WORKING GROUP 10
Mathematics education in multicultural settings

CONTENTS

RESEARCHING DIVERSITY IN MATHEMATICS EDUCATION

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CERME 5 took place in Larnaca (Cyprus), in February 22-26, 2007. This was the third time WG10 – *Mathematics Education in Multicultural Settings* met and discussed issues related to this theme. And although the designation of WG 10 slightly changed over these three CERME meetings, the focus of the group was always the same: studying, understanding, and problematizing multicultural issues and the conditions and contributions of an intercultural education. Thus, the changes in the designation of the group only illuminate its members concern in finding a designation that really corresponds to the aims of the group, illustrating how we are putting our efforts in order to achieve more accurate theoretical grounds.

As coordinators of WG 10 we tried to continue some of the discussions we had in previous meetings, namely the ones related to theoretical clarification, methods (including the researcher's role), mathematics specificity and implications for future research. This time, we had nine papers and one poster that were accepted for presentation. Unfortunately our colleagues from Denmark, Helle Alrø, Ole Skovsmose, and Diana Stentoft, could not join us in Cyprus due to the weather conditions that closed the Danish airports for some time. But as their papers are included in CERME 5 Proceedings, the questions raised by them are also considered in this introduction. We also consider the questions that emerged from the papers that were accepted for presentation but not for publication, and the poster, as they were part of the work of WG 10, in Cyprus.

Two papers explored the construct of social representations. The one by Diana Stentoft did it from a theoretical point of view; and the other paper, by Guida de Abreu and Núria Gorgorió illustrated some of the theoretical statements with the analysis of several examples. Abreu and Gorgorió's paper examined how the notion of social representation could offer useful insights into understanding practices of teaching and learning in multicultural mathematics classrooms. Julian Williams and his colleagues discussed how mathematical identities could be storied drawing on the notion of cultural models (Gee, 2001; Holland, Lachicotte, Skinner, & Cain, 1998; Holland & Quinn, 1987). The paper discussed some of the cultural models we use when building our narratives of identity and how do we identify or dis-identify with mathematics. It also aimed at understanding the institutional features mediating the learning of the students (e.g., institutional constrains, like the ones existing in English schools that do not allow students, at a certain level, to attend Maths A).

From this group of papers emerged one of the theoretical issues discussed in WG 10: the role of social representations and of cultural models in the (re) (co)construction of identities. We discussed which were the common points and the differences between social representations (Moscovici, 1986, 1989, 2000, 2001) and cultural models, their respective theoretical backgrounds, how these two constructs were used in mathematics education research related to multicultural settings, and also how they could be used in future research. We also discussed some terminology, and if the terms used were consistent with the theoretical ground that supported the research, or not. One of the issues was raised by the use of the word "*disposition*" in the instruments elaborated by Julian Williams and his colleagues, and the discussion turned around if they were talking about "*dispositions*", as the term was usually used in traditional psychology, or more of "*self positions*", as Hermans (1996, 2001a, 2001b), or Hemans and Kempen (1998) use it in the theory of the dialogical self, which could be easily connected to Williams and his colleagues' study, or even to Marková (2005) notion of social representation as closely related to the one of dialogism. This led us to the discussion about identity/identities, and about the role of narratives and metaphors related to identity/ies (Williams & Wake, 2007), or the role of dialogism in multicultural settings (César & Santos, 2006; Hermans, 2001a; Hermans & Kempen, 1998). Finally, we also tried to raise and discuss questions that related identity/ies with mathematics learning, and also with participation, namely the ones connected with constructs such as community of practice, learning community, or legitimate peripheral participation, coined by Lave and Wenger (1991), but also discussed by authors like Abreu (1999), Boealer and Greeno (2000), or Roth, Hwang, Lee, and Goulart (2005), and that were also discussed in the poster presented in WG10; or those related to an inclusive perspective and the notion of self-awareness (César & Santos, 2006).

But this group of papers also raised some methodological issues: (1) the complexity of some experimental designs, like the one by Williams and his colleagues that combines quantitative analysis, including the elaboration of data collecting instruments, with an interpretative/qualitative longitudinal study of 50 participants, including the analysis of the interviews, conceived as biographic narratives, related to mathematical activities, and observations; (2) this complexity also raised the question about how can connections be made between the two parts of the study; (3) and what are the theoretical challenges of such a study; (4) how methodological decisions are influenced by demands/ pressures that come from the policy departments, external funding agencies and political power; (5) in studies including families, up to what point can we go? And as we enter in those families lives/homes, sometimes becoming part of their practices, up to what degree do we shape what is happening? What are the ethical issues that need to be considered in these studies?; (6) what is the role of theoretical studies? What contributions can they provide for the theoretical clarification? What constructs need further studies?; (7) what is the researcher's role in quantitative studies? And in qualitative studies?.

The paper by Alrø and her colleagues also raises theoretical issues, namely those related to the notion of *learning landscape* (Alrø, Skovsmose, & Valero, 2005). The authors provide a discussion of the diverse dimensions of this notion and also some examples about its use in educational settings, namely in multicultural schools. Unfortunately these colleagues could not join us in Cyprus, and for this reason the group considered that these theoretical and empirical issues would be part of the agenda for another meeting, probably in CERME 6.

The poster presented by Rojas and Prat referred to two case studies and aimed at discussing: what is a community of practice; the contributions of situated learning (Lave & Wenger, 1991) to understand multicultural classes and the practices that are implemented in mathematics classes; the role of participation and the different kinds of participation that may emerge from different types of teachers' practices; and the role of negotiation and norms (Yackel & Cobb, 1996; Gorgorio & Planas, 2005), within different types of practices. The two cases were contrasting ones and they illuminated diverse evidence.

The poster presentation also raised some methodological questions from other colleagues: How do we select case studies, and why? or how we conduct and report analysis? But these questions were not discussed as we had no more time. Thus, the group decided they were important issues that could be discussed on a web basis, or/and in the next CERME meeting of WG 10.

Moreira's paper raised questions related localisation/ local mathematics vs. globalisation/ global mathematics. From these two notions – local and global mathematics – emerged several questions, raised by the WG 10 members. Some of the questions focused on the relation between these two notions and the theoretical background of ethnomathematics (D'Ambrósio, 1997; Gerdes, 1996), and critical mathematics. According to Moreira's interpretations of these two notions, the gypsies communities were given as an example of local communities that assumed particular ways of mathematical reasoning, and used particular solving strategies for mental arithmetic computations (Cadeia, 2006; Ferreira, 2003; Pires, 2005). Globalisation was seen as a way of enabling access to communities outside the local gypsy community, and to the different scales of society.

The discussion of this paper also raised some methodological questions: How does research have access to the practices of some particular cultural groups? How do we interpret does practices? How do we communicate/ write about them? Are there any particular methodological features when we are studying these particular local communities and assuming an ethnographic approach?

Georgiou's work focused on a different issue: the nature of the tasks presented in text-books, namely pseudo-problems. The group discussion focused on the following points: (1) the approach that seemed more adequate to her argumentation was critical mathematics, and this was not explored; (2) the nature of the task is one important feature, but how the tasks are explored is even more essential, as a task, by itself,

could not make sure that classroom practices and discussions (or the lack of it) would be meaningful for the students; (3) the differences between social representation and/or cultural models and the way students interpret the tasks that are presented in text-books; (4) the differences between research and discourse, and practices, namely in different countries, with different cultures; (5) the maths problems that are usually seen as stories and, in multicultural classes, some of these stories may not be meaningful to all students, stressing the need to use diverse stories in order to respect and value diverse cultures. This last point raised the issue that it is quite difficult to provide word problems – or any other mathematical task – that were meaningful to a lot of students, and the need to consider the teacher's potential to explore the tasks in different ways, so that they could become more meaningful and also more representative of a critical perspective of mathematics.

The methodological issues raised by this work stressed the need to contextualise the text-books – or the schools, classes, students – in order to allow the reader to interpret what is stated by the researcher. Another discussion that came from this work was about assessment, and the relations between text-books and exams, or other forms of general assessment.

Thaqui's and his colleagues work focused on the complexities of implementing a multicultural mathematics curriculum, in a culturally diverse society, where different school systems co-exist. Three Kosovan schools from different cultures, including a different language, religion, educational systems, etc., were used to illustrate different practices of mathematics teaching. The complex theoretical concepts and operationalisation of the study raised several questions that made the group discuss about interpretative/qualitative approaches, and the validity and reliability questions they raise. From this study also emerged other methodological questions: What is the importance of comparative studies, among cultures in the same country, or even among countries? How should they be designed? What is asked from a researcher in order to be able to be effective in this type of research (e.g., Should he speak and know well the different cultures under study? Should his/her interpretations be confronted to other interpretations from an outsider?).

These questions were also related to the paper presented by Aubrey and her colleagues, as this was a comparative exploratory study among Chinese, English and Finish children (60 months old) about number sense. They used *Utrecht Early Numeracy Test* (UENT) by van Luit, van de Rijt, and Penning (1994), that comprised 4 numerical and 4 Piagetian-type sub-tests. Several studies do comparisons among children from different countries, namely studies connected with number sense and numeracy, using different types of instruments to assess them (see, for example, Aubrey, Godfrey, Kavkler, Magajna, & Tancig, 2002; Aunio, Ee, Lim, Hautamäki, & van Luit, 2004; van de Rijt, Godfrey, Aubrey, van Luit, Ghesquière, Torbeyns, Hasemann, K., Tancig, S., Kavkler, M., Magajna, L., & Tzouriadou, 2003).

This study raised several methodological questions, such as: (1) What is a representative sample, namely when we are comparing different countries? How do we have reliable samples?; (2) Is the sample more representative of a country – when collected at a specific town/area – or from that town/area?, i.e., aren't there, sometimes, more significant cultural differences within a country than when we compare different countries, but from similar socio-economic and cultural backgrounds?; (3) How are minority cultures shaped by these comparative studies?; (4) What would we learn if we did a comparative study in a school, but comparing different communities (e.g., in an English school to compare the Chinese, Portuguese, English and Pakistan communities)? (5) How can we have reliable instruments, namely when we are using children from very different cultures – which is related to the problems of translation and adaptation.

These methodological questions were also related to some theoretical ones: (1) What does it mean being English? or Chinese? or Finish?; (2) What other elements shape the number sense? Are they more or less important than nationality?. These questions made the group briefly remember and discuss international studies such as TIMSS or PISA, and their impact on the populations that were ranked according to those results.

The WG 10 members considered that quantitative studies could illuminate problems that could then be studied more in-depth by using interpretative/qualitative methods. But we also discussed other difficulties and constrains of interpretative/qualitative studies: (1) Not all practices are available; (2) As researchers, what should we do to be aware of cultural differences among countries, schools, institutions and to report on them in an adequate way in our research?; (3) When we are characterising a particular cultural group (e.g. gypsies; Chinese) how can we avoid skipping into stereotypes, creating new discriminations and differences for those who do not follow the pattern we are describing?; (4) How can we get familiar the unfamiliar?; (5) How can we have a better understanding of inequities? Should we just understand them better or should we be able to design intervention studies?

One interesting feature to analyse comparative data would be to have participants from different countries going to one another countries involved in the study and using diverse cultural lens to analyse the data. This could be done both for quantitative and/or qualitative studies.

In short, WG 10 discussed studies reporting research related to: (1) students/children; teachers; parents; text-books. These studies were developed in different settings: classrooms (case studies, developed in different countries and cultures); different countries (cross-countries comparison); using analysis of documents (text-books); or doing a theoretical review. Thus, the units of analysis were also diverse: (1) a person (e.g., a teacher); (2) a social group (e.g., gypsies); or (3) a country (e.g., China, or England). We analysed and discussed two comparative studies (one from Kosovo, using three case studies; and a cross-countries comparison); one mixed study (quantitative and qualitative); (4) several case studies; and (5) one analysis of text-

books. From the theoretical point of view, the main issues we discussed were about identities, social representations and cultural models, situated learning, participation and engagement, teachers' practices, dispositions vs. I positioning, and what do we mean by traditional, when we use this designation.

For its future agenda, the WG 10 considered that we should keep discussing theoretical issues and methods in order to clarify them. We would like to become more effective organising the group between CERME conferences. We raised several possibilities: (1) creating an internet forum and/or a webpage; (2) circulating papers and discussing them; (3) writing collaborative papers; (4) participating in other events that also have a group dedicated to this theme (e.g., EARLI Conference; ISCAR Conference; The Socio-cultural theory in educational research & practice conference), and thata we could circulate information about these events; (5) discussing data and confronting our interpretations; (6) in CERME 6 one hypothesis that was raised was to have two or three key papers, to circulate them, and then to respond to them; (7) to create links with other groups.

The final discussion of the group was once again related to its name. We all agree that calling it ***Cultural Diversity and Mathematics Education*** would correspond better to the scope of the group, as it would include multicultural issues, but also other forms of diversity, inequities and cultural minorities. Thus, this is the name that will be used in CERME 6.

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SOCIAL REPRESENTATIONS AND MULTICULTURAL MATHEMATICS TEACHING AND LEARNING

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This article discusses how the notion of social representations can offer useful insights into understanding practices of teaching and learning in multicultural mathematics classrooms. Drawing on both the literature and empirical studies three specific questions are addressed: What are the dominant social representations that permeate the multicultural mathematics classroom? How do these social representations impact on the multicultural mathematics classroom practices? What are the spaces for changing these practices through becoming reflective and critically aware of these representations?

INTRODUCTION

The notion of social representations can offer useful insights into understanding practices of teaching and learning in multicultural mathematics classrooms. We are interested in a critical perspective on social representations (Howarth, 2006) as a notion that has the potential to explain and address contemporary issues in education in ethno-culturally diverse societies. In our paper, we will exemplify different social representations that circulate in multicultural mathematics classrooms, and what is more central, we will examine the processes through which teachers and students "take on" (Howarth, 2006), make use of, and transform specific social representations to mediate their practices. This analysis is necessary if the theory is to be used in promoting changes that are to contribute to improve the mathematical learning in multicultural classrooms. The analysis draws on both classical and emerging perspectives on social representations theorising. To begin with, we consider social representations as defined by Jodelet (1991):

> "Social representations are images that condense manifold meanings that allow people to interpret what is happening; categories which serve to classify circumstances, phenomena and individuals with whom we deal, theories which permit us to establish facts about them."

It is not difficult to see this notion applied to mathematical practices or mathematical learning. People interpret what happens around them as mathematical when it fits their image of what counts as mathematics. Teachers will interpret who is a good student according to their image of what learning mathematics consists of. What is more complex is to examine the impact of these images on the dynamics of teaching and learning in the mathematical classroom, especially as far as the multicultural ones are concerned. For this analysis we draw on the view that a social representation "is not a quiet thing" (Howarth, 2006). From this perspective, one could better refer to

social re-presenting, since according to this view a social representation is conceptualised as an action, as something people do.

> "Social re-presentation, as a socio-cognitive practice (…..), is something we do in order to understand the worlds in which we live and, in doing so, we convert these social representations into a particular social reality, for others and for ourselves" (Howarth, 2006, p. 68).

Social representations are not just something one uses to inform one's practices, but something that becomes part of one's reality. Social representations become constitutive of what we do, in the same way as cultural artefacts. A counting system is constitutive of our mathematical practices. When one does a calculation, it is one with cultural tools who does it (Wertsch, 1998). In the same way, when one makes sense, interprets, re-presents what is happening around one, one does this with social representations (Abreu & Elbers, 2005; Gorgorio & Planas, 2005).

ON MULTICULTURAL MATHEMATICS CLASSROOMS

The increasing ethno-cultural diversity of schools and classrooms is a phenomenon well recognised in our current world. In this global scenario, certain minority groups still have not found their place at school and, in particular, within the mathematics classrooms. Many youngsters from immigrant and minority groups underachieve academically in school mathematics. In this way, the learning opportunities that society should provide for all children through institutionalised education are far from being equal to every child. Diversity has challenged the existing practices, as schools are far from achieving their ideal goal of equipping youngsters with the mathematical knowledge and skills that will give them access to equal social opportunities. The ideas that we, as researchers, and the reader of this paper may have of a multicultural mathematics classroom are diverse. These ideas are linked to images of what culture, mathematics and learning are. Our representations may also be different from those of the teachers serving multicultural mathematics classrooms. Next we re-visit some key dynamics through which social representations operate, and examine how these can be used to make sense of "immigrant people". Following this, the remaining part of the paper addresses three specific questions: What are the dominant social representations that permeate the multicultural mathematics classroom? How do these social representations impact on the multicultural mathematics classroom practices? What are the spaces for changing these practices through becoming reflective and critically aware of these representations?

ON THE DYNAMICS OF SOCIAL REPRESENTATIONS AND IMMIGRANT PEOPLE

For Moscovici (2001) we create representations in order to make familiar what is strange, disturbing, uncanny (p. 20). This is why the notion of social representation is so central when discussing the challenge that immigration represents to society and, in particular, to schools. One brief look at the news in the mass media in many

European countries will immediately show how immigration is constructed as one of these disturbing phenomena. People express worries about immigrants using their public spaces, taking their jobs, etc. The social representations that emerge to make sense of the unfamiliar are not neutral and disinterested. They are constructed on the basis of the existing social reality, what is familiar to the social group, and involve a positioning towards the object. It is as if they are created with the function of protecting the existing practices, and defending existing identities (see for instance, Jodelet, 1991, Howarth, 2006). This process is evident in the emerging social representation of what an immigrant is like, and we will argue and illustrate later in this paper that this impacts on the construction of the immigrant learner. The image of what an immigrant is does not apply equally to all the people that legally fit into this category. Social representation theorists agree that there can exist more than one social representation to interpret the same phenomenon. Social representations are not only plural but dynamically created and transformed. Moscovoci argues that changes in culture are at the basis of changes in representations.

> "They are inextricably linked. One of the key differences between the various societies and cultures is the degree to which a representation penetrates their institutions, their beliefs, their relations, their behaviour, so as to constitute their true reality" (Moscovici, 2001, p. 19).

They become part of the individual reality through processes of socialisation, without the individual being part of the process through which these representations were constructed. Many studies show that individuals are often not aware of the dominant representations that shape who they are and how they act until they are confronted with differences. It is at this stage that they may start becoming reflective on their social representations, and open the way to change. An overview of alternative representations and the logic of their social distribution is needed for people to be able to use their 'agency' (Raudsepp, op.cit.). Nevertheless, some dominant representations are so coercive that they prevent the individual, even when challenged by practical evidence, questioning them through reflection and finding alternative meanings.

WHAT ARE THE DOMINANT SOCIAL REPRESENTATIONS THAT PERMEATE THE MULTICULTURAL MATHEMATICS CLASSROOM?

The few studies that have explored teachers' experiences of teaching mathematics in multicultural classrooms in different countries of Europe -England, Spain, Portugal, Italy - clearly show that a set of dominant representations permeates their practices (Abreu & Cline, 2003; Cesar & Favilli, 2005, Santesteban, 2006). These are often dominant representations that are anchored in traditional a-cultural and a-historical views of the nature of mathematical knowledge and of its learning. These studies also show initial evidence of some teachers becoming aware of limitations in the way they account for the mathematics learning of students from other cultures. Next we will examine some dominant representations.

The Universality of mathematics

The "universality of mathematics" can be seen as a dominant representation, which is shared by the majority of members of a highly structured group, which is uniform and coercive. Those who have representations of mathematics as a universal subject, tend to believe that when immigrant students learn the host culture language, they will learn mathematics in the same way as the local students. They express views, such as "We are lucky because the language of mathematics is universal, despite needing normal language"; "All immigrant students, have done the same mathematics. In fact, the Arabs taught us the numeric system that we use". (Santesteban, 2006, p. 79). An expression of this representation of the universality of mathematics underlies teachers' views that the only obstacle to learning mathematics in multicultural classrooms is the language of instruction. For some teachers this is the first and only crucial obstacle to learning that the immigrant student faces. When asking teachers: "Do you believe there are any aspect of our educational system that could represent an obstacle to the learning of mathematics of immigrant students?", Santesteban found that the following were typical answers: "The only obstacle is the language"; "It does not help, that Catalan is the language of the curriculum, and Castellano the everyday language spoken among immigrants" (Santesteban, 2006, p. 94).

When teachers represent mathematics as a universal subject, even if challenged by reality, they find it difficult to move forward on it. In her studies in Catalonia, Burgos (2007) worked with a teacher of a reception class who was very much committed to his students' learning. While in his class, the researcher observed that he was teaching the metric system for capacity units to a Chinese boy that had just arrived and could not speak Catalan. When asked if he knew if the boy had already been taught the units of capacity, or if at least, whether he knew if in China they would use the metric system, he answered 'I suppose so, mathematics is international… and besides that, if not he would have told me!' (We wonder how the Chinese boy would have managed to tell him!). This teacher has had children from many different origins in his classes. He has decided voluntarily to teach them mathematics. Nevertheless, he had never asked himself whether mathematics in the school was the same in other places of the world. He had taken for granted this representation of mathematics. Teachers, through their own years of schooling and through teacher education, have been exposed to a 'fossilized' representation about the universality of mathematics which is taken for granted in the social practice of teaching mathematics.

Learning takes place in the brain

Strongly associated with the representation of mathematics, as a universal subject, there is a dominant representation that its learning is dependent on a child's ability. This universal construction of children takes priority over ethnic and cultural backgrounds, as expressed by teachers:

> "As far as I'm concerned, you teach Maths according to the child's ability – not their ethnic minority... I don't treat them any differently to any other… and I wouldn't expect

them to behave any differently to any other child." (Primary school maths co-ordinator; Cline et al., 2002, p. 101, England)

This type of representation draws fundamentally on the notion that "the child's ability", located in the individual, is the key determinant factor in their mathematical learning. A variant of this representation incorporates other individual dimensions of students such as their needs, motivation, interests, pace of learning, and capacities. Santesteban's (2006) offers many examples of this representation. When teachers were asked: "Do you believe that immigrant students should be treated differently in mathematics classrooms?" they would too often answer: "Any classroom has an implicit degree of diversity, which makes students' learning (immigrants or not) a function of their interests, pace and capacities" .

HOW DO DOMINANT SOCIAL REPRESENTATIONS IMPACT ON THE MULTICULTURAL MATHEMATICS CLASSROOM PRACTICES?

In learning about their world people will take on 'presentations' of that world and re-interpret them (Howarth, 2006). These presentations are socially constructed according to the person's experiences and knowledge and become 're-presentations'. Re-presentations become a mediator in the person's interpretations and actions in the world. Social representations are not a quiet thing! This section gives examples of how dominant representations infiltrate classroom practices legitimating certain forms of knowledge, participation and identities. Based on Zittoun et al. (2003 three different ways of drawing on social representations, as symbolic resources, to mediate our thinking and action are examined. These three ways are "unreflective uses", i.e., when social representations are 'used' without any conscious grasp of choice and what it enables; "becoming-reflective uses", when one starts becoming conscious of the possibility of choosing different aspects of representations for different outcomes; "reflective uses", when choices are made on the basis of knowing about possible uses.

Unreflective uses of social representations

In mathematical classes the culture of the key social actors, pupils and teachers, becomes present in their actions. Distinct cultural knowledge is manifested in the use of cultural artefacts, of symbolic nature, such as numeric systems and algorithms, which mediate learning and teaching processes. This mediation can be seen in the use of the division algorithm (Gorgorió et al., 2006), which is distinctively expressed in Spanish speaking classes around the world. It may seem of little relevance that division is differently represented in Ecuador and Spain. However, this was indeed the cause for David's difficulties in the mathematics classroom. David, a boy from Ecuador, arrived in Barcelona aged 11. In Ecuador he was considered a good student, he never had difficulties with the basic algorithms, and when he arrived already knew how to divide using decimals. When he started school in Barcelona, the teacher was teaching how to carry out divisions with decimals, with the usual format in Spain.

David solved the exercises in his homework using the algorithm he learned in Ecuador. His homework was marked all wrong, and he told his mother, Monica, that he did not understand why his divisions were not correct. Monica, who worked as a cleaner, but had completed her university degree, did the divisions again with David using the methods they were both familiar with. She wanted to make sure David was doing his divisions correctly. At a parents' evening, Monica met the classroom teacher, and asked him how he was teaching division so that she could understand and help David. When she understood the teacher's method, she ventured to tell him how they were taught in Ecuador. Without paying too much attention to her, he replied "In Ecuador you do it wrong". When Monica recounted the episode to the researchers, she perfectly justified her way of carrying out divisions. When she completed the explanation, with a very serious expression, she asked the researchers "Do we do it correctly? Would it be possible that in Ecuador everybody did it wrong?" The relevant question is not whether there are any differences in the representation of the algorithm of the division, but how teachers react to the differences. It is unlikely that David's teacher believes that in Ecuador they teach division wrongly. What may have happened is that the teacher did not pay careful attention to Monica's explanation, and assumed that an immigrant cleaner did not know mathematics. David's case illustrates that the cultural nature of mathematical knowledge is more than the differences in the artefact in itself, and links to social representations of the process of learning mathematics, of who knows and who does not know mathematics, and what counts as correct ways of doing mathematics.

Becoming reflective of social representations

Above we gave an example of an unreflective use of the social representation by a teacher. It was unreflective because the teacher took for granted that the algorithm would be the same, and did not stop to figure out that the logic embedded in the different algorithm was correct. O'Toole and Abreu (2004) described a similar process in their study of learners in multiethnic mathematical classrooms in England. They examined the case of Monifa, a 10 year-old daughter of a Black African family, who developed awareness of the differences between the mathematical practices of her father and her teacher. When recounting an event where the teacher tried to convince her that her father's solution was not appropriate, she said: "I wasn't too keen but I understand my dad's more so I went with my dad. But she's my schoolteacher in school, so...." Monifa's way of coping would be to stick to each mathematical practice according to the context. But, as she explained, the practices of the school and home often made demands on her that made her feel as if she were "two people": "It's like I'm two people at the same time and its just hard". Case studies such as David's mother and Monifa's illustrate processes in which immigrants are confronted with different representations of what counts as school mathematical knowledge, and how they deal with the issues this poses for their cultural identifications, by becoming-reflective of uses of social representations.

WHAT ARE THE SPACES FOR CHANGING PRACTICES THROUGH BECOMING REFLECTIVE AND CRITICALLY AWARE OF REPRESENTATIONS?

In the contexts we described social representations of what counts as school mathematical knowledge are being re-interpreted and re-constructed. But, these re-interpretations are being carried out in a power context, where the minority group has no option, except to find a way of explaining why what they consider legitimate school mathematical knowledge is not accepted. One has to ask why teachers who become aware of differences in their multicultural mathematical classrooms still do not re-interpret their social representations. Our answer to this question is tentative as it seems to us that there are a variety of reasons underlying this problem. Teachers do become aware of differences in the performance of their immigrant students. Teachers also become aware of challenges in teaching immigrant students, and often report not having any special training to teach multicultural classrooms. In the absence of this information, they use their existing representations of learners and school mathematical knowledge to inform their practices. This results in: (i) an attempt to assimilate the immigrant student into the social representation of a monocultural learner; (ii) a neglect of cultural differences in mathematical practices in favour of what they perceive to be universal, (iii) an interpretation of the process of learning from a psychological perspective that focuses on the individual, ignoring the fact that learning is also a cultural and social process. Implied here is the suggestion that for teachers to become reflective and critically aware of the representations that mediate their practices a "space for challenge" must be created. We conclude by arguing that spaces for changing practices through becoming reflective and critically aware of social representations of learning in multicultural mathematics classrooms can be created by design, with attention to the topic in professional teacher training.

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LANDSCAPES OF LEARNING IN A MULTICULTURAL MATHEMATICS CLASSROOM

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Students' motives for learning mathematics cannot be understood by looking solely at mathematical classroom activities. We discuss this claim in a multicultural context using the notion of 'landscapes of learning'. This notion serves as a theoretical and methodological tool that both defines a research perspective and sketches a field of empirical research. In this paper we want to focus on the notion and illustrate its usefulness when researching mathematical learning in multicultural contexts. We draw on data and results of an empirical study on student's foregrounds with 45 teenage students in two 8th grade multicultural classes in Denmark. We show the dialectical relationship between each dimension of the landscape and the whole of it; and how, as a whole, it can help us coming closer to better theorisations.

A LANDSCAPE OF LEARNING FOR RESEARCHING MULTICULTURAL CONTEXTS

Research in mathematical learning in multicultural classrooms has tended to highlight language as a dominant aspect (e.g. Kaiser, 2003). Although important understandings have resulted from that type of research, we notice that selecting one particular aspect does not allow grasping the social complexity of multicultural classrooms. In our research project "Learning From Diversity" (Alrø, Skovsmose and Valero, 2003, 2005) we have engaged in the study of mathematical learning in multicultural classrooms from a socio-political perspective, interested in both a theoretical and an empirical examination of mathematics education practices in settings where cultural diversity, conflict and dialogue coexist. Such an examination is of extreme relevance in the current situation in Denmark where, similarly to many other European countries, dealing with the cultural diversity of immigrant population is being constructed as a problem and an obstacle for learning, and not as an enriching possibility. In Denmark no attention has been paid to mathematics education in multicultural situations. However, most of what comes to be available for teachers and researchers are results from general education in which immigrants' lack of linguistic competence in the Danish language is a dominant explanatory factor for the lack of successful participation in schooling.

As a response to that situation, we have engaged in the construction of thinking tools that allow us to move from existing simplifications of the predicaments and challenges of mathematics learning in multicultural situations. We have proposed the notion of *landscapes of learning* as a tool to guide us in exploring the empirical field. The notion has a double meaning. First, it represents an interpretation of (mathematics) education as a complex network of social practice that is constituted by different interrelated dimensions. Second, it makes possible to identify specific –

but correlated – dimensions of an empirical field to do research. Thus, it brings together a research perspective and a research field.

Among the many dimensions one could choose, we have decided to select those that recent research with a socio-political perspective emphasised as aspects needed to be considered for a better understanding of the social complexity of multicultural mathematical classrooms. Therefore we operate with the following dimensions of a learning landscape: (a) Students' foregrounds as an experienced socio-political reality (Skovsmose, 1994; Gorgorió & Planas, 2001). (b) Students' construction of identity and of cultural diversities (Sfard & Prusak, 2005). (c) Teacher's perspectives, opinions and priorities of teaching (César & Favilli, 2005). (d) The content of learning, in our case the mathematical content for classroom interaction (Powel, 2002). (e) Tools or resources for learning that students might have available. (f) Classroom interaction among students and between students and teacher (Gorgorió et al., 2002). (g) Family and parents who influence students' priorities (Gutstein, 2003). (h) Friends, who are reference groups for the students (Bishop, 2002). And (i) public discourses about immigrants, schooling and multiculturalism (Martin-Rojo, 2003).

The landscape of learning so defined allows engaging in different kinds of empirical studies addressing one or more dimensions at the time and tackling one or more particular questions. For example, two other researchers associated to our team have chosen to focus on students' identities (Stentoft, 2007) and on family and parents' priorities (Baber, 2007). In other empirical studies, Helle, Ole and Paola have also explored the landscape from classroom interaction and mathematical content dimensions. The results of these studies are under elaboration and will not be mentioned here.

In what follows we illustrate the power of the landscape of learning when researching multicultural mathematics classrooms, drawing from results from an empirical study focused on the foregrounds and motives for engaging in mathematical learning (for details on this study, see Alrø, Skovsmose and Valero, forthcoming) of a group of 45 teenage students in two classrooms in a multicultural Danish public school. In this paper we refer shortly to the design of the particular empirical study from which we draw some data. We then show how the landscape of learning is put into operation in analysing inter-views with the students where we explored students' motives for learning; and we conclude with some remarks about how we see the perspective advancing towards a theory of (mathematical) learning in multicultural settings.

AN EMPIRICAL STUDY ON STUDENTS' FOREGROUNDS

Mælkevejen Skole is a primary and lower-secondary school situated in a suburb of a bigger Danish city. This school hosts students from 29 countries from all over the world. We established collaboration with two 8th grade classes of 24 and 21 students respectively. Half of them come from countries such as Iceland, Ireland, Iraq, Greenland, Lebanon, Malaysia, Somalia, Turkey and Vietnam. The two teachers, a young man and a very experienced man, are both Danish. Despite the diversity of

students, at the time of the research there were no artefacts displayed in the classroom that indicated diversity, except for the names on the students' pigeonholes. From the surface, this classroom would be a typical "Danish" classroom.

The study was carried out in five phases. First, Helle observed the mathematics lessons during a couple of weeks with the purpose of getting in contact with students and teachers, and of having a background information about the classroom practices, though not for collecting data about classroom interaction. Second, Helle invited the students to imagine themselves in ten years and think about how their life would look like. Then students were asked to write a story about themselves and their imagined future lives. Third, students answered a questionnaire related to their present school experiences in general and to school mathematics in specific. From these three phases, the research team produced questions and guidelines for an inter-view (Kvale, 1996) with each student about his or her foregrounds. In the fourth phase, Helle conducted inter-views with students in order explore the students' foregrounds with respect to mathematics. Finally, the research team inter-viewed the two teachers and asked them to give a guess of what the students would like to do after finishing 9th grade. The intention of this conversation with the teachers was having a background material about how the teachers see the students' foregrounds.

From the different information obtained in the five stages, we concentrate on the inter-views with students and on the students' own narratives about themselves and their future possibilities. The inter-views were audio-taped and protocols about the conversations have been produced in order to make a profile of how students see their school experience and which motives they bring for learning mathematics. The analysis of the inter-views allowed us to illustrate some of the dimensions of our learning landscape. In what follows we show how the different dimensions are relevant for interpreting issues concerning students' motives for learning in multicultural settings. Some dimensions, however, were not touched upon during the inter-views: "Teacher's perspectives, opinions and priorities" and "Classroom interaction". Thus, we are not going to present an empirical evidence of the model; rather, an empirical indication of the relevance of including a variety of dimensions when interpreting students' motives for learning mathematics.

THE DIMENSIONS OF THE LANDSCAPE OF LEARNING IN OPERATION

Students' Foregrounds

Students' foregrounds can be defined as a person's interpretation of his or her learning possibilities and 'life' opportunities, in relation to what the socio-political context seems to make acceptable for and available to the person (Skovsmose, 2005). From the inter-views, it is possible to get an insight into some of the students' hopes and expectations for the future.

Minh likes to go to school. He considers school and mathematics to be very important for his future life, as he wants to become a banker or estate agent: "Something with mathematics - I love numbers."

Natascha is very fond of moving things around in her room and making things better and more beautiful. She estimates the size of the furniture and she calculates and makes drawings of the room. She has just got herself a new room of 25 m2, so there is a lot of work for her to do. Natascha wants to be an architect or estate agent. She knows that for her dreams to come true she is supposed to be very good at mathematics. And so she is. In the case of Natascha there seems to be a close connection of her interests, skills and hopes for the future, i.e. her foreground. And mathematics certainly plays an important role in this foreground.

Ryan and Kim both want to become computer scientists. They are well aware that it takes a lot of mathematics, though, to become a computer scientist. In the first place it is a precondition to get good marks in mathematics. "But you also need mathematics for programming and for estimating codes and things like that," Ryan says. They think that being a computer scientist is a well-paid job, but the important thing is having a job that you like.

Thus, many students have a clear imagination of their future education and professional life that is connected with certain ambitions including mathematics. However, a large group of other students do not know if they are going to use mathematics for the profession that they go for, but most of them mention that mathematics is important "for almost everything". Laura, for instance, wants to become a dentist, and she thinks that she is going to use mathematics, but how and why she cannot imagine at all. The same counts for Monica who wants to be a psychologist.

These examples illustrate that the notion of students' foreground is important in order to understand students' motives for learning.

Students' identities and cultural diversities

As mentioned, Mælkevejen Skole hosts students from many different countries and cultures. In the inter-views, however, none of the students touch upon that subject by themselves. And when Helle asks what difference it makes most of the students claim that: "It makes no difference!" and "It is no big deal!" They have obviously no focus on cultural diversity. But when Helle asks about advantages and disadvantages of being a multicultural class some students express an opinion. Natascha likes the presence of children from all over the world: "I think everybody should have the right to go to school and to learn, no matter where they come from, what colour they have or what language they speak. Everybody should have the right to go to school and get on with their lives. Cultural differences do not mean anything to me." When asked of advantages she says that it is nice to hear about different cultures, for instance how they celebrate Christmas in Vietnam. Tanja thinks that it is exciting with different cultures in school. "You can learn from each others' different ways of doing things".

Although the students seem to be very including and emphasise that multicultural diversity "doesn't mind", it is interesting to observe how their statements often correspond with an 'us-and-them' discourse: "We just talk to them", Gro says. "Maybe their opinions are different, but that doesn't mind. People are just not the same", Hilda adds. Actually, Hilda herself is from Iceland and strictly taken not Danish, but obviously she does not include herself in the 'them'-category. But according to the students, multiculturality is not much in focus either during the lessons or in the breaks. Minh puts it this way: "It is not important to me. We are all human beings, aren't we?"

The students are well aware of cultural differences, but they seem to agree that to take specific notice is not important. Directly addressed on this subject, however, they can only come to think of a very few examples of "learning from diversity". This indicates that they represent the 'sameness' approach in the Danish public discourse about integration, which implies making 'them' just like 'us' (Alrø, Skovsmose and Valero, 2005). This is an important observation in order to interpret what is happening in the classroom: in particular the phenomenon that diversity is not used as any resource for teaching and learning. In other words the dimension of students' identities and cultural diversities is important as the students' approaches to learning reflect how they identify themselves in a cultural setting. And as we have seen one possibility is to try to neglect cultural differences.

Mathematics

Many students refer to shopping when asked about their use of mathematics in their daily lives. Some of them also refer to weighing and measuring while cooking and baking. Quite a few students, though, state that they only use mathematics when doing their homework.

Many students mention that they find mathematics interesting when it is easy (i.e. when they can find out), but boring when it is too difficult. Tanja does not like mathematics, but actually she does not know why. "Maybe it is because I like to talk a lot, and mathematics is not a talk subject." She thinks she is doing OK in mathematics, but she spends a lot more time on it than the other students, she says. She uses mathematics in her everyday life, especially if she is going to buy things "for instance if there is sale". Some professions might need a lot of mathematical knowledge, she says, but she is not interested in such professions.

Michael wants to become an architect. He likes to draw straight lines, and he is fascinated by the thought that the durability of a house depends on the drawing. Ahmed claims that he wants to be a programmer: "In the future everything will have to do with computers", he says. He thinks that mathematics is easy and that he is good at it, but he is not so fond of it.

Mathematics is important in order to get admission to a higher education. Many students stress this point. For those students, however, it is not mathematics per se, but mathematics and good marks as a ticket to the education they want. Razia is not

sure what it would take to become a nurse, but she suggests biology, "and then you use mathematics, I think. You use mathematics almost for everything." Razia is not able to point to any mathematical content of a nurse's job. Thus, mathematics plays an inferior role in Razias foregrounds. But being good at mathematics and doing well in school are certainly important parts of her educational plans.

It seems obvious that ideas about mathematics make part of students' motives for learning mathematics. However, the students' ideas about how mathematics might relate to their aspirations are rather confused. The value of mathematics operates in an instrumental form, not as a motive based on an insight about mathematics gained as part of the mathematics lessons.

Tools

A few students touch upon the computer as a tool in mathematics education. Ida and Hannah agree that computers are not important in mathematics education. "They can do nothing more than a calculator", they claim. "You do not make any progress in mathematics by using a computer ... you don't gain more knowledge ... it is just a tool." Lisa adds to that: "You can only use Excell. You cannot hand in your homework on a computer for instance."

None of the students talk about having access to tools. In a Danish context students consider access to tools including computers as a natural good.

Family

Minh does not think that his parents think he is good at mathematics. They do not tell him so, but he does not feel good enough and he seems to be sorry about that. Minh's parents are very ambitious on behalf of Minh. The father has got a grill-bar, and he wants his son to get a well-paid job, so he wants him to become a doctor. Minh himself would rather get a job that he is interested in and happy about, but he knows that his father will get disappointed if he chooses "only" to become a banker. Considerations about being a medical doctor have passed Minh's mind.

Paul wants to become a car mechanic or an engineer. Or he would rather say that his mother wants him to become an engineer. He himself has no ambitions of becoming an engineer. He likes cars a lot and would rather like to become a mechanic.

Razia is well aware of the importance for a Muslim woman of being educated. She does not want to end up like her mother: "... my mother, she has no education. So she says, 'I do everything for you. You are having an education. You should not become like me.' [...] Because in Iraq, if you don't get an education, then you marry early. My mother got married when she was 14 years old. And therefore she is afraid." In Iraq it is rather difficult for a woman to get educated, especially in the part of the country where Razia lived. So she is quite clear of the fact that she wants to get educated in Denmark before she goes back to Iraq in the future.

In different ways parents seem to have a major influence on the students' learning motives. The students' ways of engaging in their schoolwork might be in some

harmony with their parents' priorities, or it might represent a reaction. But it is difficult to ignore the parents' perspectives as making part of an interpretation of the students' attitudes towards learning.

Friends and peers

"I like to go to school because of my friends and because it is important to learn in order to get on in life", Natascha claims. Natascha's point coins the most frequent answer among these 15 year olds: The most important thing about going to school is to get friends and to meet with your friends. Therefore, the breaks are considered to be most important. Like Natascha some other students also refer to the importance of learning, but the role of friends seems to occur immediately to the students' minds when asked about school.

Some claims about going to school may relate to the fact that we have to do with teenagers. "I don't like going to school, especially not in the mornings. I am all too tired. But I like being together with my friends", Peter says. Johnny confirms this attitude. He thinks the worst thing about going to school is that it takes so long. "You have to be there all day. There are lots of other things you could spend your time on."

Friends can have good as well as bad influence on students' preferences. But the solid stress of the importance of being with friends when going to school also indicates an important influence of friends for students' learning motives.

Public discourses

A couple of the students touch upon the public discourse on multicultural issues. "We learn about other cultures, but we do not look at them differently. For instance, we have heard about the Ramadan, and we have had food from foreign countries. This is very good, so that you not believe it is like in the news... that they are just somebody who kills other people or things like that. It is good to know how they really are. The picture in the news has nothing to do with the children we know", Susie says. She argues against the media discourse of immigrants as mostly being criminals. The children in her class are not like that.

Another strong issue in the public discourse is the 'headscarf-girls'. Razia who wears a headscarf due to religious reasons comments on that: "I know that...most Danes do not like the headscarf ... it is about racism and so on."

These examples indicate that prejudice that as expressed in the public discourse might have serious consequences, especially for immigrant students' learning motives.

THEORISING ABOUT LEARNING MATHEMATICS IN MULTICULTURAL SETTINGS

The number of dimensions we have suggested for a landscape of learning expands considerably the scope for investigation, which has characterised much theory building within mathematics education research. Our point is that learning

mathematics in a multicultural setting cannot be understood by looking solely at, for instance, mathematical classroom activities or language.

The landscape is developed with a particular reference to learning mathematics in a multicultural setting. However, it is our hope that it could serve as a way of structuring an understanding of learning in many other situations than the one we have addressed. We find that the dimensions we have presented could turn into dimensions of a theory of learning. They represent a way of seeing learning, and seeing learning in contexts. The examples above illustrate that students' motives for learning mathematics is not particularly connected to mathematics as a subject or to the school mathematics tradition.

In the inter-views only a few students pointed to the relevance of specific mathematical content for their future life. Only a few expressed a direct connection between school subjects and relevant professional skills. Michael who wants to be an architect finds mathematical drawings very meaningful; Ryan and Kim who are going to be computer scientists know that they are going to use mathematics, but they mention no application of mathematics in that area. The view is more general: "You use mathematics for almost everything" and external: "It is important to get good marks in mathematics". This means that the students' motives for learning mathematics can easily become *instrumental* (Mellin-Olsen, 1987). The importance of getting good marks is the only plausible value to assign to the school subject, if no everyday practice including mathematics is explored. As mentioned, Laura wants to become a dentist, and Razia wants to be a nurse, but none of them can imagine anything about how mathematics could make part of this, even though they assume that mathematics could be relevant. The possible relevance of mathematics seems to be well-hidden in the school mathematics tradition. As a consequence, it might become difficult for some students to see that mathematics could play any significant role in their future. If meaning for students somehow is related to their foreground, then instrumentalism could become a strict implication of the fact that mathematics appears as a clean and isolated subject. Apparently they have not experienced transitions between classroom practices including mathematics and out of school practices including mathematics. None of them mentions that kind of connections.

When the students have left the classroom, there is nothing left which reflects that we are in a multicultural setting. No pictures on the notice board reveal that there are students with different cultural backgrounds. One could think of diversities as a resource for learning and understanding. One could also consider diversities as a cause of possible troubles and difficulties (Alrø, Skovsmose and Valero, 2003). Through our visits to the school we did not see diversities made explicit in the classroom. Our overall impression is that diversities were both hidden and ignored, which was also reflected in the inter-viewing with the students. The students in our study claim that cultural differences are not important, although when asked they reproduce the 'them-and-us' discourse, which is a common construction in the Danish

public discourse. One implication of this discourse is that integration of newcomers is understood as an assimilation process that implies making 'them' just like 'us'.

We certainly have to be careful not to claim too much about the adequacy of a learning landscape as providing an adequate framing for a theory of learning. However, any such adequacy depends on what one sees as the task of theorising. The landscape of learning is something we are researching, but it is also constituted through the way we are researching. In the study above, some dimensions became visible, others did not. Thus the dimensions of the landscape of learning represent our way of looking at the landscape, and at the same time the dimensions constitute the landscape. The argument for choosing one and not a different dimensioning cannot be justified in a direct way. However, there might be an indirect justification that has to do with the observations, which a particular dimensioning of a landscape of learning makes possible. It has to do with the insight that might be crystallised. So our 'justification' of dimensions is to be found in the potentials for theorising about learning mathematics in multicultural settings. If a different dimensioning provides other observations, or other observations provide new dimensions this would be most welcome. Even though one empirical study might reveal 'something', it will never reveal 'everything'.

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Young Children's Number Sense in China, England and Finland

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This exploratory study aimed to examine and compare the influence of culture, age and gender on young 120 Chinese, 118 English and 116 Finnish children's number sense, centred on age 60 months. It used the Utrecht Early Numeracy Test (UENT) by van Luit, van de Rijt &Penning (1994), that comprised 4 numerical and 4 Piagetian-type sub-tests. Ordinal logistic regression of scores for specific sub-tests revealed girls more likely than boys to get higher scores for a comparison task with the Chinese sample outperforming the Finnish and the English sample still less successful. A more mixed picture for general number knowledge emerged with no gender effect. This time the English sample did better than the Finnish though the Chinese were still best. Differences in language, teaching and culture are discussed.

Cross-cultural differences in early number sense

This study examines the number sense of children aged five years from England, Finland and China in terms of age, gender and culture. We have fully explored the theoretical and conceptual underpinning of 'number sense' elsewhere (Aunio, Hautamäki, Heiskari & van Luit, 2006; van der Rijt, Godfrey, Aubrey, van Luit, Ghesquière, Torbeyns, Haseman, Tancig, Kavkler, Magajna & Tzouriadou, 2003). Suffice it to say that number sense entails operating with quantities and number-word systems. Case and colleagues (Case & Okamoto, 1996; Griffin & Case, 1998, for example) argue that the central conceptual structure of number develops through stages. In the pre-dimensional period up to four years, children have a global quantity schema that can address questions about 'more' and 'less' and an initial counting schema that permits ascertaining how many objects there are in a set. At around six years, the schema merge in an early central numerical structure or 'mental number line'. At each stage a reciprocal relationship exists between general and specific skills.

Meanwhile, cross-national comparisons of young children's mathematical skills have shown that the mathematical performance of Asian children is better than that of their non-Asian counterparts (Ginsburg, Choi, Lopez, Netly, & Chi, 1997; Huntsinger, Jose, Liaw, & Ching, 1997; Miller, Smith, Zhu, & Zhang, 1995: Zhou, Cheng, Mottram, & Rosenblum 1999; see Song & Ginsburg, 1987 for contrasting results). For example, young Chinese children consistently outperform their western peers in rote and object counting, concrete and mental calculation, and in the use of sophisticated strategies in mathematical problem solving. Differences in language (Fuson & Kwon, 1992; Zhou & Boehm, 2001), teaching (Stevenson, Lee, & Graham, 1993) and cultural ethos (Campbell & Xue, 2001; Caplan, Choy, & Whitmore, 1992; Jose, Huntsinger, Hintsinger, & Liaw 2000; Stevenson, Lee, & Graham, 1993; Tuss & Zimmer, 1995) have been considered to be factors underlying superior mathematical performance in Asian children. However, since cross-national comparisons of mathematical performance have mainly focused on children aged six years and above, our knowledge of such differences in young children's number sense is limited. One possibility to understand the cross-national differences in mathematics is to study such skills when teaching at school has not yet, or only recently started. Taking up this challenge our aim was to measure the early number sense of five-year-old children before or in the very early stages of formal teaching. The children were located in two European and one Asian context.

Gender differences

There are contradictory research results in respect of children's mathematical performance and gender. Some research shows that girls and boys possess identical primary numerical abilities (Dehaene, 1997; Nunes & Bryant, 1996). The researchers analysing English National Curriculum Key Stage 1 progress (children aged five to seven years) have mostly reported girls

outperforming the boys in basic arithmetic (Demie, 2001; Gorard, Rees, & Salisbury, 2001; Strand, 1997, 1999). More precisely, Strand (1999) reported that girls performed better than boys at age five years in baseline measurement and at age seven in end-of-Key-Stage-1 assessment), but that the progress of boys was bigger than that of girls between the two measurement times. Carr and Jessup (1997) reported contradicting outcomes, as in their first school year, boys and girls may use different strategies for solving mathematical problems, but there is no difference in the level of performance. Gender differences in numerical skills at preschool age have attracted little attention (Torbeyns, van de Noorgate, Ghesquière, Verschaffel, van de Rijt, & van Luit, 2002; van de Rijt *et al*., 2003), and it would therefore be worthwhile to check for such differences in young children's number sense.

Research Questions

On the basis of prior research we formulated the following questions:

- Do children from the three countries have a similar pattern of development in number sense?
- What is the influence (if any) of schooling, age and gender?

In terms of design, the intention was to provide some quantitative modelling that would form the basis for a later stage that would aim to capture social and cultural practices in real-world settings in the different national contexts.

Method

Participants

The English and Finnish participants had taken part in a previous European study and the Chinese participants were selected to match the European children as closely as possible. Educational settings were selected from urban and mixed rural areas and in such a way that they were as representative as possible in terms of size of institution, socio-economic and ethnic background of parents. Finnish children were

from the Helsinki area, Chinese children from the Beijing area and English children from an area south-east of London. Table 1 shows the total sample size was 354 pupils, 169 girls and 185 boys approximately 61 months in age. The sample size was too small to be regarded as nationally representative.

Table 1: The sample

Sample size		**China**	**Finland**	**England**	**Total**
	Girl	61	52	56	169
	Boy	59	64	62	185
	Total	120	116	118	354
Mean age		**China**	**Finland**	**England**	**Total**
	Girl	61.5	60.8	59.3	60.5
	Boy	61.7	60.6	59.8	60.7
	Total	61.6	60.7	59.5	60.6

Materials

The children's number sense was assessed using the UENT (van Luit *et al*, 1994) Form A. This includes 40 items measuring 8 topics of young children's numerical and non-numerical knowledge of quantity, including the concepts of comparison, classification, one-to-one correspondence, seriation, the use of number words, structured counting, resultative counting and general understanding of numbers. From a theoretical point of view, the first four subscales relate to the logical principles often identified as the key factors underlying children's understanding of quantities and relations or 'relational skills' (Piaget, 1966), whilst the rest of the test focuses more explicitly on the use and understanding of numbers or counting skills (Fuson, 1988; Gelman & Gallistel, 1978). Translation of the instrument into English, Finnish and Mandarin (Chinese) and its use in a number of other national contexts in European and South-east Asian has been reported elsewhere, for example, Aunio *et al*, 2006 and van der Rijt *et,* 2003.

Procedure

The tasks that involved manipulation of small cubes, interpretation of picture material and paper and pencil matching exercises were carried out in a

quiet area in a one-to-one situation of adult with child. Experienced testers were used in each context. Children enjoyed taking part with the activities and these took around 20 to 30 minutes to complete.

Results

Using confirmatory factor analysis with MPlus, about equal potential for robust results was found for (a) looking at total scores (one factor) and (b) looking at a combination of the four relational topics and a combination of the four numerical topics (two factors) that showed reliability in excess of .70.

When mean total scores are plotted against age in months (Figure I(a), the Chinese sample clearly outperforms the two European samples. When the randomness in the raw data is smoothed out by linear regression (Figure I(b)) the pattern is even clearer. Progression with age appears to be about the same in all three samples but Chinese children were on average about a year ahead of the Finnish, who in turn were about half a year ahead of the English.

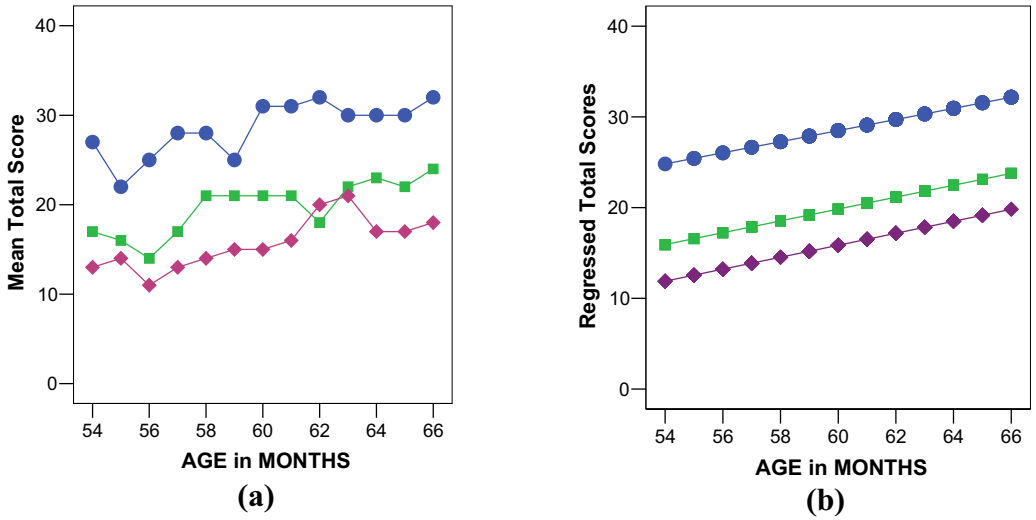

Figure I: Mean Total ENT Scores by age and country (a) from raw data and (b) smoothed by linear regression.

Only in the case of Finnish children was there any evidence for a difference in performance between boys and girls, in favour of boys. The pattern for relational scores shown in Figure II is very similar to that for total scores except the gap between Finnish and Chinese children is equivalent to

about half a year and the gap between English and Finnish equivalent to about a year. Once again there was no evidence for a gender effect except in Finland.

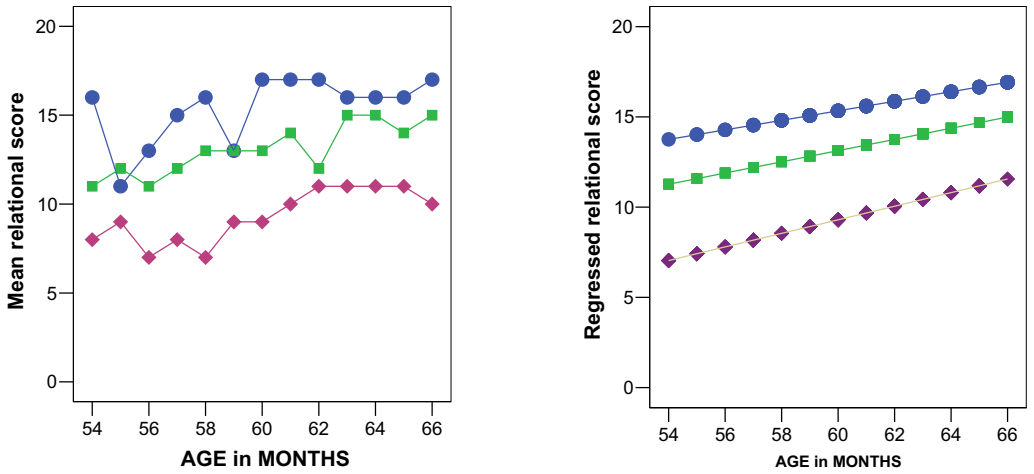

Figure II: Mean Relational Scores by age and country (a) from raw data and (b) smoothed by linear regression. ○ China □ Finland ◇ England

For numerical scores, as shown in Figure III(b), the performance of Finnish and English children is virtually indistinguishable. Chinese children appeared to be about twenty-two months ahead of English children and Finnish boys and seventeen months ahead of Finnish girls. Gender differences were almost entirely confined to Finnish children. Superiority of Finnish children over English is seen only for the relational topics.

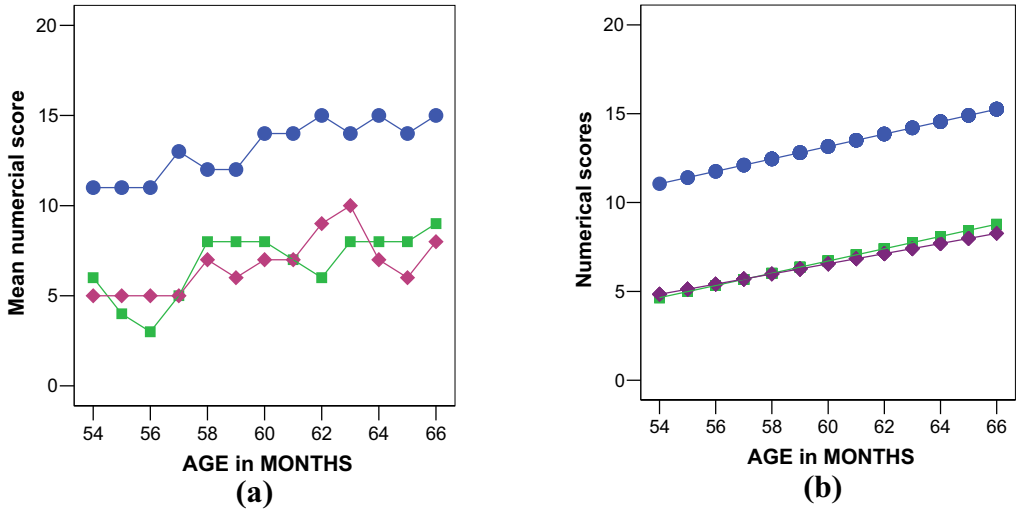

Figure III: Mean Numerical Scores by age and country (a) from raw data and (b) smoothed by linear regression. ○ China □ Finland ◇ England

Discussion

In answer to the first question, children from the three countries did *not* appear to have a similar pattern of development for either comparison (a relational task) or general number knowledge (a numerical task). As our previous work has suggested, it appears that UENT measures two different aspects of the number-sense construct with the relational skills being *less* influenced by teaching and language difference and, hence, culture than counting skills. This interpretation is consistent with Case's notion of a central conceptual structure with specific numerical skills that build upon general numeral skills and are affected by social, environmental and cultural factors, hence, teaching. This brings us to the second question. There is at least some indication that very early exposure to numerical instruction may account for differences in number skills in the English and Chinese context but if general skills in contrast are more dependent upon maturation, how do we account for differences in the relational skills or the high performance of Finnish girls? Whilst Case (1996) suggests that the development of number sense is a dynamic combination of progress in general and specific skills, general skills may also be influenced by teaching. However, differences in preferred activities and/or opportunities to engage in these tasks by boys and girls in early childhood could also lead to different experiences and different patterns of development of number sense. It could also be the case that too early an emphasis on specific number skills that is *not* combined with general number skills as may occur in the English context, disturbs the reciprocal relationship between the two and affects the development of number sense.

In summary, first of all, samples from the three countries progress with age in number sense but the pattern of differences among children from the three nations suggests that there are other sources of influence at play. Secondly, these findings are in line with our previous work suggesting that the nature of the relational and numerical skills (as measured by the UENT) are

different. Thirdly, if numerical skills are more influenced by schooling or language differences than the relational tasks, it may well be that the earlier school start age of Chinese and English children is reflected in their higher numerical scores. This is borne out in our earlier work (van de Rijt *et al*, 2003) where English children were found to have an early bias to numerical skills. Moreover, the particularly high scores for Chinese children may be explained in part by the systematic number words in the Chinese language facilitating the development of understanding of numbers, counting and the underlying ten-base system (Fuson & Kwon, 1992). Fourthly and finally, the Finnish gender differences are of particular interest, as these did not emerge in our previous work.

Most importantly, our interpretation of these findings should be constrained by the limitations of the study. First of all, the sample sizes were very small and at best can be only indicative of what might have been found in a larger sample of children. Identified influences of language, culture and schooling at this stage can at most be regarded as tentative.

As the next stage of our work will take us into early childhood settings to observe mathematical activities taking place it may be wise to avoid speculation regarding the origins of these differences. The hope is that case studies from each nation may shed further light on the cross-national differences that emerged.

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FILLING THE GAP BETWEEN GLOBAL AND LOCAL MATHEMATICS

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The changing cultural contexts of today's societies demand new roles of schooling but also require new attitudes and ways of interaction with different social groups, their knowledge and values. Teachers are central figures in the educative process that mediate knowledge and social interaction between individuals in classrooms that are more and more multicultural. The main argument of this paper is that in order to facilitate students' understanding of mathematical knowledge it is crucial to develop teaching approaches that enable students to draw in their local mathematics. To address this issue, I analyze recent investigation focused on the mental calculus of Portuguese gypsy children. Next, I relate it with situations that require global mathematical knowledge and with ways of teachers' training to interconnect different kinds of mathematical knowledge.

INTRODUCTION

Since the 1970s, there have been studies aiming to show the importance and relationships of cultural, social and linguistic factors in the learning and teaching of mathematics. These studies consider the socio-cultural dimension of students' experiences and backgrounds to better understand and critically examine the increasing complexity of the pedagogical processes in multicultural classes. Research reveals that different cultural patterns of socialization and language systems lead to different perceptions in mathematical concepts and relations (Barton, 2001), and different engagements in socio-cultural practices to support school (Abreu, 2005; Moreira, 2002). In addition, research has also pointed out that to make mathematics a familiar subject, students must participate in mathematical accounts, and take their own realities as locus and motivation for mathematical representation and problem solving (Secada, Fennema &Adajian, 1995; Gorgorió & Planas, 2001; Knijnik, 1996). In addition, ethnomathematical studies have shown that social groups are the locus of "proper mathematics" that is necessary to study along with school practices to optimize mathematical comprehension of all social groups (D' Ambrósio, 1997; Gerdes, 1996a)

Moreover, comprehending the relevance of mathematics in our daily lives and its multifunctional roles in contemporary society is a central issue in schooling and a complex phenomenon that affects mathematics education. As Skovsmose and Nielsen (1996) noted "The globality and universality of mathematics education became a barrier for the students in seeing the relevance of mathematics in relation to their daily life and their society" (p. 1260). I now propose to focus attention on the connection between the universality and locality of mathematics, and on teachers' role in promoting this connection.

GLOBALIZATION AND EDUCATION

Several studies have shown that global culture is interpreted and used in different ways by local cultures (Appadurai, 1996; India & Rosaldo, 2002). "Globalization" refers "to the intensification of global interconnectedness, suggesting a world full of movement and mixture, contact and linkages, and persistent cultural interaction and exchanges" (India & Rosaldo, 2002, p. 2). Ultimately, these studies show how the global dimension is already present in the local cultural reality, and how each particular community is linked to a broader level of culture.

In the global culture, schools have experienced complex changes. Not only do they have to train students for economic and professional purposes, they also have to spread the technological paradigm as well as promote a democratic rationality among students to participate in society as informed and critical citizens. These goals aim at promoting competence to deal both with daily life situations and globalization by gathering examples and ideas to use from the daily life of students as well as from more distant and abstract situations.

However, education is a complex process with several protagonists (located in the family, peer groups, schools or community groups) having their own epistemologies, strategies and technologies of learning. In fact, each social group has ideas about what a new member has to know and how he or she should behave to be considered a member of the group. Not only does the group have ideas about educational content, but also how this content should be taught. Group educative theory, or its way of thinking about how to educate, leads to actions that unchain a practical form of making to learn what should be learnt (Gee, 1990; Heath, 1983; Lave & Wenger, 1997, 1991; Reed-Danahay, 1996; Street, 1993). Therefore, within each social group learning is culturally situated and linked with a set of specific behavioral and social processes that are fundamental to the learning success. These processes are sustained by the group in regard to the new members, and emerge as fundamental to frame not only the act of learning, but also the content matter of learning itself. Thus, social groups familiar with schooling possess already social processes to frame the school content and also have acquired the adequate manners to learn them (Bourdieu, 1979).

MATHEMATICS

From the 17th century, mathematics has become a symbol of global society, indispensable to development and globalization, and a crucial tool for the universalization of knowledge. We can understand the impact of mathematics on society at large by observing how contemporary societies are mathematically behaved and dependent. This, in turn, demands mathematical literacy to interpret social and natural phenomena and to critically participate in social life (Bishop, 1991; Skovsmose, 1994).

On the other hand, since the 1970s, the research findings from the field of Ethnomathematics have pointed out i) how different cultural groups possess

particular ways of approaching mathematics ii) the social, cultural and political nature of the variables and processes involved in Mathematics Education and iii) the complexity of the articulation between the mathematical knowledge based in primary culture and that promoted by schools, highlighting the dissociation of the school mathematics from daily life.

If the dynamics of globalization is taken into consideration to discuss mathematics in multicultural classes, global processes are re-interpreted and re-appropriated by local culture. To deal with them means to construct linkages between ways of thinking and acting locally and ways of thinking and acting in other dimensions of society. Nevertheless, the use of mathematics as a tool for globalization and its role in the universalization of knowledge continues to be incomprehensible to individuals, in general.

Consequently, since globalization is a fact or life, if people do not develop the competencies to access its process of constructing knowledge, they hardly might function in contemporary society. To be capable of critically participating in society, to know how to relate local interests to a global society requires to have an explicit idea about how to express throughout mathematical concepts and how they are used in problems which may affect some or all the humankind.

In short, we need to focus our attention simultaneously, both on the importance of mathematics at a local level, and on its relevance to global phenomena. On one hand there is the need to introduce students to a global way of dealing with mathematics because mathematics is explicitly related to economic development, technology, and democratic competence, and is present in different ways in different communities--what I call “global mathematics”. On the other hand, there is the need to be local in the sense that people need to develop their own ways of knowing and learning and to keep their local culture alive, including their mathematics. Both are indispensable to creating updated mathematical meaning in contemporaneous societies.

Ethnomathematics is an effective medium between local mathematics and global mathematics and to critically interpret the interactions between the local and global dimensions of society. In this regard, some illuminating experiments where carried out, for example, by Knijnik (1996) working with the landless community in Brazil.

LOCAL MATHEMATICS

The disconnection between the school mathematical curriculum and students’ daily lives has been largely discussed inside the community of Mathematics Education. As I am arguing, part of this dissociation is related to the connection between local and global knowledge, which also includes different strategies and technologies of knowledge.

A consensus about the necessity of school mathematics contextualization, demands that students actively participate in it, and that the teacher, in a dialogical process, “involve students in a permanent problematization about their existential situations”

(Freire, 1985, p. 56), in order to conduct the application of mathematics on the contexts of students' experiences and thinking. Otherwise the use of mathematics continues to be restricted to "another world" that neither belongs nor is appropriated to "our world".

To comprehend the necessity to learn about global mathematics, students need to have their own authorship in the construction of mathematical knowledge in classes. This means that students should be actively involved in the search of contexts recognized by them as relevant to mathematization by means of their rational thought originating in their own experience and group context. In other words, the interest of knowing mathematics needs to combine interpretations from the local and in the global contexts to benefit future society, and each social group.

The dialogue between global and local mathematics encourages students to investigate their own culture and mathematical activity; consequently the role of teachers to legitimize cultural knowledge and to help students in this process of mathematical contextualization is of great importance.

TEACHERS AS RESEARCHERS

To enunciate clearly the teachers' professional competencies to face both the challenging of mathematics teaching and the social transformations of both the current classroom and society is a complex task. There is a consensus in considering that mathematical knowledge, conceptions about learning and teaching, the comprehension of students´ thinking and problem solving are the main issues to changing approaches to teaching mathematics (Fennema & Franke, 1992).

Regarding the specific challenges of teaching mathematics in multicultural classrooms, Cesar and Favilli (2005) observe that "teachers recognize the need for innovative practices, namely intercultural ones, but they do not change their practices according to the cultural roots and diversity existing in their classes" (p. 9). In respect to teachers' education and their professional development, the perspective of Ethnomatematics poses as a central issue the importance of the appropriation of theoretical-methodological tools to help teachers to comprehend the diversity of mathematical activity. Namely, to integrate local mathematics in teaching, into the organization of teaching practices and in the elaboration of didactical materials and tasks. As Gerdes (1996b) notes, teachers education should include training

> "to investigate ideas and practices from their own cultural, ethic and linguistic communities and to search ways of constructing their teaching from them on (…) and to contribute to the mutual comprehension, the respect and the valorization of (sub) cultures and activities" (p. 126)

Domite (2004) also highlights that one of the major contributions of Ethnomathematics to teachers' education is to bring into focus "students", ways of legitimizing their knowledge and the possibilities "to work on *learning's from*

outside school and from school" (p. 420). In this framework is central the idea of the teacher of mathematics as an ethnomatematical researcher (Stillman & Balatti, 2001).

Another important issue to teachers' education for multicultural settings is the role of schools in education. Considering that the development of knowledge reflects social, cultural and power relationships, and that schools are a place of knowledge transmission, it is necessary to reflect upon the social goals of school knowledge, questioning both the criteria underlying the choices of what to teach in schools and the challenges to face cultural diversity.

In short, the role claimed for teachers both as researchers of local mathematical practices and as critical mediators of the different ways of knowing and learning mathematics, requires theoretical and methodological training. Moreover the use of local mathematical ideas and practices into the curricula as tools for intervention in a global world assumes that teachers are capable not only of knowing about them but also of contextualizing them locally and discussing their potentialities and limitations.

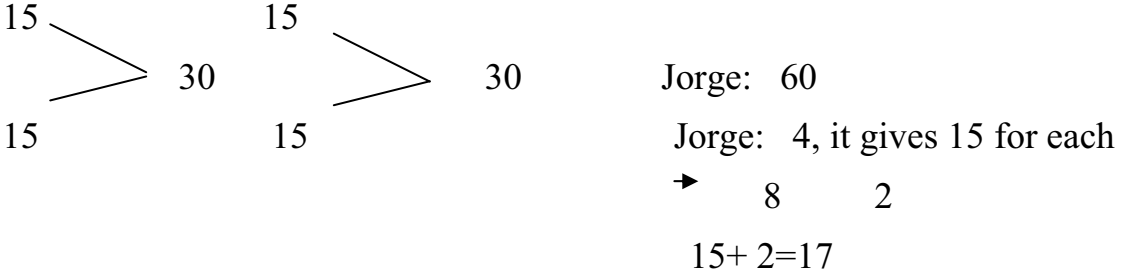
EXAMPLE: Portuguese gypsy children and mental calculations

Elementary teachers have illustrated that the predisposition to mathematics is notable among Portuguese gypsy children, despite their high dropout rate. Recent investigations developed in three Portuguese gypsy communities in different locations show that, in general, not only do gypsy children prefer mathematics, but also that they have fewer difficulties than in other school subjects (Cadeia, 2006; Ferreira, 2003; Pires, 2005). Using participant observation methods, from one to two years, these investigations documented Portuguese gypsy children's strategies of mental calculations and gather elements of their relationship with school mathematics. The data were collected among children, in their gypsy communities, and in fairs and schools (playgrounds and classes).

For example, Jorge, a nine-year old gypsy child enrolled in third grade, who used to go to the fairs with his parents during weekends or after school, not only used to help his parents by making known what to sell, but also helped by doing the calculation of the prices. His father trusted fully in his son calculations.

The following situations, fully described in Pires (2005), show how Jorge, actually, does his mental calculations.

To calculate 68: 4 =, Jorge reasoning was the following:



In the division, 369:3=, Jorge gave the answer right way: “123”. In regard to the division 643:2=, Jorge took a few seconds more than in the previous situation to give the answer: “321”, explaining that “it remainders 1”. When was asked, by the researcher, to explain his reasoning and to try to write down his thinking, he wrote the following notes:

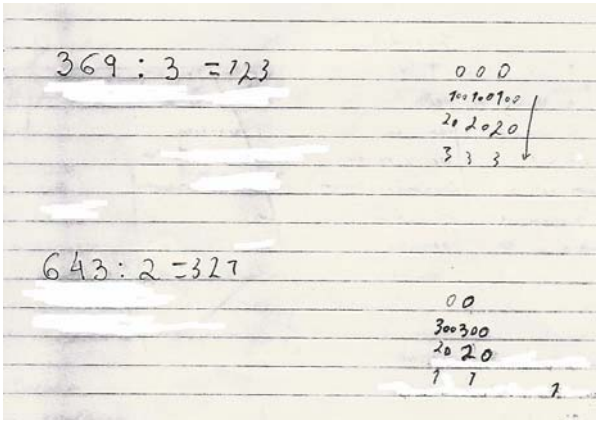


Figure 1 – Jorge notes in his notebook

Source: Pires (2005, p. 155).

On the right side, it is possible to observe his reasoning in both divisions, which was to divide by three (in the first division) and by two (in the second) each decimal class of each number. This is, to perform “369:3”, the three cents, in 369, was decomposed in 100+100+100; the six tens was decomposed in 20+20+20, and the nine units in 3+3+3. To perform “643:2” the six cents were decomposed in 300+300, the four tens in 20+20 and the three units in 1+1, resting one.

Another example, taken from Ferreira (2003, pp. 85-86), tells us about Gustavo, a ten years old boy enrolled in fourth grade, who, after giving good answers based on mental calculations to questions such as: the double of 12, the double of 25, the triple of 63, when the researcher asked him to solve the following operation, the child wrote 4010.

$$\begin{array}{r} 25 \\ +25 \\ \hline \end{array}$$

Gustavo’s answer reflects his reasoning which was to add twenty plus twenty, and them five plus five.

The following example taken from Cadeia (2006, pp. 89-90) illustrates how Cristiana, a twelve-year old girl enrolled in third grade, calculates.

Researcher: Cristiana, tell me how much someone should pay for three pieces, seven euros and seventy-five cents, each?

Cristiana: It does not exist. What a strange price!

Cristiana: Ok. How many pieces?

Researcher: Three pieces, seven euros and seventy-five cents, each.

Cristiana: Five plus five plus five make fifteen. Now, sixty plus sixty makes one hundred and twenty plus sixty makes one hundred and eighty. One hundred and eighty plus fifteen makes …one hundred and ninety five.

Researcher: One hundred and ninety five corresponds to what?

Cristiana: The amount of the three pieces.

Researcher: Did you forget something?

Cristiana: Ah teacher, wait a moment. The one hundred and ninety five cents makes one euro and five cents. Now, three pieces, seven euros each. It makes…seven plus seven makes fourteen plus seven makes twenty-one. It seems that little song that our teacher teaches us; seven monkeys and you are one.

Researcher: Are you calling me a monkey? Go ahead. Finish your calculation.

Cristiana: Twenty-one euros plus one euro, it is twenty-two euros and ninety-five cents.

Researcher: You make it. Was it hard?

Cristiana: It was very hard. Let me rest for a while.

The main base of gipsy children's learning processes continues to be done inside their families and communities. Learning is done especially by observation and reproduction of what they hear and see, while taking an active role and participating in the practices of their communities. Within familiar and effective contexts and throughout oral transmission, knowledge, practices and techniques are observed, experienced and imitated by children, who in this way learn and reproduce the knowledge of their social group. Actually, the education and socialization of gypsy children are not only mostly developed inside their communities but significantly children go along with their parents to work, especially to markets and fairs (Cadeia, 2006; Ferreira, 2003; Fraser, 1992/1995; Okely, 1983/1993; Pires, 2005; San Roman, 1980).

These three children's contacts with commercial activities in their communities immerse them in contexts of mathematical activity, resulting in knowledge and agility towards mental calculus. Therefore, Portuguese gypsy children' predisposition to mathematics, observed by their elementary teachers is grounded in their communities-based practices. Moreover, these three investigations also show that the strategies of mental calculations used by these children were not considered in classes.

As far as the research can tell us, we consider that the example of the mathematical predisposition of Portuguese gypsy children, especially in regard to mental calculations is a good example of the dissociation that exists between global and local

mathematics. Mathematical process used in these children' communities were not linked with the process used in school. If these mental strategies to calculate can be efficient in markets and fairs situations, they are impossible to carry out when problems involved another magnitude of numbers, for example. Cristiana was already sensitive to this situation when she observed that seven euros and seventy-five cents was a strange price for a piece.

FINAL REMARKS

Different social groups become pedagogically interesting for school based educative processes if schools learn how to use their resources in classrooms. In addition it is necessary to connect the local need of mathematics with global necessities.

In this perspective, only a school mathematical attitude that opens up the communication with local experience of mathematics allows negotiation among abstraction, mathematical content and local mathematics to move on into the role of mathematics in the process of the universalization knowledge. Students' authorship in the contextualization of local mathematics is crucial to confer meaning and utility to mathematical behaviors inside and outside school and to develop imagery where mathematics is present.

Agility in mental calculations, appropriated by Portuguese gypsy children in their community-based educative process deserves to be legitimized in classes and explored to enrich the learning of multicultural mathematics. Namely, such agility should be used to show the importance of local mathematics, how it operates and how it can be improved. Moreover it is also a good example to explore the need for a global mathematics. Nevertheless, in this particular case, if teachers are not aware of children's mental calculation processes and do not use them to know more about the role of local mathematics in mathematical knowledge in contemporary society, a good opportunity to educate the citizens of the world is lost. Therefore, teachers' training programs should include the need to educate for the ethnomatematical research teacher. This aspect deserves the utmost attention. Not only it is crucial to develop approaches to teaching that enables the students to draw in their local mathematics for the students' own learning, but also teachers, and students with their help, need to be able to search local mathematical practices bring them inside classes with other social practices where mathematics is essential. From elementary school, classrooms are the proper places to start bridging the gap between local and global mathematics, by presenting opportunities, grounded in local examples, to children understand why and in what occasions their local mathematical knowledge is good to solve problems and to plan and organize activities, or has limitations since, ultimately, they need to work out more complex problems and perform global mathematics inside and outside his community.

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MULTIPLE IDENTITIES IN THE MATHEMATICS CLASSROOM: A THEORETICAL PERSPECTIVE

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ABSTRACT

This paper explores perspectives on multiple identities in the mathematics classroom. Initially, identity is placed in relation to education to set the framework for the paper and it is argued that research grounded in specific categories, e.g. class, race or gender come with some limitations. Theories of discourse and social identity are considered for investigating the mathematics classroom. This leads to considerations of how dynamic identities may influence classroom discourse and vice versa and how identities can be related to possibilities and limitations in students' learning of mathematics. The paper concludes with some brief comments on doing research from a postmodern perspective where structures, if any, are fragile and where the notion of a final result no longer exists.

IDENTITY IN EDUCATION

Issues of identity and diversity feature heavily in debates on education. Perspectives on culture, gender, class, age and ethnicity are continuously held accountable for students' performance in the classroom or in the argumentation for design of educations and teaching materials (Appelbaum, 2002; Halsey, Lauder, Brown, & Wells, 1997; Moore, 2004). In educational discourse students are often placed in pre-defined identities which are used to determine or forecast their performance in school and higher education and predict obstacles on the way. Swanson (2005), in her study of discourse and political context in the mathematics classroom, points to the ongoing constructions of student identities as recontextualised elements of "outside" socio-political discourses are brought into the mathematics classroom. Swanson gives the example of black students being constructed as disadvantaged and as a result they are offered different possibilities for learning mathematics than those students not identified as disadvantaged.

Numerous studies have focused on African-American and Hispanic performance in school. Studies often attribute performance to ethnic backgrounds, the educational situation of indigenous people or the inclusion or exclusion of girls and women in education. These types of studies are centred round the defining characteristics of a student as coloured, indigenous or female and provide a framework for analysis rooted in pre-established and artificially fabricated identities from which the subject

under investigation cannot flee. Although these studies often shed light on serious matters, for example, how and why girls are excluded from mathematics education (Walkerdine, 1998), they can only tell one very specific side of the story, i.e. the story pertaining to an individual as black, female or indigenous.

The remaining part of this paper will examine an alternative approach to identities and categories as they come into play in the mathematics classroom and as they effect and are effected by discourse. This approach departs from a poststructuralist emphasis on identities as dynamic and constantly changing (Bauman, 1991; Bauman, 1996). This approach means a shift from thinking about identity as something we have – something tangible and constant, to thinking about identity as something we achieve or acquire along the way and discard or reject when they are no longer salient. This approach on the notion of identity has in recent decades given rise to new theoretical perspectives in the fields of social psychology (Turner, Hogg, Oakes, Reicher, & Wetherell, 1987) and discourse analysis (Fairclough, 1992; Gee, 2000-2001; Lewis & Ketter, 2004). Considering identities from this perspective can be seen as utilising a lens slightly out of focus. Focus is no longer on particular identities but is rather a blurred landscape of multiple identities, where discourse determines our immediate point of reference.

IDENTITY IN MATHEMATICS EDUCATION

Recently issues of identity have made their way into mathematics education and mathematics education research. Multiple socio-political factors have been recognised as influencing discourses surrounding mathematics education (Gorgorió & Planas, 2001; Powell & Frankenstein, 1997; Valero, 2002). Interpreting mathematics education through specific socio-political parameters can help us understand in detail how mathematics education is perceived in society and by individuals in terms of traditional socio-political factors and how these perceptions are brought all the way into the context of the mathematics classroom or the curriculum design process.

As was pointed out above, narrowing the lens to only focusing on one or a few socio-political factors we risk overlooking identities made relevant in e.g. classroom discourse and determining for individuals' or groups' inclusion or exclusion in mathematics education. As has been noted by Cobb and Hodge (2002), "*A crucial limitation of these institutionalized categories in our view is they do not necessarily correspond to people's own sense of identity*" (p. 258). Gee (2000/2001) note that "*A focus contextually specific ways in which people act out and recognize identities allows a more dynamic approach than the sometimes overly general and static trio of "race, class and gender*" (p. 99). Skovsmose (2005) illustrates how race can make educational researchers (and curriculum designers, politicians and other decision-makers) blind to influences other so obviously making the environment for teaching and learning mathematics unattractive. He points out that students in a run-down

school building with poor resources could have other motives than race for not engaging in mathematics education in a highly motivated manner.

Gee (2000-2001) presents four supplementary and interrelated ways to address the formation and sustainability of identities: nature identity, institution identity, discourse identity and affinity identity. These perspectives pertain to aspects of micro and macro level analysis but are all dependent on discourse and dialogue, i.e. identities enacted and recognised or "*being certain kinds of people" (Gee 2000 – 2001 p. 110)*[1]. Discourse is socially formed although each individual's engagement with various discourses forms individual trajectories and individual ways of narrating identity.

Swanson's example of disadvantaged students could be viewed as an expression of nature identity, as skin colour – being black – is determining for possibilities in mathematics education. As also noted by Gee it takes a particular discourse and dialogue to express this nature identity. Similarly, Skovsmose's example points to a nature identity of race and skin colour as determinant for researchers' thinking about coloured students. This led researchers to ignore factors of an uninviting or even hostile school environment in their investigations of students' performance in mathematics (Skovsmose, 2005). This ignorance occurred precisely as focus was kept on one specific aspect of identity, without taking other discourse identities into account. Given the unattractive school environment, were the students in a position at all, where learning mathematics was an option?

Much research has already been carried out in the field of mathematics education. Identities are often utilised which can be linked to Gee's notions of nature identity and institution identity i.e. race, gender or genetic dispositions as nature identity and class, background and institutional affiliation (as students, pupils or teachers) as institution (or societal) identity. To a lesser extent have the notions of discourse identity and affinity identity been explored and the question posed; which identities are brought out in the discourse of mathematics education and how do they influence our possibilities for teaching and learning mathematics? To examine this question it is necessary to shift focus of investigation from pre-ascribed identities to the discursive settings found in mathematics education. A natural place to begin is the mathematics classroom, although it should be noted that discourse identities will inevitably influence discourse and interaction of any kind and are not restricted to classroom interaction.

IDENTITY IN THE MATHEMATICS CLASSROOM

Moving from the abstract of mathematics education to the specificity of the mathematics classroom we enter onto centre stage for continuous enactment and recognition of discourse identities. Here identities are created, accepted, negotiated and rejected all in the process of teaching and learning mathematics. This is where

Gee's four notions of identity are expressed, elaborated and changed. To gain further insights into the processes leading to the representation of particular identities we can turn to the social identity theory which was later developed into the Self Categorisation theory. Social identity theory was first established by Henri Tajfel and John Turner. Tajfel (1982), and Turner (1987) provide a complex theory determining for our being and acting in any given discourse. Turner (1987) characterises social identity theory as:

> .. those aspects of an individual's self concept based upon their social group or category memberships together with their emotional, evaluative and other psychological correlates, e.g., the self defined as male, European, a Londoner, etc. It assumes that people are motivated to evaluate themselves positively and that in so far as they define themselves in terms of some group membership they will be motivated to evaluate that group positively, i.e., people seek a positive social identity (Turner et al., 1987).

This definition focuses on group membership but also opens up to the possibility that individuals do not always identify themselves as members of a particular group. Turner further elaborated on the relations between self and group as well as group influences in his self-categorisation theory stating that:

> The basic hypothesis is a cognitive (or social-cognitive) elaboration of the nature of social identity as a higher order level of abstraction in the perception of self and others. (Turner et al., 1987).

The self categorisation theory is based on a basic assumption that the individual possesses an infinite number of self-concepts and that the use of these self-concepts is mediated by the context at hand i.e. the discourse. The self categorisation process appears at different levels of abstraction and consists of a cognitive grouping of stimuli (or inputs) perceived to be similar to one group and in contrast to other groups of stimuli thus perceived to be different. For example, a group of mathematics teachers identifying themselves as mathematics teachers and are consequently in contrast to a group of biology teachers. The self concept can be in operation at multiple levels of abstraction and therefore the individual is not automatically and constantly part of a social entity or group. The individual may equally categorise himself as different to any available group in a given discourse (Turner et al., 1987). The self-categorisation theory does not assume a social cognition to form, but rather that one, as an individual, identify with different inputs in different discursive settings and in the process find it appropriate to bring different self-concepts into the discourse.

Instead of providing pre-fabricated identities, the self categorisation theory provides us with some tools to investigate discourse as it unfolds with its ongoing negotiation of identities. With its focus on social cognitive processes the self categorisation theory can not in itself be used to address discourse, but offers an insight into the processes of individuals and groups leading to their interaction in a particular

discursive setting i.e. a setting where some students learn mathematics and some do not.

Although rooted in the field of social interaction and focused on the dynamics of interaction the self categorisation theory has been criticised for being established primarily as a result of psychological experiments and for its emphasis on individual cognitive processes. This criticism has primarily come from scholars working in the field of Membership Categorisation Analysis where dynamics of interaction and identities are explored through analysis of everyday talk-in-action (Edwards, 1998). Condor and Antaki (Condor & Antaki, 1997) have demonstrated how theories of social cognition may contribute to discourse analysis, proposing that although rarely related to discourse studies the concern of social identity theory with collective self-presentation and the function of stereotyping in relation to social justice may be of value when analysing discourse. However, Condor and Antaki also emphasise the theory's focus on;

> …the mechanisms behind, and the consequences of, positioning oneself as a member of a social category (Condor & Antaki, 1997, p. 332).

The transcript below serves as a brief example of how the dynamics of identities and discourse may affect and be affected through mathematics classroom interaction. The sequence was recorded as part of a case study in a mathematics lesson for students at a Danish teacher training college. The study is conducted based on primarily naturally occurring talk-in-interaction (sound recordings of classroom interaction) which are subsequently analysed with a view to uncover identities established in the classroom discourse and their influence on the teaching and learning of mathematics.

T: Peter and Andrew. (Teacher calls students to the blackboard to present their work on a mathematics exercise).

L: So remember Peter it it is the critique we received and the compliments and so. So try and show it.

P: Yeah. But you made exactly the same mistake as I did sitting and talking. By in the middle of class you started to correct me. So we both did it. But it was very fun- funny.

L: Yeah. Just pass on the crap.

P: No, I am just sharing it. (laughs)

L: Oh yeah!

P: I I also take my part of…

L: Someone has to tell you that you that you need to keep quiet.

P: Yes.

L: But we should not discuss this now. We can do that afterwards. (P: laughs) What! It's true!

T: (sharply) L Keep quiet!

L: But it's not. But it's P who started it.

T: Sssshhhh

P: So I would like to …. (proceeds to present mathematics assignment).

The sequence illustrates how different identities or self categorisations can be brought out in classroom interaction within a short period of time. Two students are called to the blackboard to present their work on a mathematics assignment. In calling the students to the blackboard the teacher identifies the students as mathematics students. This identification is however interrupted as a student in the class reminds one of the presenters about some feedback they have previously received regarding their interaction in class. This identification which is accepted by the presenting student draws on past experiences and changes the immediate classroom discourse entirely. The discussion continues between the two students until the teacher interrupts by ordering one student to keep quiet. This leads the presenting student to return to his identity as a mathematics student and resume the presentation of the mathematics assignment.

In this example at least two discourse identities are brought out in classroom interaction. The identity of being a mathematics student and an identity pertaining to some past experience of feedback shared between two of the students. Although the two distinct identities pertain to and might appear relevant due to some past experiences i.e. experiences of mathematics and a particular mathematics assignment and experiences of shared and somewhat negative feedback on classroom interaction, bringing out the identities in class have some immediate implications for the current classroom discourse. The presentation of a mathematics assignment is interrupted until the teacher intervenes and Andrew the student also presenting appears excluded from this discourse identity. Thinking in terms of Turner's self categorisation theory Peter can be said to shift between two self categorisations and affiliations to others. First he is associating with Andrew as presenter of an assignment. Next he is associating with L in their shared past experiences of feedback, and finally he re-associates with Andrew as he proceeds to present.

As illustrated here by utilising theories of discourse and identity we may gain a better understanding of the dynamics of classroom interaction and the fragility of the mathematics student identity. It only takes the comment of one student to bring "disturbance" to the discourse identity at hand and the presentation of a mathematics assignment. The example further opens up to issues of how students bring their different backgrounds and past experiences into the classroom interaction.

IDENTITY AND POSSIBILITY

Considering discourse as a scene for creating, negotiating, rejecting and accepting fluctuating identities raises issues of possibility and impossibilities in relation to mathematics education. Which are the consequences of multiple identities in the mathematics classroom? How may we as teachers or researchers address this

complexity in an attempt to create positive teaching and learning environments? And how does discourse influence students in their learning of mathematics?

How we construct ourselves and others in our social interaction, and for example in the mathematics classroom, will inevitably influence our possibilities and limitations in the interaction process. If we construct ourselves as keen mathematics learners rather than bullies we are more likely to engage in a meaningful communication about mathematics. If we construct ourselves as a football fan or a mother in the mathematics classroom interaction, will it be meaningful to us to discuss issues of mathematics?

The construction or availability of identities is according to Turner (1987) determined by our past experiences and our current motives or what Skovsmose (2005) has labelled background and foreground. Identity thus involves not just our past experiences – what we know and believe but also our (perceived) possibilities for the future what we expect or wish. This in turn complicates the notion further as how we think of the future and what we choose to emphasise about our future can change at any moment in time, as we receive new information, as we process our experiences, as we encounter new scenarios. Identities are fragile and consequently possibilities for meaningful mathematics education are not relying on stable factors such as resources available or design of curriculum. Possibilities are as much relying on students actively involved in a discourse where identities include a positive attitude towards learning mathematics.

Returning to the above extract of a mathematics lesson at a Danish teacher training college the example can to some extent inform us of classroom possibilities and impossibilities both for the students actively interacting and for the class as a whole. Peter shifts his discourse identity away from that of presenter of a mathematics assignment on to one relating to past experiences of feedback. In doing so he also momentarily removes the possibility of presenting the mathematics assignment and to some extent he abandons Andrew. Shifting away from the identity of presenter also means that during the sequence Peter and L's discussion and utilisation of identities makes it difficult for others in the classroom to engage in the planned mathematics activity of presenting a student assignment. Eventually, the teacher resolves this situation of impossibility and non-mathematics identification by asking L to keep quiet, which leads Peter to return to an identity of presenter of a mathematics assignment and resume his presentation.

The sequence above provides an illustration of how easily students can remove themselves from identities relating to mathematics into identities of in this case identifying with some past experiences of feedback. The extract further points to students' backgrounds and foregrounds as these continuously inform classroom discourse and work as a constant potential. They constitute a potential for altering

discourse and therefore also for influencing possibilities and impossibilities (Alrø, Skovmose, & Valero, in press).

Possibility and impossibility in mathematics education are closely related to discourse and identity. This adds a new dimension to what it means to create positive teaching and learning environments. Naturally, there must be a focus on teaching materials as well as the ability of the teacher to communicate the mathematics content to the students. Added to this could be the constant awareness of classroom discourse and its effect on students' involvement in mathematics. Given the fragility of discourse and identity maintaining an identity in the classroom appearing positive to all students and embracing the teaching and learning of mathematics seems a great challenge as well as an ever changing challenge.

IDENTITY IN THE POST MODERN

Viewing a mathematics classroom – or any classroom for that matter – and seeing the presence of multiple identities and the complexities of their emergence and equally sudden disappearance can be both confusing and relieving and certainly may pose consequences to our perspective on the teaching and learning experience.

Bauman (1991) has described the post-modern man as a vagabond or tourist seeking to untie himself from life's restrictions. Where the man of the modern era sought to build an identity the man of the post-modern has as his mission not to commit himself to any particular identity over prolonged periods of time. In the era of the post-modern the curiosity of the social construction of human interaction remains on centre stage of research and is often expressed through various forms of discourse analysis and social psychology.

Some might call it naive to engage in research free of pre-ascribed identities or categories. And some will no doubt pose questions as to how one chooses to ignore underlying and hidden factors influencing classroom interaction. The response to such plausible and valid hesitations in accepting an alternative approach is that just as hidden or underlying factors need to be brought out into the open in our efforts to understand the dynamics of education, so does the actual classroom interaction with its identities, categories and discourse deserve a space in our constructions of education. I therefore choose to view perspectives on underlying factors and perspectives on actual classroom interaction not as incompatible but rather as supplementary. In addition perspectives open up to the possibility of locating identities and categories relevant to classroom interaction but not immediately imaginable by educational researchers. This could be identities pertaining to students' unique backgrounds and foregrounds yet crucially relevant in classroom discourse.

Exploring discourse and identity from a postmodern perspective is an uncertain undertaking. First, it is impossible to predict the identities and categories eventually proving relevant in the research. Second, the dynamics of identity and reality makes the notion of fixed results a notion of the past. Some may suggest that research of this

nature is without validity and can only shed light on particular moments in time only to be immediately outdated. Placing an emphasis on fixed categories and identities this is inevitably the case, however shifting focus to process rather than result, a postmodern take on identity, discourse and interaction could provide a new understanding on processes continuously creating and changing discourse and thus students' possibilities for learning mathematics.

NOTES

1. Gee (1999) has previously used the notion of situated identity when relating identity to discourse.

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STORYING MATHEMATICAL IDENTITIES WITH CULTURAL MODELS

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When we build our narratives of identity in interviews, we make use of "cultural models", culturally normative rules, schemas, ways of describing people, activities and ways of being. In this study we identify cultural models used by two mathematics learners in the stories they told us about their lives, including their experiences of learning mathematics and their disposition to study mathematics in the future. Each model provides a tool for identifying or dis-identifying with mathematics. While some models might be thought more 'positive', and others more 'negative', students positioned with strong imagined life stories or ambitions can turn these models to their own purposes. Thus maths is 'hard' can sometimes be storied as 'challenging and fun' rather than 'too hard and boring'. The significance of the educational culture is discussed.

INTRODUCTION

In part of our Widening Participation project *Opening doors to mathematically-demanding programmes in Higher Education (HE)* (www.lta.education.ac.uk/TLRP.html) we are exploring the ways in which young students tell of their disposition to go into further study, particularly in Higher Education, and particularly to study courses in which mathematics might be relevant. One part of the research involves a survey and *quasi experiment*, in which measures of 'disposition to study in HE' and 'disposition to study more mathematics' are constructed as outcome variables. However, to complement this quantitative study we are also conducting multiple case studies of students as they progress through a year or so of Further Education (usual age 16-19). We are interviewing the students about their 'background history' (including whether they would be the first in their family to enter HE, etc.) their experiences with mathematics and disposition towards future study; their current attitude to their experiences in mathematics classrooms is also of interest, as this helps reveal how their identity work makes use of or is influenced by pedagogy.

Our project is informed by the Cultural-Historical Activity Theory (CHAT) perspective on identity in practice, the corpus of which is usually attributed to the original Marxist troika of Vygotsky, Leont'ev and Luria, as interpreted and developed in multicultural contexts its later generations by Cole, Engestrom and others. (See Williams & Wake, 2006a,b; Roth, in press; also Roth, Hwang, Lee, and

Goulart, 2005; and websites such as www.edu.helsinki.fi/activity/people/engestro/). Many CHAT scholars such as Wertsch, Wells and others also incorporate Bakhtinian and even Hallidayan discursive and dialogical constructs of voice into the theory (see eg Holland 1998; and many works by Gee, such as Gee, 1999, 2001). Even Bruner's account draws powerfully from the Vygotksyan perspective (Bruner, 1996).

Identity is here seen as emerging from engagement in joint object-orientated activity; socio-culturally mediated 'activity'. This is because, as Vygotsky pointed out, the use of 'psychological tools' (such as language) in practice is always double edged, what is used in social interaction comes reflexively to be used internally, on the self. Thus, one 'becomes' what one 'does' and, importantly, one comes to 'think' what one 'says': reflexivity and self- and social positioning ensures that we become the player that we perform (see Jenkins,2004, for an account that integrates the social theory of Goffman, and Barth). While 'activity' is always driven by a 'community' motivation the individual is always 'positioned' (by self and others) in a division of labour, held in place by 'rules' governed by cultural norms and expectations. Holland and Quinn (1987) originally developed the notion of 'cultural model' to describe the culturally derived rules and schema used by students to tell stories of themselves and their everyday activity- tools for doing their identity. Gee and others have expanded the concept to include everyday cultural concepts and conceptual frameworks that govern what we can perceive, but also what we can tell. Thus, cultural models provide a resource – and a constraint – for reflective identity work.

We find the more recent notion of a kind of cultural landscape of 'cultural models in figured worlds' evocative (see Holland et al., 1998); one's narrative of identity can be told as a path through our available 'figured world'. Thus, a student might tell of their identity as someone who "likes to work alone … and always needs to know there is a 'right' answer." Such self-identification might lead to such a student coming to be regarded – and to regard themselves - as a potential mathematician, if the dominant cultural model of doing mathematics in the culture/subculture is that of a lonely, black-and-white activity. Many in our culture tell of mathematics as a 'hard' subject; a student who wants to position themselves at a distance from mathematics may say "mathematics was too hard for me". But for another, it turns out, mathematics is hard 'but challenging'.

Thus, in our theoretical framework, cultural models such as 'maths is hard', 'maths is black and white', 'maths is lonely' or 'maths is for geeks' (all models we have found in our interviews) are models that students learn in practice from social activities in general and classrooms in particular, and they can provide tools for students doing identity work of various kinds. Students are presumably not totally free to narrate themselves as they wish using these models, first because they may not wish to align themselves with a particular position (e.g. 'being a geek') and second, because positioning oneself is always a 'claim' that may be subject to social confirmation, or dispute. For us then, it is an empirical question to answer: "how can/do students draw

on cultural models in telling stories about their identity (in relation to mathematics)?" and "how can/do these stories produce/reproduce a disposition to study mathematics, or not?"

Our methodology is informed by this theoretical framework: we are pursuing interviews with 50+ students (three each, transcribed) over the course of a year or so during which their early pre-university course studies gather pace and when they make decisions about university applications. These students were selected to ensure that 'middle' attaining students likely to drop out of maths at advanced level and those 'first generation into Higher Education' families are included. In interviews we ask about their biography, their dispositions and future intentions (we also have survey instrument data including dispositions that serve to locate these students against the canvas of a larger population). We also ask about their engagement in mathematics classes (some of which we observe). In the following we examine two cases of students with low-ish grades who both chose to do advanced mathematics despite having a difficult time at their previous secondary school: in their interviews they used some of the same models as resources but in very different ways, partly reflecting their very different experiences in their current College mathematics programmes. These are chosen for analysis because they provide such a contrast: one engages with mathematics in her classroom as a sociable activity in which she is always 'having a go'. The other is withdrawn and even isolated in his class, positioned as being 'struggling'.

One might think that the common cultural model 'maths is hard' is one that is used by low attaining students to speak of their dropping of the subject, and this is the way the model comes to be used by the student who is becoming isolated from mathematics. However, for the 'engaged' student it becomes used as a positive resource for narrating herself as an engaged mathematics learner.

GEMMA'S STORY

Gemma will be the first from her family to go to university. In fact she cannot name anyone she knows in her circle of friends and family who has been to university. But there is no question in her mind that she WILL go, she says, "I've been going to uni since I was 8". She has lived 'locally' all her life in a community that has all the 'poorest' social indicators. Her principal and teacher described, with almost ironical pride, the local community as sitting regularly at the bottom, or near the bottom, of every league table of performance and social index of deprivation the government has produced. Gemma tells us that her mother's work as a cleaner and shop worker is stressful, which has helped to motivate her as "I see my mum, like, working in a shop and cleaning and I don't want to do that, so that's kind of influenced me in my own work not to follow that path cos she gets stressed out and stuff". Gemma tells us several times that her mum has been very supportive of her and encouraged her ambitions all her life (as has her mother's partner). She did well at Primary school: "I was always into books at school and I was always levels ahead". She said that getting

level 5 – a very high grade - at age 11 in the National tests was an important marker for her. She experienced her Catholic primary school as relatively – compared to secondary – 'inspiring'.

At age 8 she decided she wanted to become a marine biologist so she could work with Orca whales: "I've just always taken a fancy to Orcas, ... Killer whales, ... Free Willy is my favourite movie (laughs)." Even though her mother thought she would 'get bored' of this particular ambition, Gemma has stuck with it and her mum has continued to support her; she got advice during secondary school from the 'connections' service and knows exactly what she has to do in her AS and A level grades in science and maths to get to university and then to do a PhD in Marine Biology. She knows she will spend 6 years at university and which one she wants to go to for her studies, as it has a connection with research into Orcas. In fact she tells us that the field she will need to follow to get to work with Orcas is more specialised, those who study big sea animals are called marine mammalogists, and "you have to be one of the top ones" to get into it.

Her experience of secondary school was very mixed, with classrooms being boring and classroom behaviour off-putting. The teaching was often uninspiring and she lost interest or a while: "From when I went to secondary school I lost interest in quite a lot of my study … at (Primary school) there was more passion in it while at secondary school it was just "you've got to get through this.." In contrast to her self-ranking in mathematics at Primary school, she says now "I wouldn't class myself as that good but maybe a bit above average". However, she describes maths as being 'challenging' rather than hard: "… there was a lot of noise in the class … [disruptive?].. yes; but I enjoyed it and it was a challenge as well …"

Gemma got a modest grade in her final mathematics examination at age 16, and then did a statistics course, not being allowed to do the higher mathematics course for some reason. She would be considered a 'high risk' according to statistical trends at post-16, and in many Colleges she would not be allowed onto the Advanced Mathematics course. She says she was worried she might not be able to cope with the Algebra on her current course but actually feels she is doing well, and is enjoying maths now, "... it's a good system here: it gets the whole class involved and you get to hear how others do it and if it's better you can learn" which she compares to the dominant teaching practice in secondary school which is "boring and you forget it…"

Her attitude to mathematics seems to have undergone a transformation since going to 16-19 College: "I am liking Maths as much as I like Biology which is my favourite subject … so I'm getting really… liking it compared to before." She explains why: Maths is more 'engaging' and she can express her opinion and hear what others have to say – she even mentions the interactive work with white boards and posters... and maths is now described as 'fun'.

This story – we call it a 'restorying' as it has been constructed by us from her interview - tells us several interesting things about her identity in relation to further

study, Higher Education and Mathematics. While her family background does not provide any 'role models' of people who went to university, she has developed an ambition that her family supported, and her mother's 'stressful' experience of work has influenced her positively. Her childhood ambition has been nurtured by family and shaped by the education service, and has matured into a career ambition. She knows she will need maths, challenging/hard or not, to achieve her ambition. But recently her enjoyment of mathematics seems to have returned and we can speculate that this will help in some way: we will see when we meet her again in six months time.

The model of a marine biologist and the inspirational film "Free Willy" seems central to her particular story. Many students of this age, just beginning at 6th form of Further Education (FE) College, simply say "everyone I know is going to university" or even "I like science so I'll probably do something in the sciences but I am not sure what yet." However, the principal of Gemma's College told us that stories such as Gemma's are not unusual: one boy who had been on a work placement developed a specific ambition to work on a particular machine in an aircraft laboratory, and had worked it out that he would need to have a degree in Aeronautical engineering to get into the necessary training programme in the aerospace industry. Thus we speculate that this might be a more common characteristic of some narratives of students from class backgrounds that do not provide many personal 'models' of university graduates in their community or family circle.

The particular way that Gemma constructs herself as having a positive relationship with mathematics might be relevant here, we speculate: her early imagined lifestory of 'becoming a marine biologist' comes together with her success in Primary school, and "getting a level 5". As she develops her plan in secondary school she finds out that mathematics will be important to becoming a science student at university. This apparently positive synergy could perhaps have been expected to be dampened or even destroyed by a dull experience in secondary school, but (i) she is at least as good as the average of her peers in mathematics, and (ii) her family – especially her mother – encouraged her, and she finds energy from the thought that she does not want the stress of her mother's life as a manual worker.

IMAGINING A LIFE STORY: THE JOURNEY METAPHOR

We have speculated that Gemma's positive disposition to study mathematics is in part at least sustained by her imagined lifestory of a university science career leading to a career as a marine biologist. We see other important sustaining resources as well, such as her success in Primary school, her relationship with her peers and her recent positive experience of maths learning. These resources are important in providing her story with energy, and we imagine some were central to her in Primary and others later in Secondary school. But we suggest that in her story now there is a prominent central 'leading thread' in her work-ambition: this seems to be the central element

that provides for a 'positive disposition' towards mathematics, which in turn encourages her to see maths as a challenge rather than too hard.

Let us consider for a moment the way a life story like Gemma's comes to be imagined: and also, by way of contrast, let us consider how the story, with much the same resources, might have been different. Gemma accepts, but makes use of the notion that mathematics is 'hard' for her: rather she posits maths as 'challenging' and she likes a challenge. But we know that for other students, mathematics is 'hard and dull', or 'too hard' and thus becomes something to be avoided. Within another lifestory, Gemma might well have adopted this notion as a means of representing a different disposition, of telling a story of a different person and imagined life. Lakoff and Johnson (1999) analysed the metaphor of 'life as a journey' and this seems apt to our present analysis. The 'imagined journey' in Gemma's case has a clear beacon in the distance, envisioned for her initially in film, and through other media later. In her narrated 'troubles', she meets many obstacles along the road, but also resources: she avoids the secondary school abyss and identifies a challenging climb there. There is an imagined slough of despond in future manual labour that she strives to avoid. In some cases perceived obstacles turn out to be friendly, what was said to be 'hard' to do becomes perceived as 'challenging'. Yet each 'resource' has a potential downside: if one experiences failure too often perhaps 'challenging' will become 'too hard', and the immediate part of the journey too difficult.

On the face of it a 'cultural model' seems to afford a way of overcoming an obstacle on one's life path through one's cultural landscape or 'figured world' and as such is a bridge for all who want to pass that way. But some models are not like that; turning an obstacle like "maths is hard" into an affordance as in "maths is challenging and I like a challenge" may be more possible for some students than others. It is as though different students are offered different landscapes to do their life journey through, that is, different social groups are offered different figured worlds. The educational institution and classroom, and pedagogy appear to provide different tools also. To illustrate this, we now present the narrative of another student, Lee, who constructed a less positive account of his experience in studying mathematics and his disposition to continue doing so.

LEE'S STORY

Lee also hopes to be the first in his family to go to university, although unlike Gemma, he says his mother has a fairly high status job [Deputy Manager in a bank] which he describes as 'quite up there'. Lee is also less clear than Gemma regarding his future career trajectory stating that he wants to "get a good job" but "to get a good job, you got to go to Uni". When asked what he will study, he answers "I am not sure, one of the subjects I am doing now at A level probably" [Politics, Psychology, Sociology and Maths].

Lee says that he was one of "the clever ones" at primary school, with Maths as his strongest subject. When asked if he liked maths, he says: "Yes. I was always good at maths, pretty much. (…) I never struggled in maths, it was pretty easy. I got like top levels and stuff, in primary."

However, at secondary school he says he lost interest: "No, it's not something that interests me. I am good at it, but it didn't interest me. I don't see the point of it" and identifies his relationship with his maths teacher (the same one throughout secondary school) as particularly problematic. Despite his lack of interest, Lee got a Grade B at Higher level in his final exam and chose to take up Advanced maths when he started college because it would "look good for university". However, he began to struggle with the subject and as a consequence, was encouraged by his teacher to transfer courses from Mathematics to 'Use of Mathematics' as an easier option.

> "And I was doing... all right in maths, and then I just…I don't know. I was like that in all subjects because when you…like do a step up to college, it's quite different to secondary, isn't it? […] Because it's more work, it's like less time spent on one particular way, […] And I just weren't used to it. […] And then, like the math teachers, …they were saying if you are struggling now you won't pass your January mock, you know there is an exam in January. So if you struggle…and because like I got a couple of bad results in the tests, like practice tests kind of thing. I got to kind of lost it one day. […] So it was like December, November time, he said 'I think it's best if you do Use of Maths which is like this course', so I was like 'Oh, it's way too late now'. He said 'you'll do all your exams in May, June, whenever it is', and he says 'you will do it at the Uses and you should do coursework, someone like you will do that easy and pass quite easy'. […]

Lee says that he thinks the reason he struggled was because he did not keep on top of the workload and it was no longer possible to "just go through the course and pass it like …quite easily like I did (before)". But the Use of Maths course has also been problematic for Lee and he predicts that he will "completely fail the course". He has tried dropping out a number of times and has remained in the class only because his teacher has insisted he stay. This, he feels is a waste of time:

> "[…] I have to do 4 exams, and I know I can't do it, I have not done the coursework, so…it's just a waste of time for me. I could have concentrated more in my other subjects."

Lee particularly identifies his isolation from the rest of the class as central to his negative experience:

> "[…] like I sat over there, because that's where my chair was because all other seats were taken, I am just sat there and it's like just pop in alone, day dream, because it's no like nobody to tell you, you've got to…work kind of thing. Because there is no pressure to finish."

When asked why his relationship with maths deteriorated, Lee draws on the cultural model of 'maths as too hard'. He says "it was like, you got all like harder stuff

coming up obviously, like formula, and stuff like that" and that "I just don't see the point". Indeed, the fact that the Use of Maths (UoM) course was presented to him as easy but turned out to be harder than he expected is central to his account of why he is going to fail.

> "So when I came here [UoM] I wasn't interested in the first place and then it was harder, I wasn't interested. [ ]…I think if I'd really gone on on it [the previous maths course] I would have been able to pass at least, but then I got told that this course is …you were going to get UCAS [university] points and stuff, and that it was quite easy…"

He also questions the relevance or "Use" of maths to his future plans in life:

> "Like, they say it's real but I don't want to know about how much coffees and coffee (are?), …[ ] No, it's not relevant to me. I don't need to know that. [ ] But I don't need to know how to do trigonometry, in everyday use. So I don't see that as real life context. Unless I am going to be like…whatever you need to use it for. Because maths is just like equations and stuff like that, and numbers, […]…you either like maths or you don't. I don't like it, so…that's how I see it."

The way Lee constructs his story regarding his relationship with maths highlights how not all students are equally positioned in terms of drawing on particular cultural models. Like Gemma, Lee draws on the notion that 'maths is hard' but instead of using this to positively align with the subject as 'a challenge', he uses it to position himself away from maths because it is 'boring' and 'not relevant to me'. We suggest this more negative use of 'maths as hard' as a cultural model may relate to Lee's disposition as marginalised from maths in college. In both his Maths classes, Lee has been positioned as a 'struggling' student by his teachers and even his physical location, seated on the margins of the UoM class (as a late entrant), appears to have exacerbated a sense of increasing isolation. Thus, we argue that the way Lee uses the notion that 'maths is hard' in his lifestory is an attempt to reflexively distance himself from the marginalised social position his teachers have assigned him, and which he has accepted.

This suggests that crucial to the use of cultural models in students' narrative accounts is *how they are positioned* by teachers and institutional policies, e.g. who 'can do' and 'can *not* do' mathematics. Gemma, who in another college would have been prevented from taking Advanced mathematics, is given the space to construct her narrative around the notion that mathematics is 'challenging,' providing potential energy to 'maths as hard' that could otherwise become 'maths is too hard'. Meanwhile, Lee finds himself marginalised from mathematics due to his insititutional position as a 'struggling' student and consequently finds a way to construct a narrative which reconciles this sense of 'not belonging'. But in another sense, the idea that mathematics as 'too hard, irrelevant and boring' may also provide potential energy to Lee's identity work - as one who does not see any 'use' for mathematics, as a non-mathematician, perhaps one who will study politics etc. (there is not enough of a biography in the above account to see how this will work out positively yet).

These two narratives illustrate how in understanding life stories we need to look at the affordances and constraints of the relevant cultural models that students can use, and in each case ask how they might be resources for representing distinct dispositions, e.g. expressing students 'leading' ambitions on the one hand, or obstacles and troubles for students on the other. We suggested that one leading social influence might be a career ambition; in other cases we have tried and failed to detect any ambition or motive, with students apparently 'living in the moment'. We have seen some signs that such students are likely to be particularly suggestible to short term and local influences, including their social experiences of the mathematics classroom .

However, not all students have equal access to the world of cultural models we have identified: while they have some agency in shaping their positioning, or disposition, they are not freely and equally positioned in society, in their institution and in their classroom. One student finds herself with no 'real' graduates in her family to serve as 'models', but looks instead to the imaginary, fantasy world of films for a marine biologist graduate. Another student told us she has just one example of a graduate in her family, and that the person concerned had a bad time, ending a course with huge debts. She is unsure of whether she should risk the experience, but is continuing at College anyway because there is nothing else to hand other than working in a shop.

The College institution too offers some access to Maths with a relatively risky, weak, previous grade whereas others deny the students this. Our fieldwork provides explanations: some Colleges are themselves positioned differently from others in terms of funding and the student 'market'. In order to maintain a high reputation (mediated through league tables of performance indicators) some Colleges refuse to recruit weak students to mathematics courses, while other Colleges opt out of this competition (or for other reasons are not so competitively positioned) and go for an 'open access' institutional policy that then puts a premium on making mathematics accessible to a wider group of learners. Similarly some classrooms offer a 'sociable maths' that is 'fun' and interactive, where for others the dominant model involves 'working on your own'. In this respect many students' interviews have suggested there is hope that pedagogy can really make a difference: apart from providing role models, the classroom or College institution may also offer multiple models of learning and hence 'ways of being' a learner of mathematics. This is what the project is currently investigating.

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