must be seen as emotionally valuable. The basic feeling of "interest" provides, in principle, the positive energy required for discovering new things. In evolutionary terms, humans are open to discovering their environment, to acquiring new experiences – i.e., to learn (Wimmer and Ciompi, 1996). The basic feeling of "interest" also motivates learning processes. This readiness, which exists in principle, to discover the "world", to learn new things, can be disturbed by negative experiences in relation to earlier learning processes. In such cases, negative feelings slow down new learning processes and can turn into an obstacle – an affective obstacle – for further progress. Let us now discuss the assimilation and accommodation process with respect to affects. For Piaget, the simplest form of assimilation is a process that only requires the application of an existing cognitive scheme to a new situation. Repeated application leads, in the cognitive case, to stabilisation of the cognitive scheme. The person thereby acquires a routine for solving a special problem. In the affective case, the origin of a scheme as the product of a successful problem solution process is linked to positive feelings. Successful thinking processes and learning processes are delightful. But repeated application of a scheme leads to "emotional neutralisation" of the positive feeling, in some cases to negative feelings (to solve routine tasks is often seen as boring). We know from our experiences in everyday life that these actions are often routines, and that these routines are combined with a low level of emotionality (Ciompi, 1999). But to handle routine tasks can nevertheless lead to affective reactions, for instance if the process is disturbed by slip-ups. The emotional reactions in this case are local, situational and mostly do not last long. Such local affective reactions accompany all processes of acting and thinking but only repeated positive or negative experiences lead to the affective part of the affective-cognitive scheme or a global affect (Goldin, 2000). On the other hand, we ought to note that every successful application of a scheme to a new situation leads to an extension of the application field and to more information about the affective side of the scheme. Thus meta-affect arises.

If we consider reciprocal assimilation from the affective perspective, we find a much more complicated situation. The integration of subschemata into a more general scheme, or the assimilation of a scheme into a more general scheme, is a very complex process, cognitive as well as affective. This process has close connections with the problem of context-related learning and school learning, as well as routine

and non-routine problem-solving processes. The problem of context-related learning versus school learning is founded in the question of the relationship between the special and the general, the concrete and the abstract. This is currently an area of intensive study, especially in the case of mathematics learning. Here, the clash is between mathematics as a special tool to be applied only in a special field, and mathematics as a general tool for many problem situations (Lave and Wenger, 1991; Evans, 1999).

Mathematical problem-solving is one of the key problems in mathematics education (Schoenfeld, 1985). With regards to cognition, solving non-routine problems necessitates competence in generalisation and abstraction. With regards to affect, Goldin (2000) has sketched the problem-solving process from the affective point of view. A process such as reciprocal assimilation needs strong support from the basic feeling “interest” as well as self-confidence in oneself as a problem solver. (For insights into the complexity of concepts of self and their dynamic influence on affect and cognition in mathematics learning, see (Malmivuori, 2001)). This is a crucial point because the problem solver also has to handle frustrating situations. Bringing such situations to a successful conclusion is only possible if the learner has a positive conception of him or herself. But the process is very sensitive to failure, and can, following unsuccessful results, lead to negative global traces of local affect (Goldin, 2000), which influence learning strategies as well. The process of abstraction and generalisation, a central requirement for successful problem-solving, is one of the most difficult steps in a mathematical learning process. It is one of the origins of negative beliefs and attitudes in relation to mathematics.

3. Meta-affect and affect logic

Affect logic postulates an affective-cognitive reference system consisting of affective-cognitive schemata that control our action and thought. This reference-system is the result of assimilation and accommodation processes and is hierarchically structured by the affective and cognitive component. The concept of meta-affect refers to this hierarchical structure as well as to its affective and cognitive part. For further insights, we ought to look at the situation from the neuroscientific point of view. According to this view, there exist two different systems, cognition and emotion. Both exist as a result of biological evolution, with the aim of aiding the individual’s survival (Wimmer and Ciompi, 1996; Damasio, 1999; LeDoux, 1998; Roth, 2001). Although located in different parts of the brain (Damasio 1999; LeDoux 1998; Roth 2001), there are connections between both systems that allow interactions. A very important consequence of the existence of these two systems is that we have to distinguish between “feeling” and “knowing that we have a feeling” (Damasio, 1999; 26); or “emotional reactions” and “conscious emotional experience” (LeDoux, 1998; 296).

Furthermore, we should note that although all processes on the neuronal level are unconscious, some of these processes lead to conscious results. We are aware only of

these conscious parts of the processes. For remembrances, too, two memory systems exist with respect to emotions: an implicit emotional memory and an explicit memory of emotions (LeDoux, 1998). The implicit emotional memory operates unconsciously, is strongly connected to arousal systems and may often lead to bodily reactions. The explicit memory of emotional situations contains all the conscious knowledge of emotional situations, emotional reactions to objects, persons and ideas etc.. The most important consequence of this is that this memory system is part of the cognitive memory and there is no distinction between a remembrance of an emotion and a remembrance of cognitive content (LeDoux, 1998). The fact that memory of emotions is cognitive has important consequences:

- 1) We have knowledge about our feelings, their origin and their effect. This knowledge is stored in memory systems as cognitive knowledge.
- 2) Memory of emotions is open to “rational” manipulation. That means we are able to think about our emotional remembrances, and that all verbal statements about emotional facts are controlled by cognition.
- 3) Knowledge of our affect with respect to objects and situations allows us to handle our affect at least in controlled situations (see Goldin’s example of the roller coaster experience (Goldin, 2002; 62)).
- 4) Humans are able to “construct” their remembrances in a way that they are able to live with this memory. Part of this process is forgetting unpleasant facts more easily than pleasant ones: our memory has suppression mechanisms to handle unpleasant remembrances (Roth, 2001).

Turning now to meta-affect, assimilation and accommodation processes lead to affective-cognitive schemata. The affective component is stored in two memories: in the implicit memory that works unconsciously but influences our action and thought (Damasio developed the concept of “somatic marker” to explain this (Damasio, 2004; Brown and Reid (2004))); and in the explicit memory that stores all the knowledge of affect with respect to people, objects and situations. Affective-cognitive schemata always contain both the unconscious and the conscious components. Repeated assimilation and accommodation processes in relation to a special problem leads to consolidation of the unconscious reactions, as well as to more and more conscious knowledge of feelings and emotional reactions. It provides information on the outbreak of emotional reactions and allows the development of strategies for handling such situations. Malmivuori (2001, 2004) describes the functioning of self-concepts by distinguishing further an unconscious and a conscious share in the regulatory process: “Thereby, affective regulation represents lower level or more automatic self-regulatory processes with weak self-control beliefs or personal agency and lower state of self-awareness, while active regulation of affective responses relates to enhanced self-control beliefs and high personal agency with efficiently integrated self-regulatory processes and promoted self-awareness” (Malmivuori, 2004; 117). Assimilation and accommodation processes are also a necessary prerequisite to develop hierarchical structures – meta-structures. This meta-knowledge allows us to

use our affects in a conscious way. As we know, emotional reactions are also used by humans to produce desired results in social processes (Goldin, 2002).

In summary, we have seen that meta-affect arises as a consequence of the hierarchical structure of the affect logical schemata. Our cognitive knowledge of our affect allows us to control our actions in affective situations. Successful handling of affective situations stabilises by assimilation the affect logical schemata, and consequently beliefs, as our cognitive window to emotions (Schlöglmann, 2004; Goldin, 2002).

4. Meta-affect and learning strategies

4.1. Interview, and a preliminary analysis

First, let us consider the following excerpt from an interview with a young adult student undertaking a program to prepare participants for vocational education in the Austrian “dual vocational education system”. A mathematics course is part of this program. The student had just sat a test in basic mathematics, and the questions in the interview refer to one of the tasks in the test; namely, a percentage calculation (I. is the interviewer, M. is the student). The aim of this interview is to obtain hints regarding the relationship of meta-affect, belief structure and learning strategies.

[Task 6a: What is 20% of 500 shillings?]

I.: *Task 6a involved percentages: what did you do here?*

M.: (M looks at her paper). *I should calculate 1%, but how?*

I.: *You know 100% and want to know how much 1% is.*

M.: *Times 100, um, divided by 100, um, no times 100.*

I.: *You know 100% and want to know how much 1% is.*

M.: *500 divided by 100, I don't know, I don't know what to say.*

I.: (Gives a hint from the calculation in the test): *You have chosen the right formulation (as a rule of three). What would you have done if you had known that 1% is this many shillings and you want to know 20%?*

M.: *Yes, then I would have multiplied by 20.*

I.: *And how do you get from 100% to 1%?*

M.: *Um.*

I.: *You divide by 100.*

M.: *Hmm.*

[Task 6b: What is the total amount if 300 shillings are 20%?]

I.: *Now we come to task 6b.*

M.: *This is the same, I guess? I suppose it is 100 again.*

I.: *300 shillings are 20%.*

M.: *How much is 1%? Divided by 100.*

I.: *No.*

M.: *Times 100.*

I.: *No.*

M.: *Times 20.*

I.: *How do you get from 20% to 1%?*

M.: *80.*

I.: *No.*

M.: *I don't get it.*

I.: *Think of task 6a. How did you get from 100% to 1%? Divide by 100.*

M.: *Hmm.*

I.: *How do you get from 20% to 1%?*

M.: *Divide by 20%.*

I.: *Now you know how much 1% is. How many percent do you want to calculate? The full amount. What percentage is the full amount?*

M.: *20%.*

I.: *No.*

M.: *100%, but how?*

I.: (Explains the task.) *What do you feel is hard about task 6?*

M.: *If I knew the formula to calculate 1% and 100%, then I'd know how to do it, but as it is....*

Let us first briefly analyse the interview (for further interpretation see Schlöglmann, 2000). The student first attempts to solve problem 6a by multiplying by 100, discards this solution immediately and divides by 100, then corrects herself again by multiplying by 100. With the interviewer's help, M. would eventually give the correct answer (not in the above excerpt), but would be uneasy about the correctness of it (as we may see in M.'s first line in the second part). M.'s comments and answers indicate that she does not understand the concept of percentages.

In the second task, M. invokes the method that was successful in solving the first task without thinking about the new structure of this exercise. Because the interviewer rejects M.'s first answer, she tries a different operation (multiplication). The interviewer rejects the new answer and M. tries another number (20). The number which, in M.'s conception, is connected with percentages (100) works with neither division nor multiplication. So M. takes a number given in the question and tries to solve the problem using one of the two previous operations. Because the interviewer again doesn't accept her answer, M. tries a new strategy, linking the two numbers 100 and 20 by the operation of subtraction. (M. presumably takes subtraction as the operation because multiplication would give a number that seems too large.) M. eventually gets the correct solution, with the interviewer's help. (I would be surprised if the interviewer's explanation had improved M.'s understanding of percentages. A hint that my interpretation could be correct is the fact that M. supposes that the use of a formula would help her solve the problems correctly).

4.2. Meta-affect and learning strategy

From the affective perspective, we can identify two types of reaction in this interview. First, M. always tries to give an answer even when she doubts the

correctness of it. M. is always active. For many people, activeness is a means of reducing stress. Activeness can show an interlocutor that one is doing something and trying to solve the problem.

The interview also provides hints of M.'s learning strategy in the mathematics classroom. M.'s conception of mathematics does not include the notion that a learner of mathematics should understand mathematical concepts. On the contrary, to M., mathematics means having a formula, and the learner merely has to put the right numbers into the right positions and crank a handle. The answers in the interview suggest that M. does not have sufficient understanding of the meaning of operations like subtraction, multiplication and division. M. does not, for instance, build a model of a situation, nor "translate" such a concrete model into an abstract mathematical model; on the contrary, she combines numbers in the task using operations without thinking about their meaning in the context. We may suppose that such handling of a mathematical task is a consequence of a learning process, and has its origin in meta-affect connected with mathematical problem-solving.

Meta-affect is strongly connected with acting in emotional situations (monitoring of affect). For many students who have negative experiences with problem-solving, it is important, on the one hand, to cope with such highly emotional situations, and on the other hand, to restrict the effects of failure on their conception of self. As a result, such students develop a belief structure (Goldin, 2002) that becomes a foundation for their problem-solving ("all mathematical problems need a formula and each formula combines numbers and operations") and learning strategies ("try to learn pictures of formulas without understanding their mathematical meaning").

This leads to the question: "Is this kind of learning – learning without understanding – possible?" Neuroscientific research has found hints that such learning is indeed possible (Schacter, 1990, 1999; Tulving and Schacter, 1990). The basis of this kind of learning is a perceptual representation system (PRS). This perceptual representation system is specialised for processing the form and structure of words and objects, but it "knows" nothing about the meaning of the words and the use of the objects. The PRS and semantic memory (i.e. stores meaning) systems usually collaborate closely. Furthermore, the PRS is very sensitive to perceptual properties of target information: "...access to information in PRS is hyperspecific, probably because, unlike other cognitive memory systems, it contains no abstract focal traces" (Tulving and Schacter, 1990; 302). The belief that one is unable to understand mathematics and that one has to find a strategy for handling mathematical tasks, particularly in test situations, leads to a meta-affect that uses the perceptual representation system for handling mathematical tasks.

But we know from the characteristics of the PRS that the retrieval process is very strongly dependant on perceptual properties. We can therefore understand why changing the letters used to represent variables, or altering the word order of word problems, leads to inability to solve the "new" task. Considering the classroom situation, many teachers know that some of their students have these kinds of problems. The teachers try to help them by training them to solve routine problems

with nearly no changes in the way the problems are represented. Students get the feeling that if they look at the form of a task and of the steps of the solution process, they can succeed at least in routine situations. This success leads to a strengthening of the meta-affect. It is like a feedback loop out of control: this method of teaching and learning leads to a small measure of success in special situations; this success strengthens the meta-affect; and the meta-affect leads to demands for this teaching method (and consequently to the propagation of this learning method). The main problem here is that this knowledge is unhelpful outside school – it only helps the student to pass tests (Schlögmann, 2005).

5. References

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