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**WORKING GROUP 2:
AFFECT AND MATHEMATICAL THINKING**

Chair: Markku S. Hannula

University of Helsinki, Finland, and Tallinn University, Estonia

Co-ordinators:

Peter Opt'Eynde, University of Leuven, Belgium

Wolfgang Schläglmann, University of Linz, Austria

Tine Wedege, Malmö University, Sweden

INTRODUCTION

Affect has been a topic of interest in mathematics education research for different reasons and from different perspectives. One branch of study has focused on the role of emotions in mathematical thinking generally, and in problem solving in particular. Another branch has focused on the role of affect in learning, and yet another on the role of affect in the social context of the classroom. Affective variables can be seen as indicative of learning outcomes or as predictive of future success. The different approaches that have been used in the study of affect include psychological, social, philosophical, and linguistic. Also the range of concepts used in this area is wide, most frequently used terms have been beliefs, attitudes and emotions. Less frequently used, but not necessarily less important terms in this field include anxiety, confidence, self-esteem, interest, values, motivation, needs, goals, and identity. In Working Group 2, "Affect and mathematical thinking", we welcomed all these and still other perspectives into a discussion that aimed towards a deeper understanding of the role of affect in mathematical thinking and learning.

One of the goals of the working group was to enhance discussion in the CERME - congress and research between the congresses. At CERME 5, Working Group 2 created an atmosphere of collaboration among its 25 participants. A call for paper took place and as a consequence of a reviewing process, 20 papers from 14 countries were accepted for presentation. The congress program scheduled seven sessions, each 105 minutes, for work in the group. The co-ordinators worked out a plan for these sessions where a presentation of the key ideas and results of the accepted papers should take place, followed by a general discussion of these key ideas. The eighth session on the last day of the congress was used for a summary of activities during the conference and highlighting important research questions for the following years.

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Papers discussing similar topics were grouped together in the seven sessions under the following headings: Meta-aspects of affect; Motivation and mathematics; Affect and self-regulation; Researching children's affect; Measuring students' beliefs and attitudes; Beliefs and attitudes in mathematics learning and teaching; Changing beliefs and attitudes.

In the work presented and discussed, we located two main concerns for research and practice:

1. The structure of affective domain and its effect on mathematical activity in classroom
2. The development of affect and intentionally changing affect

THE STRUCTURES OF THE AFFECTIVE DOMAIN AND ITS EFFECT ON MATHEMATICAL ACTIVITY IN CLASSROOM

Within the domain of mathematics education, the work by McLeod (e.g. 1992) has been very influential to the conceptualisation of the affective domain. In his conceptualisation of affect beliefs, attitudes and emotions are located one dimension where one end refers to cognitive, stable and less intense affect (beliefs) while the other end refers to affective, less stable and intense affect (emotions). However, more recently several authors have argued for a need to move beyond this conceptualisation (see e.g. Hannula, 2006; Zan, Brown, Evans & Hannula, 2006).

This was also the spirit of CERME 5 group on affect. We need to clarify the definitions of the concepts we use, we need to broaden the field by introducing new concepts, and we need to be more specific about how the different concepts are related. In the final session of our working group, Peter Op 't Eynde presented a figure which summarises the different concepts that were discussed during CERME 5 (Figure 1).

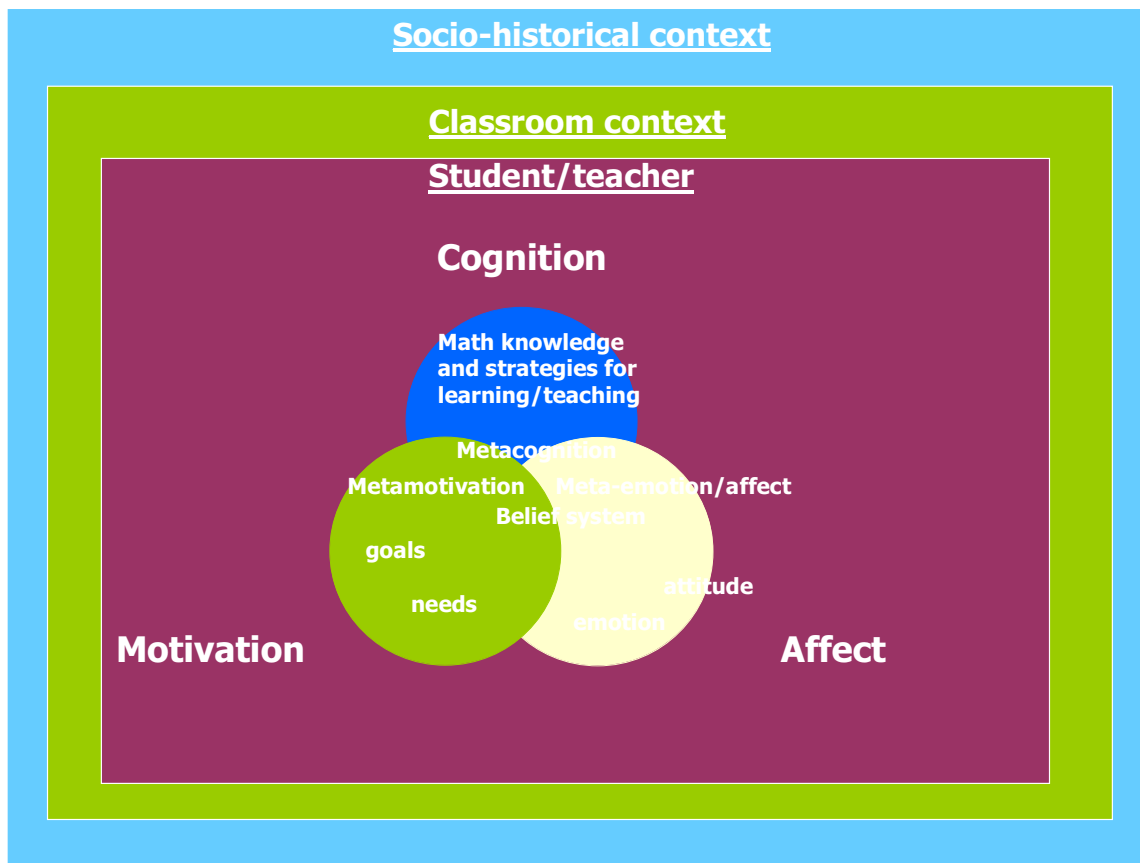


Figure 1. Structure of the affective domain

Elaborating concepts and their relationships

Several papers made significant elaboration on some of the concepts or their relationships. Most notably the concept of motivation was used in several papers. One of these was theoretical, discussing the relation between two different conceptualisations of motivation for learning mathematics: intrinsic - extrinsic distinction as defined in Self Determination Theory and Mellin-Olsen's concept of rationale for learning mathematics (Wæge). The other papers on motivation were empirical, focussing on students' motivation in mathematics (Athanasiou & Philippou; and Panziara, Pitta-Pantazi & Philippou). Motivational beliefs and goal orientations were found to be important factors but it seems that some of the constructs depend on student age (Panziara et al.).

Two papers focussed on identifying the structure of mathematical beliefs (Rösken, Hannula, Pehkonen, Kaasila & Laine; and Diego-Mantecón, Andrews & Op 't Eynde) By means of factor analysis they obtained different dimensions structuring mathematical beliefs. Both of these studies support some of the earlier factorizations of belief systems and they provide scales that have good reliability. Most notably they confirm the following aspects of mathematical beliefs:

- Beliefs about mathematics (e.g. difficulty, enjoyment)
- Beliefs about the self (e.g., goal orientations, relevance, self-efficacy)
- Beliefs about the (classroom) context (e.g. teacher's role)

The effects of affect

Several of the studies were interested in the relationship between affect and achievement. Eleftherios and Theodosios had executed a quantitative study of Greek students' beliefs and attitudes concerning mathematics and their effect on mathematical achievement. Nicolaou and Philippou looked at the relationship between self-efficacy and performance within the context of problem posing.

However, there are somewhat counterintuitive results regarding the lack of improvement in self-efficacy when performance was increasing (Marcou & Lerman). In the discussions it was hypothesized that this may be due to the fact that self-evaluations are made among peers. When the whole group is advancing - as was the case in their study - the pupils see no advancement among their reference group.

Schlöglmann discussed the two types of errors in problem solving processes: "misconceptions and errors called "slips". To explain the emergence of the latter he used the concepts of working memory and workspace, and elaborated the usually unconscious impact of affect in attention.

Contextualizing affect

In addition to recording the beliefs and attitudes of the students, it is also important to look at how the more general social variables such as social status, type of school and gender are related to differences in students' affect and achievement (e.g. Andrews, Diego-Mantecón, Op 't Eynde & Sayers; Athanasiou & Philippou; Eleftherios & Theodosios; Panziara et al.).

Another way to look at the effects of context is to study the affect of a specific group in a specific situation:

- student beliefs in a self-regulated mathematical problem-solving environment (Marcou & Lerman);
- beliefs and goals of teachers in a professional development project (Hannula, Lepik & Kaljas);
- affect in mathematics teacher students' written essays about their school time experiences in mathematics (Hoskonen).

Lange discussed how the context should be taken into account when doing qualitative studies with children. He elaborated the notion of children's perspectives on mathematics starting from children as social actors with their own ways of constructing meaning and interpreting their world, and that meaning is what children ascribe to their actions in the field of school mathematics learning.

Developing the tools to measure students' beliefs and attitudes

Many of the papers had refined some of the methods to measure affect. Some were more explicit about the methodological implications than others. From the point of view of developing qualitative methods to research affect, the discussion by Lange was most important

There were three presentations in the Working Group, where the focus was to develop or test a questionnaire to study students' beliefs. The development of such an instrument is intrinsically related to also defining the concepts and their relationships. Rösken et al. focussed on the systematic character of beliefs in a sample of Finnish upper secondary students. By means of exploratory factor analysis they obtained seven dimensions structuring this construct. Diego-Mantecón, Andrews, Op 't Eynde and Sayers presented two related papers. First, Diego-Mantecón et al. described an adaptation of the mathematics-related beliefs questionnaire (MRBQ) developed at the University of Leuven in Belgium. They were able to increase the reliability of the scales and confirm its applicability to Spanish and English secondary students. In the second paper, Andrews et al. discussed the effectiveness of the revised instrument as a means of discriminating between the mathematics-related beliefs of students from schools in England and Spain, and examined its potential for distinguishing between gender and age.

In the discussions we identified several challenges for the future. One almost classical problem is the difference between espoused and enacted beliefs. Tackling the differences between mathematics and school mathematics is another challenge. There was even discussion on how to properly deal with contradictions that characterize belief systems. It was acknowledged that it is difficult to take into account the socio-historical background of the students. The development of a multi-method approach was seen as a fruitful way to meet these challenges.

THE DEVELOPMENT OF AFFECT AND INTENTIONALLY CHANGING AFFECT

The conceptual analysis of affective domain in relation to change has led to introducing new concepts, such as self-regulation and meta-affect that are able to tap the dynamic aspects of the belief systems.

Meta-aspects of affect

Meta-affect was first introduced in mathematics education by DeBellis and Goldin (1997). Meta-emotion/meta-affect includes an awareness of the emotion as well as of the action to control and regulate it.

Moscucci discussed 'a meta-belief systems activity' on the basis of learning experimentation, where the importance of making learners aware of their belief systems regarding mathematics became apparent. Panaoura looked at the more cognitive aspect of meta-affect in her study of the impact of recent meta-cognitive

experiences on pre-service teachers' of pre-primary education self-representation in mathematics and its teaching. The stability of the students' self-efficacy beliefs about learning and teaching of mathematics was also examined as an indication of self-image.

Affect and self-regulation

Self-regulation strategies as the general term include cognitive, motivational, and emotional regulation. Some regulation is highly conscious while some of it remains inaccessible to consciousness. (For elaboration, see e.g. Hannula, 2006)

Schlöglmann discussed how affect influences attention and how this automatic (and dysfunctional) self-regulation may lead to certain types of errors in mathematics. However, students self-reports indicate that they use emotional regulation strategies in relation to mathematics learning also consciously, although not very often (Op 't Eynde).

Teaching self-regulation strategies seems to have an effect on performance but less on (self-efficacy) beliefs (Marcou & Lerman).

Discussion and challenges

In the discussions we identified several needs to deepen our knowledge on these topics. Important questions for future research are:

- The relation between meta-emotion and metacognition
- Self-regulation of emotions in learning contexts
- The knowledge and skills necessary for efficient self regulation
- Analyzing teaching practices that stimulate the development of self-regulation
- Conscious and subconscious regulation of affect and motivation

Changing beliefs and attitudes

Several studies were interested in the development of affect under specific influence, such as didactical games (Vankúš), a reform oriented mathematics competition (Wedegé & Skott), a self-regulated mathematical problem-solving environment (Marcou & Lerman) and across the transition from primary to secondary school (Athanasίου & Philippou).

Awareness and reflection were identified as powerful tools for change, but the emotional plane in many ways provides the necessary conditions (uneasiness, aha-experience, feeling of joy/safety,...).

The theory of conceptual change was suggested as a fruitful framework to study changing beliefs of pre-service elementary school teachers (Liljedahl Rolka & Rösken)

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Change of the affect was identified as one big question that interested all participants. Identifying more specifically what causes the changes is a tricky problem, as we cannot outrule the Hawthorne effect in any interventions. Moreover, we need to address the socio-historical background, for example, through using intense qualitative instruments (log books, story telling,...). We also need to study in more detail the processes of change - the interactions between (meta)cognitive and (meta)emotional processes. On the other hand, the stability of change is an important question that requires yet different approaches, such as longitudinal studies.

SUMMARY

In each CERME an effort is made to identify some emerging or important themes that might reflect the field in general, not only those studies presented in the conference. The refinement of more specific constructs has continued, as well as the linking the cognitive and the affective/motivational. Self-regulation and socio-historical perspective seem to be theoretical frameworks that are becoming increasingly important. The multi-method approach is becoming almost a norm in this area of research

The work will go on and we will have another working group on affect at CERME 6.

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EVALUATING THE SENSITIVITY OF THE REFINED MATHEMATICS-RELATED BELIEFS QUESTIONNAIRE TO NATIONALITY, GENDER AND AGE

Paul Andrews¹, Jose Diego-Mantecón¹, Peter Op 't Eynde² and Judy Sayers³

University of Cambridge, UK¹, University of Leuven, Belgium² University of Northampton, UK³

In a paper presented earlier at this conference we discussed our adaptation of the mathematics-related beliefs questionnaire (MRBQ) developed at the Catholic University of Leuven (Op 't Eynde and De Corte, 2003). The revision, like the original, yielded four factors, and a number of sub-factors, which analyses showed to be reliable and confirmatory of the complexity of students' mathematics-related beliefs. In this paper we discuss the effectiveness of the revised instrument as a means of discriminating between the mathematics-related beliefs of students from schools in England and Spain, and examine its potential for distinguishing between gender and age. The results suggest that the scale serves all the purposes well, highlighting a number of culturally-, age- and gender-related differences.

INTRODUCTION

There is a growing body of research showing the influence of students' beliefs on their mathematical learning. Such research has tended to focus on, inter alia, beliefs about the nature of mathematics, mathematical knowledge, mathematical motivation, and mathematics teaching, with each category being examined in isolation (Op 't Eynde et al, 2006). This lack of integration provoked colleagues at the University of Leuven into developing, from a warranted theoretical perspective, a comprehensive instrument for assessing students' beliefs about mathematics and its teaching (Op 't Eynde and De Corte, 2003). Called the mathematics-related beliefs questionnaire (MRBQ), the instrument was developed for use with 14 years old Flemish students and showed itself sensitive to differences in the beliefs of students in different types of school and their gender (Op 't Eynde et al, 2006). However, its cross-cultural transferability has yet to be evaluated and two of the four scales yielded by the Flemish data were found to be less reliable than expected. Such issues underpinned our decision to attempt a refinement of the MRBQ in order to improve the reliability of the scales, evaluate its cross-cultural transferability while retaining its sensitivity to variables like gender.

In a paper presented earlier at this conference (Diego-Mantecón et al, 2007) we discussed our refinement of the MRBQ, how it yielded four reliable scales, each with at least two reliable sub-scales, and exposed some of the structural relationships between different forms of mathematics-related beliefs. In this paper we report on the refined MRBQ's cross-cultural transferability through an analysis of data drawn from students in two culturally different European countries (England and Spain) at

two ages (12 and 15) as well as examining its sensitivity to variables like age and gender.

THEORETICAL FRAMEWORK

We discussed the nature of beliefs and their significance in respect of mathematical learning in our earlier CERME-5 paper. Importantly, “we may not be the best people to clearly enunciate our beliefs” since they “may lurk beyond ready articulation” (Munby, 1982: 217). That is, beliefs are, essentially, accessible only by inference (Fenstermacher, 1978). Moreover, humans organise beliefs into systems within which are primary and derivative, and central and peripheral, beliefs. Thus, beliefs comprising a system are neither entirely independent nor equally susceptible to external influence (Green, 1971). Moreover, belief systems do not require social consensus or even internal consistency (Da Ponte, 1994), making it possible not only for a belief system to be held in isolation of others but also for individuals to hold apparently conflicting beliefs (Green, 1971). From a methodological perspective, if, as Green (1971) asserts, beliefs are manifested at the level of the system then research is better focused on the study of belief systems than on isolated beliefs (Op ’t Eynde and De Corte 2003: 3).

Despite apparent clarity in respect of belief structures, there remains much ambiguity in respect of definition (Pajares, 1992, Op ’t Eynde et al, 2002). From the perspective of this paper, we take beliefs, in general, to be “subjective, experienced-based, often implicit knowledge” (Pehkonen and Pietilä, 2003: 2). In particular, students’ mathematics-related belief systems draw on beliefs about mathematics education, beliefs about themselves as learners and beliefs about the classroom context (Op ’t Eynde and De Corte, 2003). Such a definition does not deny the role of knowledge in belief construction and, along with individual differences in respect of interpretation and prior experience, explains why people construct different beliefs from the same experience.

Conventionally beliefs are examined by means of questionnaire surveys, the data from which are subjected to exploratory factor analyses which reduce large numbers of variables to sets of common factors, considerably fewer in number than the number of variables, representative of the underlying constructs (Cureton and D’Agostino, 1983, De Vellis, 1991). Importantly, in respect of validating our methods, Op ’t Eynde and De Corte (2003) argued for a principal components approach and, since our study is a development of their work incorporating a number of new or replacement items, we felt we should not deviate from this.

In this paper we attend to student’ mathematics-related beliefs in different cultural contexts. Most previous studies have been undertaken in single national contexts (Pintrich and De Groot, 1990) with few attempting explicit comparative evaluations. Indeed, “the international comparison of pupils’ mathematical beliefs still seems to be an almost unexamined field” (Pehkonen, 1995: 34). This lack of attention to the comparative dimension provokes a number of pertinent questions. For example,

does it mean that researchers working in one context assume that beliefs are so uniquely located in the context in which they were formed that cross-cultural transferability is impossible? Does it mean that researchers assume that domain-specific beliefs are held by all, irrespective of culture or context? Does it mean that researchers have simply failed to consider the significance of national context?

Where comparative studies have been undertaken - research in which Finnish students seem constantly implicated - the extent to which attempts have been made to uncover and explicate structural properties have been variable. For example, Pehkonen and Tompa (1994), in a comparison of Finnish and Hungarian students' beliefs, used factor analyses to reduce large numbers of items to "compact" proportions but, essentially, ignored the structural implications and focused attention on a comparison of individual items scores. Pehkonen (1995) describing the results of a five way study involving students in Finland, Estonia, Hungary, Sweden and the United States, discussed student responses to individual items which were then grouped according to the researcher's predispositions. Berry and Sahlberg (1996), in an examination of Finnish and English students' beliefs about learning, and Graumann (2001), in a study of German and Finnish students' mathematical views, also grouped item scores according to pre-determined categories, which they described as factors, rather than the outcomes of systematic analyses. Such studies are disappointing in their lack of attention to the structural aspects of beliefs and reliance on item comparisons.

The above indicates that comparative analyses of belief systems are problematic enterprises. Osborn (2004) has argued that comparative researchers should attend, in particular, to issues of conceptual and linguistic equivalence to ensure instrument validity across cultures. Indeed, problems of conceptual and linguistic equivalence are frequently unacknowledged in comparative research with the consequence that instruments effective in one culture fail in another - a problem experienced by Mason (2003) in her Italian adaptation of the Kloosterman and Stage (1992) instrument. Such problems, frequently a consequence of one country's researchers dominating a project's instrument development, have compromised much comparative mathematics education research (Keitel and Kilpatrick, 1999, Wiliam, 1998). Overcoming such difficulties is time-consuming and expensive. Andrews (2007), for example, has described how researchers from five European countries spent a year observing lessons and discussing each others' culturally-located beliefs about effective teaching before developing an agreed framework for describing mathematics classroom activity. However, if comparative research is to avoid many of the criticisms levelled at projects like TIMSS then such negotiation is essential.

METHOD

The original study set out to categorise "the structure of belief systems and on an identification of the relevant categories of beliefs and the way they relate to each other" (Op 't Eynde and De Corte, 2003; 3). The analyses yielded four factors in line with the theoretical perspectives informed by their reading of the literature.

Disappointingly, only two of these four scales achieved satisfactory levels of reliability and no attempt has yet been made to determine the extent to which the instrument transfers to cultures other than the Flemish in which it was developed. Our objectives were, through a refinement of the original questionnaire, to improve the reliability of the instrument and examine the extent to which it would transfer to different cultures and be sensitive to student age and gender.

The MRBQ comprised 58 items which were reduced to 40 by the original analyses. These were augmented by a further 33 drawn from various sources which were thought to complement the theoretical model developed for the original study. These sources included, *inter alia*, scales developed by Kloosterman and Stage (1992) and Pintrich and De Groot (1990). All items were subjected to the scrutiny of colleagues in England and Spain to establish conceptual and linguistic equivalence (Osborn, 2004) and ensure that each was as concise as possible.

Both versions, English and Spanish, were piloted on a small number of volunteer students. Finally, all items were placed alongside a six point Likert scale and strategically mixed. A six point scale was used in accordance with the approach of the Leuven team and because we believed that denying a neutral option would improve the quality of the data yielded. The revised questionnaire was administered in one school near Cambridge, England and three near Santander, Spain. All students in each of two cohorts (ages 12 and 15) were invited to complete a questionnaire during one of their mathematics lessons. The surveys, both of which were undertaken in the spring of 2006, yielded 405 Spanish and 220 English questionnaires. While it is clear that little generality at the level of nationality can be inferred from such small and localised samples, particularly in the light of the original instrument's sensitivity to school type within a single country, we believed that our objectives of instrument reliability, cultural transferability and sensitivity to gender and age – we were not trying to generalise but determine the sensitivity of the instrument to different populations - were largely independent of such issues.

RESULTS

In accordance with our stated intention of determining the extent to which the data reflected psychological constructs, analytical procedures commensurate with such a goal were undertaken. The outcomes of this are reported in our earlier paper and show a reliable scale with reliable subscales. However, by way of contextualising the results reported here, the reader is reminded that the analyses reported in that paper, based on an initial set of 73 items, yielded a reliable sixty-item, four-factor, scale as 13 items were rejected for the full analysis. The factors alluded to beliefs about the role of the teacher as an initiator of learning, beliefs about one's personal competence with mathematics, beliefs about the relevance of mathematics to one's life and beliefs about mathematics as a rote-learned and difficult subject. We offer no further comment on the initial analysis as our intention here is to focus on the strength of the conceptual equivalence embedded in the questionnaire and the degree to which it is sensitive to variation in beliefs.

To determine the effectiveness of the revised instrument in different contexts, separate factor analyses were undertaken on the data from each of the two countries. In both cases, four factor solutions were forced to facilitate comparison. In both cases, similar items were found to load on similar factors although, inevitably, there were some minor differences. For example, one of the factors yielded by the analysis of the Spanish data and one of the factors derived from the English data can be seen in table 1. It seems clear to us that the two factors show remarkable similarity, not only in the commonality of items but also the importance, as reflected in its loadings, of each item within the factors. Additionally, these items relate very closely to those of the first factor yielded by the analysis reported in our earlier paper and concerns beliefs about the role of the teacher as an initiator of learning.

	S	E
My teacher really wants us to enjoy learning new things.	0.825	0.808
My teacher is friendly to us.	0.802	0.775
My teacher understands our problems and difficulties with mathematics.	0.797	0.776
My teacher tries to make the mathematics lessons interesting.	0.785	0.766
My teacher listens carefully to what we say.	0.755	0.817
My teacher always shows us, step by step, how to solve a mathematical problem, before giving us exercises.	0.752	0.686
My teacher appreciates it when we try hard, even if our results are not so good.	0.739	0.780
My teacher wants us to understand the content of our mathematics course.	0.726	0.579
My teacher always gives us time to really explore new problems and try out different solution strategies.	0.663	0.659
My teacher explains why mathematics is important.	0.644	0.494
My teacher thinks mistakes are okay as long as we are learning from them.	0.516	0.473
My teacher is too absorbed in the mathematics to notice us.	0.464	0.668
My teacher does not really care how we feel in class.	0.425	0.642
We do a lot of group work in this mathematics class.		0.508

Table 1: one of the Spanish and one of the English factors with the Spanish loadings in numerical order.

Similar accounts can be offered for the remaining factors. The items associated with each factor yielded by one country's data always resonated closely with the items of one of the factors yielded by the other country's data. To assess the degree of resonance the following procedure was undertaken. A score for each factor was calculated for each student equal to the mean of that individual's scores on each of the factor's items. Correlations, the outcomes of which can be seen in Table 2, were then calculated, for the students in each country, between the country-specific factor scores and the factor scores yielded by our analysis of the international data reported in our earlier paper. These show that each of the four *country* factors correlates at a very high level with one of those from the original study. For example, the first

factor of the original analysis, concerned with beliefs about the role of the teacher as an initiator of learning, found a perfect ($\rho=1.000$) correlation with the first English factor and an almost perfect ($\rho=0.980$) correlation with the first Spanish factor. So well aligned were the respective national factors with the international that the lowest correlation yielded by this analysis was the negative ($\rho=-0.922$) between the international factor four, beliefs about mathematics as a difficult, inaccessible and elitist subject, and the fourth English factor.

		E1	E2	E3	E4	S1	S2	S3	S4
O1	rho	1.000	0.463	0.539	0.239	0.980	0.576	0.414	0.282
	p	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
O2	rho	0.463	1.000	0.621	0.132	0.546	0.938	0.583	0.252
	p	0.000	0.000	0.000	0.008	0.000	0.000	0.000	0.000
O3	rho	0.538	0.635	0.979	0.312	0.476	0.656	0.959	0.001
	p	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.981
O4	rho	-0.263	-0.105	-0.347	-0.922	-0.238	-0.377	-0.143	-0.961
	p	0.000	0.034	0.000	0.000	0.000	0.000	0.004	0.000

Table 2: correlations (ρ) and associated probabilities (p) between, on the left, English factor scores (E_i) and those of the original analysis (O_i) and, on the right, Spanish factor scores (S_i) and those of the original analysis.

The figures show that the four factors yielded by our original analysis, reported in the paper presented earlier at this conference (Diego- Mantecón et al, 2007) correspond closely to those yielded by each country's data. That is, having established that the Spanish version was as accurate as possible a translation of the English, the instrument has achieved, to a satisfactory level, the conceptual equivalence necessary for its successful use in the two countries – even the smallest correlation accounted for more than 85 per cent of the variance between the two factors concerned. Clearly, future work will necessitate evaluating the effectiveness of the scale in other countries and we are currently collecting data in Flanders and Ireland.

Testing the factors

In accordance with our objectives of determining the extent to which the revised questionnaire would be sensitive to differences in students' age, gender and nationality, factor scores - means of all items loading on that factor - were calculated for each student. These were then subjected to a variety of comparative analyses. The use of a six point scale, with a score of 1 being positive and 6 being negative, means that a mean of 3.5 represents neutrality. The following draw on the data from all 625 students from the two countries.

In respect of student age, the figures of table 3 show that across three of the four factors, students at age 12, irrespective of nationality and gender, were of the order of half a point more positive than at age 15 and that these differences were

significant at the level of $p < 0.0005$. The only factor which showed no significant age-related difference concerned mathematics as an inaccessible and rote-driven subject where the beliefs of students of both age groups tended towards rejecting the notion. Thus, in general and irrespective of nationality, younger students see greater relevance in what they study than older students, they are more positive about the efficacy of their teachers' role as the facilitator of learning and they are more positive about their own competence as learners of the mathematics.

	Teacher's role		Competence		Relevance		Inaccessible	
Age	15	12	15	12	15	12	15	12
Mean	2.94	2.30	3.38	2.86	2.47	1.96	4.19	4.28
SD	1.08	0.85	0.92	0.85	0.75	0.65	0.66	0.72
t / p	7.975 / 0.000		7.403 / 0.000		8.867 / 0.000		1.624 / 0.105	
Country	S	E	S	E	S	E	S	E
Mean	2.44	2.84	2.84	3.54	2.04	2.43	4.34	4.07
SD	0.99	0.98	0.83	0.90	0.65	0.82	0.71	0.63
t / p	4.840 / 0.000		9.833 / 0.000		6.136 / 0.000		4.899 / 0.000	
Gender	F	M	F	M	F	M	F	M
Mean	2.59	2.57	3.30	2.90	2.22	2.14	4.31	4.18
SD	0.99	1.02	0.94	0.86	0.75	0.72	0.66	0.73
t / p	0.146 / 0.884		5.499 / 0.000		1.391 / 0.165		2.325 / 0.020	

Table 3: means, standard deviations, t scores and associated probabilities for each factor by student age, nationality and gender.

The figures of table 3 also show that Spanish students' beliefs, irrespective of age and gender, were significantly more positive on the three positively directed scales and significantly more negative on the negatively directed scale than English students' beliefs. Indeed, in respect of beliefs about personal competence, the Spanish mean was 0.7 of a point more positive than the English mean which was clearly neutral and, it could be argued, bordering on the negative. That is, Spanish students view themselves as competent in relation to what they learn while the English were neutral. The Spanish students were clearly more positive about the relevance of mathematics to their lives and were more positive about the role of their teachers as facilitators of learning. They were also more negative in respect of mathematics as an accessible and elitist subject.

Lastly, the figures show that girls, irrespective of age or nationality, are less positive in their beliefs about personal competence than boys at better than the $p < 0.0005$ level although, interestingly, they are significantly more negative than boys on the negative scale concerning mathematics as inaccessible and elitist. Boys and girls are equally positive in respect of both their teachers as facilitators of learning and the relevance of mathematics to their lives.

The figures of table 4 show the results of analyses of variance performed to identify any joint effects of the three background variables of gender, age and nationality on the factor scores. The first three rows of the table, as expected, confirm the above analyses. The remaining rows allude, although there is no scope in this paper to explore them beyond merely commenting, to some interesting combined effects.

	Teacher role		Competence		Relevance		Inaccessible	
	F	p	F	p	F	p	F	p
Gender	0.00	0.951	29.71	0.000	1.36	0.244	2.59	0.108
Age	53.65	0.000	53.87	0.000	81.86	0.000	6.94	0.009
Nationality	22.66	0.000	96.72	0.000	46.94	0.000	28.16	0.000
G and A	0.57	0.450	0.32	0.573	0.34	0.558	0.66	0.416
G and N	9.47	0.002	5.99	0.015	2.94	0.087	11.04	0.001
A and N	7.94	0.005	2.23	0.136	0.01	0.933	7.88	0.005
A and G and N	4.66	0.031	0.86	0.353	0.12	0.725	1.42	0.234

Table 4: analyses of variance showing combined effects of student age, nationality and gender and factor scores.

It can be seen, for example, that gender and age have no combined effect on any of the factors although gender and nationality combine on all but beliefs about the relevance of mathematics. Age and nationality produced a combined effect on beliefs about both the teacher's role as facilitator of learning and the inaccessibility of mathematics. Interestingly, there was a weak combined effect of all three variables on beliefs about the teacher as facilitator of learning but no other factor. By way of illustration, the figures of table 5 go some way to explaining the combined effect of gender and nationality on the first and last factors.

	Teacher role		Inaccessible	
	Spain	UK	Spain	UK
Female	2.36	2.93	4.49	4.04
Male	2.50	2.74	4.22	4.10

Table 5: mean scores on two factors by gender and nationality.

In respect of the teacher as facilitator of learning, Spanish girls are the most positive and English girls the least positive of all the groups with both groups of boys, Spanish and English falling between. Similar statements can be applied to the mean scores for mathematics as an inaccessible and elitist subject. Such findings are of substantial interest and should warrant further work in the area.

DISCUSSION AND CONCLUSIONS

We believe we achieved our objectives. Firstly, the analysis of the combined data, as reported in the paper we presented earlier at this conference (Diego-Mantecón et al, 2007), revealed four reliable factors confirming the comprehensive of the instrument. The secondary analysis by country, as reported here, yielded four factors

from each country each of which correlated at almost perfect levels with those of the original analysis. This confirms that the refined mathematics-related beliefs questionnaire accomplished, at least in terms of its use in England and Spain, the conceptual and linguistic equivalence necessary for successful comparative research (Osborn, 2004). Secondly, and acknowledging that we are not trying to generalise to national populations, the significant differences in beliefs between students from the English school and students from the Spanish schools support a conclusion that the instrument is sensitive to context. Thirdly, the refined instrument has not only retained the sensitivity of the original to gender but also proved sensitive to student age. Moreover, throughout our analyses, the probability levels were sufficiently small as to make errors of interpretation highly unlikely, providing additional evidence that the refined MRBQ should provide a powerful tool for further comparative investigation of students' mathematics-related belief systems.

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STUDENTS' MOTIVATION IN MATHEMATICS AND GENDER DIFFERENCES IN GRADES 6 AND 7

Athanasiou C., and Philippou G. N.

Department of Education, University of Cyprus, chrathan@cytanet.com.cy

This study investigates changes in students' motivation in mathematics across the transition from primary to secondary school (from grade 6 to 7) and changes in motivation according to gender in the above grade levels. The analysis of 229 students' responses to a questionnaire suggests that students' motivation in mathematics declines during the transition to secondary school. Elementary school students endorse more praise and token goals and social motivational orientations whereas middle school students endorse competition goals and performance motivational orientations. Differences in students' motivation in both grade levels according to gender were examined; boys in both grade levels were found to be more performance-oriented and endorse more competition and social power goals than girls, whereas girls endorse social responsibility goals more than boys.

BACKGROUND AND AIMS OF STUDY

The term motivation comes from the Latin root of motive, *motivus*, "to move" (McCallum, 1997) and early researchers were concerned with what moves a resting organism to a state of activity. Motivation has been construed in many different ways and after many years of empirical research there is still little agreement about what motivation is and what dimensions it includes. It has been construed in terms of needs, causal attributions, affective responses, expectancies for success and self-perceptions. All these conceptions lead to a fragmentation of the study of motivation and produced a profusion of often conflicting recommendations to teachers for enhancing motivation.

Despite the fact that motivational research is fragmented and diffused, researchers seem to be reaching an agreement regarding the importance of motivation in learning and teaching contexts. Many studies indicated that, students' motivation influences or predicts the use and structure of cognitive and metacognitive strategies and students' outcomes and performance in various subjects (e.g. MacCallum, 1997, Pajares & Graham, 1999).

When teachers and motivation researchers consider of motivating students or enhancing student motivation in an educational context, they usually have a specific idea in mind about why and for what cause students ought to be motivated. All these ideas imply that questions about enhancing motivation are not necessarily questions about whether or not a student is motivated or how much motivation he or she has but what form the motivation takes (MacCallum, 1997). Thus, changing students' motivation involves increasing motivation in a quantitative and in qualitative sense (i.e. the amount and the type of students' motivation). Both changes in motivation are

intertwined with the contexts of motivation and learning, the contexts of the classrooms and of the schools in which students are located.

Many researchers have identified the transition from childhood to adolescence as a time of significant personal and contextual change and hence a useful starting point for examining motivational change (e.g. Midgley et al., 1995). More specifically, the period surrounding the transition from primary to secondary school has been found to result in a decline in students' motivation and achievement in mathematics (see e.g., Eccles et al., 1993, Midgley et al., 1995). More recent studies have conceptualised motivation in very different ways, including various motivational constructs such as cognitive, i.e. students' motivational orientation (Anderman et al., 2001, MacCallum, 1997), or affective, i.e. students' self-beliefs (e.g., Pajares & Graham, 1999, Wigfield & Eccles, 1994).

The decline of students' motivation in mathematics across this systemic transition was found to be related to certain dimensions of the school and classroom culture. Many studies suggested that there are developmentally inappropriate changes in a cluster of classroom organizational, instructional and climate variables. The dimensions of the school culture that were found to have an effect on motivation during the transition to middle school include the perceived classroom goal structure (Midgley et al., 1995, Urda & Midgley, 2003), teachers' sense of efficacy and teachers' ability to discipline and control students (Midgley et al., 1989), teacher-student relations and opportunities for students to participate in decision making (Athanasίου & Philippou, 2006).

In the majority of the above studies motivational change and its relation to the school and classroom structure during the transition from primary to secondary school was examined for students as a single group. Recent research in the area of students' perceptions of classroom environments, adds credence to the view that students do not all perceive the same environment in the same way at least on some of its dimensions. Therefore, recent studies emerged that examined motivational change according to gender (e.g. Anderman & Anderman, 1999, Watt, 2004), students' ability and perceived academic competence (e.g. Anderman & Midgley, 1997) and even according to less frequently occurring changes in the school and classroom context such as moving to a more positive teacher/student environment (e.g. Midgley, Anderman & Hicks, 1995, Urda & Midgley, 2003). With regard to gender differences, motivation was found to be domain specific with studies indicating that males and females differ in their levels of motivation for various academic subjects such as mathematics, language, sports and music (Eccles et al., 1993, MacCallum, 1997, Watt, 2004).

As far as mathematics are concerned, previous studies examining gender differences in students' academic goals provided some evidence that boys may be more likely than girls to endorse personal ability goals and to have higher competence beliefs (Anderman & Anderman, 1999, Anderman & Midgley, 1997, Roeser, Midgley &

Urdan, 1996) both in elementary and secondary school. In the above studies, boys and girls ratings of the usefulness and importance of mathematics did not differ significantly. In terms of social goals, findings from several studies indicate that girls endorse relationship and responsibility goals more than do boys, whereas boys endorse status goals more than do girls (Anderman & Anderman, 1999).

When gender has been considered across the transition from primary to secondary school, the studies yielded mixed results. For instance, Seidman et al. (1994) reported no significant difference among boys and girls in the declines in self-esteem, class preparation and grade-point average during the transition, whereas the study of Wigfield et al. (1991) found gender differences in self-concept of ability and liking of mathematics across the transition. The study of Anderman & Midgley (1997) reported that females were not more task-focused than males in mathematics across the transition, with males reporting higher mean levels of personal performance goals than females.

Jacobs et al., (2002) found that perceived competence and values with respect to mathematics generally decline through school, providing evidence for continued declines after the transition to secondary school. They concluded that boys experience greater declines in perceptions than do girls, which may be a consequence of greater declines in boys' achievement through school, which may lead to problematic outcomes for boys in terms of dropping out of academic settings. On the other hand, Watts (2004) reported that gender differences favoured boys in mathematics during the secondary school years. More specifically, boys maintained consistently higher perceptions regarding their talent in mathematics than girls across the secondary school years. In contrast, expectancies for success remained relatively stable for boys, whereas girls' expectancies declined through the first years in middle school and recovered in senior years although not to the same level as in the first year in middle school. Lastly, boys and girls had similar math utility values, whereas girls perceived mathematics as more difficult than boys most of secondary school.

The aim of the present study was to examine the developmental changes in students' motivation in mathematics across the transition from primary to secondary school (grade 6 to 7) and to investigate whether gender differences in motivation exist in these two grade levels. We report results from the first two phases of a longitudinal project that is currently being developed in Cyprus aiming to examine students' motivational change (through a range of motivational constructs such as cognitive, emotional and social) in mathematics during the transition from primary to secondary school, focusing primarily on how modifiable facets of the school culture (including classroom, teacher and social variables) influence the nature and quality of student motivation and investment in learning. Several studies have already addressed these issues, but they were concerned with a single facet of motivation or of the school culture at a time. Furthermore, what is so far missing is the consideration of the

individual differences students experience in motivational change during the transition from primary to secondary school.

In the longitudinal study, the same students will participate over a period covering two consecutive school years, with four waves of quantitative measurements (through questionnaires) including one measurement in the first school year (April) and three in the second year under investigation (October, January and April). The exact timing of the measurements is based on the organization of the school year in Cyprus where the study is conducted in combination with the Phase Model of Transitions by Ruble (1994). Three cohorts of students will be participating in the study. Students in Cohort A will experience the transition from the last year of primary school to the first year of middle school (grade 6 to 7). Cohort B will be studied over the last two years of primary school (grade 5 to 6), whereas Cohort C will be studied over the first two years of secondary school (grade 7 to 8). The methodology of the study will involve the analysis of qualitative data as well. Students will be selected for semi-structured interviews which will be designed to elaborate information from the questionnaires and to complement the information gained from the analysis of the group data.

The data of the study presented in this paper were collected in April and October 2006. Analysing these data, we sought answers and preliminary information to the following research questions:

- 1) Are there any developmental changes in specific aspects of students' motivation in mathematics across the elementary (grade 6) and secondary school (grade 7)?
- 2) Are there any statistically significant differences in students' motivation according to gender in the elementary (grade 6) and secondary school (grade 7)?

METHOD

Participants in this study were 229 students, 105 boys and 124 girls. In the first phase of the study the students were drawn from five elementary schools, whereas in the second phase the same students were attending two secondary schools.

Data were collected through a self-report questionnaire. The questionnaire comprised of 81 items measuring five dimensions referring to students': (a) motivational goals (including six specific motivational goals such as effort/task, praise, token, affiliation, competition/social power and social concern e.g. for effort/task "I try hard to make sure that I am good at my math work"); (b) general achievement goal orientations (including six goal orientations referring to valuing, mastery/global, performance-approach, performance-avoid, performance general and social goal orientation e.g. for performance-approach "I would feel really good if I were the only one who could answer the teachers' questions in mathematics in the classroom"); (c) perceived self-efficacy for doing mathematics (e.g. "Even if the math work is hard, I can learn it"); (d) self-esteem in mathematics (including two dimensions such as positive self-

esteem and negative self-esteem in doing mathematics e.g. for negative self-esteem “I often make mistakes in mathematics”); and (e) perceived instrumentality of mathematics for reaching future goals (e.g. “It is important for me to perform well in mathematics to reach my future goals”). The items referring to the first two dimensions were adapted from the Inventory of School Motivation Questionnaire (McInerney, Yeung & McInerney, 2000), whereas the other dimensions were adapted from the Patterns of Adaptive Learning Survey (Midgley et al., 2000).

The statements were presented at a six-point Likert-type format (1=strongly disagree, 6=strongly agree). The reliability estimates (Cronbach alphas) were found to be quite high for all the scales, ranging from $\alpha=0.65$ to $\alpha=0.90$.

Data processing was carried out using the SPSS software. The statistical procedure used in this study was paired samples t-test. The $p<0.05$ level of significance was adopted for these paired comparisons.

RESULTS

Table 1 presents the means of the students to the scales tapping motivation. Similar numeric superscripts within each row indicate no significant difference between the means in the same row, while variable superscripts mean significant difference.

	GRADE LEVEL			
	GRADE 6		GRADE 7	
	M	SD	M	SD
Motivational goals				
Effort/Task	4.10 ¹	0.71	4.13 ¹	0.65
Praise	3.70 ¹	1.00	3.29 ²	1.09
Token	2.27 ¹	1.07	2.00 ²	1.13
Affiliation	3.32 ¹	0.92	3.31 ¹	0.88
Competition/social power	2.19 ¹	0.96	2.49 ²	1.04
Social concern	3.84 ¹	0.93	3.77 ¹	0.85
Mot. Goal orientations				
Valuing	3.87 ¹	0.85	3.95 ¹	0.87
Mastery/Global	3.86 ¹	0.71	3.83 ¹	0.71
Performance-approach	2.59 ¹	1.03	2.79 ²	0.96
Performance-avoid	2.17 ¹	0.93	2.43 ²	0.91
Performance general	2.56 ¹	0.94	2.77 ²	0.87
Social	3.18 ¹	0.94	2.98 ²	1.02
Instrumentality	4.14 ¹	0.75	4.27 ¹	0.65
Self-efficacy	3.85 ¹	0.78	3.84 ¹	0.67
Positive self-esteem	3.76 ¹	0.75	3.64 ¹	0.77
Negative self-esteem	3.51 ¹	0.78	3.67 ²	0.69

Table 1: Mean level of motivational variables by grade level

The t-test analysis indicated that for praise and token motivational goals and for social motivational orientation in mathematics, the sixth graders' mean ratings are significantly higher than those of the seventh graders' (for praise $t=4.13$, $p<0.01$; for token $t=2.58$, $p<0.05$ and for social motivational orientation $t=2.16$, $p<0.05$).

Students in grade 7 performed at a significantly higher level than the students in grade 6 on competition/social power goal ($t=-3.06$, $p<0.01$); on performance-approach motivational orientation ($t=-2.09$, $p<0.05$); on performance-avoid orientation ($t=-2.97$, $p<0.01$); on performance general orientation ($t=-2.44$, $p<0.05$) and on negative self-esteem ($t=-2.24$, $p<0.05$).

As far as the gender differences in students' motivation in the elementary and secondary school are concerned (second research question), Table 2 presents the means of the students in grade 6 and grade 7 in each of the motivation variables. Paired t-test results ($p<0.05$) comparing pairs of means within sex in the same grade level are illustrated with superscripts in the tables below; means with similar superscripts between boys and girls within the same grade level do not differ. Differences within the same sex across grades are not examined.

	GRADE 6				GRADE 7			
	Boys		Girls		Boys		Girls	
	M	SD	M	SD	M	SD	M	SD
Motivational goals								
Effort/Task	3.92 ¹	0.67	4.25 ²	0.71	4.08 ¹	0.68	4.18 ¹	0.63
Praise	3.64 ¹	0.90	3.74 ¹	1.08	3.28 ¹	1.11	3.29 ¹	1.07
Token	2.41 ¹	1.08	2.15 ²	1.06	2.22 ¹	1.22	1.81 ²	1.01
Affiliation	3.23 ¹	0.96	3.39 ¹	0.89	3.50 ¹	0.92	3.34 ¹	0.84
Competition/social power	2.36 ¹	0.97	2.06 ²	0.93	2.60 ¹	1.02	2.38 ²	1.05
Social concern	3.61 ¹	1.07	4.04 ²	0.74	3.61 ¹	0.90	3.90 ²	0.78
Mot. Goal orientation								
Valuing	3.94 ¹	0.81	3.82 ¹	0.89	3.99 ¹	0.81	3.91 ¹	0.92
Mastery/Global	3.84 ¹	0.69	3.89 ¹	0.73	3.85 ¹	0.68	3.80 ¹	0.74
Performance-approach	2.66 ¹	0.97	2.53 ¹	1.07	2.90 ¹	0.94	2.70 ¹	0.97
Performance-avoid	2.27 ¹	0.95	2.08 ¹	0.91	2.55 ¹	0.90	2.32 ¹	0.91
Performance general	2.66 ¹	0.98	2.48 ²	0.91	2.93 ¹	0.81	2.64 ²	0.91
Social	3.23 ¹	0.88	3.13 ¹	0.98	2.93 ¹	1.13	3.02 ¹	0.90
Instrumentality	4.06 ¹	0.77	4.21 ¹	0.72	4.25 ¹	0.71	4.28 ¹	0.59
Self-efficacy	3.82 ¹	0.75	3.86 ¹	0.80	3.85 ¹	0.65	3.83 ¹	0.69
Positive self-esteem	3.71 ¹	0.75	3.80 ¹	0.75	3.66 ¹	0.75	3.63 ¹	0.79
Negative self-esteem	3.64 ¹	0.71	3.42 ²	0.82	3.69 ¹	0.68	3.66 ¹	0.70

Table 2: Mean level of motivational variables by gender for students in grade 6 and 7

The analysis revealed a gender difference favouring boys both at the elementary and secondary school on token motivational goal ($t=1.93$, $p<0.05$ for boys in grade 6 and

$t=2.77$, $p<0.01$ for boys in grade 7), on competition/social power motivational goal ($t=2.31$, $p<0.05$ for boys in grade 6 and $t=1.75$, $p<0.05$ for boys in grade 7) and on performance general motivational goal orientation ($t=1.60$, $p<0.05$ for boys in grade 6 and $t=2.45$, $p<0.05$ for boys in grade 7). On the contrary, girls mean ratings in both grade levels are higher than those of boys on social concern motivational goal ($t=-3.51$, $p<0.01$ for girls in grade 6 and $t=-2.56$, $p<0.05$ for girls in grade 7).

For students in the elementary school (grade 6), boys mean ratings were found to be significantly higher than those of girls on negative self-esteem ($t=1.99$, $p<0.05$), whereas girls mean ratings were found to be significantly higher than those of boys on effort/task motivational goal ($t=-3.50$, $p<0.01$).

DISCUSSION

The purpose of the present study was to explore the developmental differences in students' motivation across the transition and the differences in motivation according to gender in primary and middle school.

The results confirmed the conclusions of previous studies about the decline in students' motivation in mathematics during the transition to secondary school (e.g., MacCallum, 1997, Urda & Midgley, 2003). More specifically students in secondary school (grade 7) are more performance oriented both in terms of demonstrating ability and avoidance in demonstrating lack of ability in mathematics. Also seventh graders endorse more social goals in terms of seeking social power, status and comparison. Given the differences in the perceived school culture, the goals teachers have for their students and the instructional strategies they use in their classrooms that studies suggested (e.g. Athanasiou & Philippou, 2006, Urda & Midgley, 2003), it is not surprising that seventh grade middle school students adopt personal goals that are more performance and competition focused than do sixth elementary students. Furthermore seventh graders self-esteem in mathematics was significantly lower than the sixth graders self-esteem, indicating that students' self-esteem decreased following the transition to secondary school, a finding that is supported by other studies (e.g. Wigfield et al., 1991, Wigfield & Eccles, 1994). On the contrary students in grade 6 (elementary school) are more socially oriented in terms of expressing a willingness to help other students in their math work and are seeking more praise, recognition and tangible rewards for their math work. The latter is not surprising given the practices that are used in the elementary school and the fact that teachers very often provide rewards to students who do the best work in mathematics.

As far as gender is concerned, the results of the study indicated that boys in both grade levels are more performance oriented, seek tangible rewards, social power and status for math work more than girls in the same grade levels. On the contrary, girls in both grade levels are more socially oriented than boys in terms of helping other students in their math work or of adopting social motivational orientations. In other

words, girls endorse relationship and responsibility goals more than boys in both grade levels. These differences were consistent with those that have been reported elsewhere (e.g. Anderman & Anderman, 1999, Roeser, Midgley & Urdan, 1996) and suggest that boys may be more likely to engage in social comparison than are girls. It is may be even possible that the endorsement of social responsibility goals is associated with an increased focus on academic tasks as is the case for girls, whereas endorsement of social goals for maintaining social status are associated with increased focus on the self and performance as is the case for boys.

Lastly, boys mean ratings in sixth grade were significantly higher than girls on negative self-esteem in mathematics. That is boys in the sixth grade are expressing more worries about their capabilities in doing math work than do girls, whereas girls mean ratings in the same grade level are significantly higher than those of boys on effort and task motivational goals meaning that are more willing to expend effort to improve their math work. The same results were found by other studies investigating gender, personal goals and self-esteem (Anderman & Midgley, 1997, Roeser, Midgley & Urdan, 1996). What would be interesting in future studies is to see how the endorsement of performance/mastery goals and self-esteem of boys and girls is affected by the goals stressed in the classroom. Since studies revealed that elementary schools stress mastery goals more and performance goals less whereas middle schools stress performance goals more, it will be of great importance to study the influence of the classroom goal structure on the endorsement of personal goals and self-esteem for boys and girls.

Lastly, it is interesting to note that boys and girls in both grade levels did not differ significantly in their ratings regarding the instrumentality or value of mathematics for the future. This is quite logical, taking into consideration that in Cyprus in both grade levels teachers and parents stress the importance of mathematics in everyday life and for the future education of students.

The findings of the present study highlight the developmental changes in students' motivation in mathematics and the differences in motivation according to gender in the elementary and secondary school. Longitudinal studies, like the one that it is currently developed, addressing these issues can assist in unravelling the complexity of motivational change during the transition from primary to secondary school. In these studies however, motivation must be studied as a multifaceted construct (with the inclusion of cognitive, affective and social constructs) and along with the influence of the school culture and context in the process of motivational change in mathematics. It is important to examine not only "how" students' motivation change across the transition but also "why" students' motivation change, through the examination of the impact of contextual factors (including various dimensions) on students' abilities to negotiate the demands associated with systemic transitions.

Furthermore, longitudinal studies must consider the individual differences students experience in motivational change regarding mathematics during the transition from

primary to secondary school. Eccles and her associates (1993), infer that the transition affects all students in the same way. This is a very strong and by no means undebatable assertion; we hypothesise that if there are differences in students' perceptions of their classroom environment, which should be really expected, then it is possible that students perceive the transition differentially. Therefore there is a need to determine which school and classroom environments are more appropriate for different groups of early adolescents. The latter is one of the aims of the longitudinal study that is currently being developed.

There is also a need to understand not only the effects of what is most prevalent in classrooms but also try to determine what the most facilitative environments are, even if they are uncommon, in order to test the effects of these environments on the nature of change in student motivation. In this way we can begin to understand not only the effects of the most prevalent environmental changes but also the effects of less frequently occurring changes. Such information will be useful for teachers, educators, counsellors and policy makers to make systemic transitions easier so that fewer students are lost. These preventive steps can include the identification of the dimensions of the school culture that have a positive or a negative impact on students motivation and the strengthening of the support structures provided to students either by their family or by the school (transition programs).

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REFINING THE MATHEMATICS-RELATED BELIEFS QUESTIONNAIRE (MRBQ)

Jose Diego-Mantecón¹, Paul Andrews¹ and Peter Op 't Eynde²

University of Cambridge, UK¹ and University of Leuven, Belgium²

In this first of two papers we describe an adaptation of the mathematics-related beliefs questionnaire (MRBQ) developed at the University of Leuven in Belgium (Op 't Eynde and De Corte, 2003). The original instrument, developed to provide a theoretically warranted and comprehensive measure of students' mathematics-related beliefs, yielded four scales, each reflecting a different construct. However, only two of the scales achieved satisfactory levels of reliability and the instrument has yet to be tested on students outside Flanders. We describe here how the MRBQ was refined to yield four conceptually different and reliable scales, and ten subscales, with a sample of Spanish and English secondary students.

INTRODUCTION

In this paper we present an adaptation of the mathematics-related beliefs questionnaire (MRBQ) developed at the University of Leuven, Belgium (Op 't Eynde and De Corte, 2003). The original team's intention was to develop, from a warranted theoretical perspective, a comprehensive instrument for the assessment of students' beliefs about mathematics, and its teaching and learning. The MRBQ has been used in a Flemish study and has shown itself sensitive to differences in the beliefs of students in different types of school and student gender (De Corte and Op 't Eynde, 2003; Op 't Eynde *et al.*, 2006).

However, two of the four scales yielded by the original factor analyses achieved only moderate levels of reliability with no factor analytic attempt made to determine any subscales. Moreover, the MRBQ was developed for and evaluated on Flemish students with, as yet, no evidence to suggest that it is transferable to other contexts. This paper reports on an attempt to refine the MRBQ in order to determine empirically the structure of Spanish and English students' mathematics-related beliefs. In so doing we aim to extend our understanding of the ways in which students' mathematics-related beliefs impact on their engagement with the subject and its ultimate learning. The sensitivity of the yielded instrument to variables such as student age, gender and nationality is discussed in our second paper.

THEORETICAL FRAMEWORK

Over the last few years researchers have to come to understand that cognition and metacognition are necessary but not sufficient psychological functions for effective learning and that affective factors are "important constituent elements of learning" (Op 't Eynde *et al.*, 2002; 14). Indeed, Kilpatrick *et al.* (2001) have argued that mathematical proficiency comprises five intertwined strands of which one, productive disposition, refers to affective rather than cognitive or metacognitive interactions with the subject and embraces beliefs about mathematics, notions of

self-efficacy, motivation to study, attitudes towards study and so on. Such findings resonate with De Corte *et al.*'s (2000) suggestion that a *mathematical disposition* comprises five similar qualities of which one, mathematically-related beliefs, addresses the learner's subjective conceptions about mathematics education and beliefs about the self both as mathematician and member of the class, school and wider community. In short, there is a growing awareness that affective factors have a significant impact on all aspects of mathematical learning. Moreover, learner affect is influenced by a variety of factors including the context in which the learner lives and is schooled, experience and perceptions of ability, and the day-to-day classroom interactions determined by learners' motivation (Kloosterman, 1988), self-confidence (Middleton and Spanias, 1999) and self-efficacy (Schwarzer, 1992).

Acknowledging these issues, the Leuven team developed the mathematics-related beliefs questionnaire with the objective of categorising "the structure of belief systems and on an identification of the relevant categories of beliefs and the way they relate to each other" (Op 't Eynde and De Corte, 2003: 3). In their review of the literature, Op't Eynde *et al.* (2002) identified three main categories of belief-related research which informed the development of their instrument. These were beliefs about mathematics education, beliefs about the social context and beliefs about oneself as a learner of mathematics. In so doing, they were attempting to reconcile the work of researchers in cognitive, motivational and affective research traditions who frequently "operate in relative isolation from each other" (Op 't Eynde and De Corte, 2003: 3).

On beliefs and knowledge

Beliefs operate at two levels (Green, 1971, Abelson, 1979). At the lower level are single beliefs characterised in four ways; they may pertain to the existence of entities outside the believer's control, represent an idealistic alternative world, have both affective and evaluative components, and derive from a person's experiences (Abelson, 1979; Nespor, 1987). They are "deeply personal, rather than universal, and unaffected by persuasion. They can be formed by chance, experience, or a succession of events" (Pajares, 1992: 309). At the second level they are clustered into belief systems which may be held in isolation from other belief systems, making the holding of conflicting beliefs possible (Green, 1971). They are filters through which experiences are interpreted (Pajares, 1992) and tools with which humans protect and promote themselves (Snow *et al.*, 1996).

In respect of beliefs and knowledge, although Pehkonen and Pietilä (2003) argue the distinction is fuzzy, Abelson (1979) and Nespor (1987) suggest that beliefs are non-consensual and consequently disputable, while knowledge is generally verifiable. That is, beliefs are individual constructs while knowledge is essentially socially constructed (Op't Eynde *et al.* 2002). Furinghetti and Pehkonen (2002), in an attempt to clarify the situation, distinguish between objective and subjective knowledge as a means of distinguishing between the formalised and collectively agreed knowledge that is mathematics, and the individually constructed, experiential

and tacit knowledge of the individual. In this paper we understand beliefs, in general, to be “subjective, experienced-based, often implicit knowledge” (Pehkonen and Pietilä, 2003: 2) where, in particular, “students mathematics-related belief systems are constituted by their beliefs on mathematics education, beliefs about the self, and beliefs about the class context” (Op ’t Eynde and De Corte, 2003: 4). In respect of their theoretical framework, the three categories of beliefs were constituted by smaller sub-categories which are not independent but closely interact (Op ’t Eynde and De Corte, 2003). In particular, beliefs about mathematics education comprised three subcategories concerning mathematics, mathematical learning and problem solving, and mathematics teaching. Beliefs about the self comprised five subcategories relating to: intrinsic goal orientation, extrinsic goal orientation, self-efficacy, task-value and control. Finally beliefs about the class context focused on student beliefs about ways in which their teachers interact and teach them. Importantly, they focus also on beliefs concerning their teachers’ motivation to teach them.

METHOD

As has been indicated above, the analysis undertaken by Op ’t Eynde and his colleagues yielded four factors providing a certain consistency to their theoretical framework and reflecting, unlike most other studies, the structural relationships between the underlying psychological constructs. However, only two of the four associated scales achieved a satisfactory level of reliability as measured by the Cronbach alpha, and no attempt had been made to determine the extent to which the instrument would prove successful in cultures other than the Flemish in which it was developed. Thus, our goal was to refine the original questionnaire with a view to improving the reliability of the four factor scales, examining possible subscales, as well as determining the extent to which the instrument would assess beliefs in different educational cultures.

The original instrument comprised 40 items. These were augmented by a further 33, intended to supplement and improve the original scale, drawn from various sources including the scales of Kloosterman and Stage (1992) and Pintrich and De Groot (1990). All items were subjected to the scrutiny of several colleagues in both England and Spain in order to establish not only conceptual and linguistic equivalence (Osborn, 2004) but also to ensure that each was as concise as possible.

Both versions, English and Spanish, were piloted on volunteer students before the items were placed alongside a six-point Likert scale, strategically mixed and grouped into fives in order to facilitate response. A six-point scale was used in accordance with the approach of the Leuven team and because we believed that forcing a decision would improve the quality of the data yielded. The revised questionnaire was administered in one school near Cambridge, England and three near Santander, Spain. All students in each of two age groups (12 and 15) were surveyed during a mathematics lesson. The Spanish survey was undertaken in April 2006 and the English in May of the same year.

RESULTS

Girden (2001, p. 7), who writes about the evaluation of research articles, states that “the questions relating to results of the study focus on appropriate analysis of the data”. In accordance with our stated intention of determining the extent to which the data reflected psychological constructs, analytical procedures commensurate with such a goal were undertaken. Firstly, a reliability analysis was performed on the whole data set of 625 responses (220 English and 405 Spanish), and yielded a pleasing $\alpha = 0.934$. Moreover, the removal of no single item would have improved the reliability and so all 73 items were included in an initial principle components factor analysis. Initially the data, subject to the Cattell Scree test (De Vellis, 1991), suggested a five-factor extraction but this proved difficult to interpret. However a four-factor solution appeared straightforwardly interpretable and, as will be shown below, robust. To increase the factors’ interpretability, they were rotated; the two most commonly used methods are ‘orthogonal rotation’, which produce factors that are independent, and ‘oblique rotation’ in which the factors are correlated (Bryman and Cramer, 2001). Factors from this study were subjected to orthogonal rotation since they were presumed to be unrelated. Interestingly, the four-factor solution accounted for 38.9 per cent of the total variance which, although not high, compared favourably with the 38.3 per cent obtained by Op ’t Eynde and De Corte (2003). In respect of factor loadings, 13 items failed to reach an acceptable 0.4 on any scale and were dropped from the analysis. This left a 60-item scale with a reliability of $\alpha = 0.939$. Each of the four scales was subjected to further analyses to identify any subfactors. Each produced two or three and these are reported below alongside the factor loadings of each of the four initial factors. For ease of communication, the loadings for each of the four factors are not in order but ordered by the subfactors.

Factor 1

The items and their loadings on the first factor and its subfactors can be seen in table 1. Every item in this factor, although some are slight variants of the original items, could be found in the first factor identified in the original analyses of the MRBQ. Our interpretation is that many of the items address the encouragement of a productive disposition towards mathematical learning as much as they do the learning itself. For example, a number of items, as indicated by words such as *enjoy*, *appreciates*, and *friendly* address the affective domain while others, as indicated by words like *understand*, *explore* and *explain*, seem focused explicitly on the cognitive. Op ’t Eynde and De Corte (2003) concluded that this factor concerned student beliefs about their **teacher’s role** and we would not disagree with that. The Cronbach alpha for this scale ($\alpha = 0.921$) was identical to that reported in the original study. The secondary analysis yielded two satisfactorily reliable subfactors. The key phrases of the first (1.1) allude to the ways in which teachers attend to their students’ meaningful learning ($\alpha = 0.924$) and those of the second (2.2) address affective dimensions and perceptions of teacher interest in the student ($\alpha = 0.734$). This secondary analysis shows that factor 1 seems to be divided into two subfactors

rather than three as was stressed in the original study. However, Op't Eynde and De Corte (2003) did not conduct a second analysis to confirm the three sub-categories their framework suggested would comprise factor 1, having assumed that the items it comprised would define them sufficiently.

	1	1.1	1.2
My teacher really wants us to enjoy learning new things.	0.812	0.839	
My teacher understands our problems and difficulties with mathematics.	0.788	0.771	
My teacher tries to make the mathematics lessons interesting.	0.772	0.770	
My teacher appreciates it when we try hard, even if our results are not so good.	0.756	0.744	
My teacher always shows us, step by step, how to solve a mathematical problem, before giving us exercises.	0.730	0.737	
My teacher listens carefully to what we say.	0.769	0.730	
My teacher is friendly to us.	0.785	0.708	0.418
My teacher always gives us time to really explore new problems and try out different solution strategies.	0.660	0.698	
My teacher wants us to understand the content of this mathematics course.	0.665	0.694	
My teacher explains why mathematics is important.	0.594	0.665	
We do a lot of group work in this mathematics class.	0.401	0.532	
My teacher thinks mistakes are okay as long as we are learning from them.	0.505	0.532	
My teacher is too absorbed in the mathematics to notice us.	-0.553		-0.845
My teacher does not really care how we feel in class.	-0.523		-0.828
Alpha	0.921	0.924	0.734

Table 1: loadings on the first factor and its two subfactors

Factor 2

The second factor seems focused on the student's perception of his or her ability to succeed with mathematics. There are elements of the cognitive as reflected in words like understand, but the majority of items appear focused on the affective domain in general and self-efficacy in particular. There is a considerable overlap with the second factor that emerged from the analysis of the MRBQ, but we would argue that the factor here is considerably less about the significance of mathematics, as in the original analysis, than it is about personal **competence**. Indeed, the use of expressions such as *I think*, *I can*, *I prefer*, and *I am certain* all point towards some sense of mathematical self-efficacy. The alpha coefficient for this factor ($\alpha = 0.915$) compared favourably with that of the original 0.89.

The following is an examination of the three subfactor solution. The items of the first subfactor seem concerned with a student perception of enjoyment in the intellectual demands of mathematics or mathematics as pleurably demanding. The second subfactor, which is clearly not unrelated to the first, relates to absolute or intrinsic mathematical competence while the third addresses an issue of relative or

extrinsic mathematical competence. The first and second subfactors showed high reliability with $\alpha = 0.917$ and $\alpha = 0.812$ respectively, while the third showed a moderate reliability with $\alpha = 0.667$. The original study suggested only two subfactors, task-value and self-efficacy. However, as in factor 1, a second analysis was not undertaken to validate them.

	2	2.1	2.2	2.3
I think that what I am learning in this class is interesting.	0.601	0.850		
I like what I am learning in this class.	0.594	0.836		
I'm very interested in mathematics.	0.655	0.807		
I like doing mathematics.	0.678	0.757		
I prefer class work that is challenging so I can learn new things.	0.522	0.683		
I expect to do well on the mathematics tests and assessments we do.	0.583	0.605		
I prefer mathematics when I have to work hard to find a solution.	0.565	0.596		
I find I can do hard mathematics problems with patience.	0.462	0.534		
I am certain I can learn how to solve the most difficult mathematics problem.	0.556	0.534		
I don't have to try too hard to understand mathematics.	0.634		0.835	
Compared with others in the class, I think I'm good at mathematics.	0.692		0.638	0.416
I think I will do well in mathematics this year.	0.647	0.432	0.581	
I understand everything we have done in mathematics this year.	0.701	0.475	0.572	
I can usually do mathematics problems that take a long time to complete.	0.501		0.476	
I can understand even the most difficult topics taught me in mathematics.	0.678	0.443	0.452	
By doing the best I can in mathematics I try to show my teacher that I'm better than other students.	0.411			0.852
I try hard in mathematics to show the teacher and my fellow students how good I am.	0.404			0.769
Alpha	0.915	0.917	0.812	0.667

Table 2: loadings on the second factor and its three subfactors

Factor 3

The majority of the items of the third factor, although several address issues of learning, seem focused on student perception of mathematical **relevance**. This can be construed in terms of successful participation in the real world, mathematics as an intrinsically worthwhile subject or mathematics as a service to other occupations. The underlying structure of this factor resonates more closely with the second factor,

significance of mathematics, identified in the original study. Our view is that neither this, nor any of the other factors that emerged from our analysis, reflects the “social activity” dimension underpinning the third factor identified in the original MRBQ analysis. The alpha coefficient obtained ($\alpha = 0.875$) was an improvement on the 0.65 of the original study.

	3	3.1	3.2	3.3
Mathematics has no relevance to my life.	-0.459	-0.789		
Studying mathematics is a waste of time.	-0.547	-0.763		
Mathematics is a worthwhile and necessary subject.	0.645	0.593	0.404	
I study mathematics because I know how useful it is.	0.509	0.584		
Knowing mathematics will help me earn a living.	0.638	0.578		
I think mathematics is an important subject.	0.628	0.533	0.500	
I think that what I am learning in this class is useful for me to know.	0.590	0.423		
Mathematics enables us to better understand the world we live in.	0.493		0.757	
Everyone can learn mathematics.	0.407		0.679	
Mathematics is used all the time in people's daily life.	0.616		0.677	
If I try hard enough I understand the mathematics we are taught.	0.422		0.551	
I can use what I learn in mathematics in other subjects.	0.416		0.424	
Discussing different solutions to a mathematics problem is a good way of learning mathematics.	0.425			0.819
I think it is important to learn different strategies for solving the same problem.	0.572			0.745
Time used to understand why a solution works is time well spent.	0.581			0.533
Routine exercises are very important in the learning of mathematics.	0.429			0.404
Alpha	0.875	0.821	0.814	0.741

Table 3: loadings on the third factor and its three subfactors

Taking a look at the three subfactor solution, we see the following. The first subfactor seems related to a sense of mathematics as personally relevant, while the second subfactor addresses issues of global relevance. That is, the first seems to concern the personal relevance of mathematics while the second asserts the collective relevance of mathematics. The third subfactor appears related to a student perception of the different strategies in the learning of mathematics and problem-solving. As above, each subfactor proved to be reliable, with $\alpha = 0.821$, $\alpha = 0.814$ and $\alpha = 0.741$ respectively for subfactors 1, 2 and 3.

Factor 4

The items of the last factor, which is negatively oriented, point towards beliefs about mathematics as a rote-learned and difficult subject. In some respects there is a resonance with mathematics as a domain of excellence that emerged from the original MRBQ analysis. However, the items of the factor identified here allude more towards mathematics as an inaccessible subject than it does excellence. The reliability coefficient for this last scale ($\alpha = 0.764$) compared very favourably with the original 0.69.

	4	4.1	4.2
If I can not solve a mathematics problem quickly, I quit trying.	0.467	0.800	
Only very intelligent students can understand mathematics.	0.461	0.754	
Only the mathematics to be tested is worth learning.	0.432	0.609	
Ordinary students cannot understand mathematics, but only memorise the rules they learn.	0.524	0.549	
If I can not do a mathematics problem in a few minutes, I probably can not do it at all.	0.448	0.548	
It's a waste of time when our teacher makes us think on our own.	0.440	0.435	
My teacher wants us just to memorise the content of this mathematics course.	0.549		0.690
Mathematics learning is mainly about having a good memory.	0.447		0.635
There is only one way to find the correct solution to a mathematics problem.	0.457		0.489
Everybody has to think hard to solve a mathematics problem.	0.445		0.485
My teacher thinks she/he knows everything best.	0.418		0.444
Getting the right answer in mathematics is more important than understanding why the answer works.	0.555	0.418	0.425
My only interest in mathematics is getting a good grade.	0.427		0.401
Alpha	0.764	0.741	0.615

Table 4: loadings on the fourth factor and its two subfactors

The two subfactor solution seems the more robust with the first subfactor seemingly emphasising a belief concerning mathematics as something unattainable to all but the able child. The second seems concerned with a view of mathematics as a fixed body of knowledge which requires little but a good memory. A summary description could be mathematics as rote-derived knowledge. As with the other factors, the original study did not demonstrate the existence of smaller scales in factor 4. This secondary analysis, however, showed the existence of two possible subscales: subfactor 1 with $\alpha = 0.741$ and subfactor 2 with $\alpha = 0.615$.

CONCLUSIONS

In their earlier paper Op 't Eynde and De Corte (2003) described the mathematics-related beliefs questionnaire, developed as a research-warranted and comprehensive

instrument for assessing students' beliefs. They demonstrated the reliability of two of its four yielded scales. We have shown that the original instrument is amenable to further development and, in so doing, derived a four-factor and a ten-subfactor solution with each scale proving highly reliable. We have also shown that the same structure is held by students of two different nationalities, suggesting that it may be held by students in other European countries. Further iteration of the development process will, however, be necessary to identify the structure more thoroughly. In short, the revised instrument has achieved our initial objectives, although any future work will concentrate on further instrument development, the evaluation of its effectiveness in a wider range of educational systems and further clarifying the relationship between beliefs and attainment.

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MATHEMATICS TEACHERS' DESIRE TO DEVELOP

Markku S. Hannula, Madis Lepik & Tiiu Kaljas

Tallinn University, Estonia

As part of the reform movement the importance of teachers' professional development has been acknowledged. Here we report of the beliefs of teachers at the beginning of a professional development project in Estonia, where they participate in a community of inquiry. We found out that although the teachers had large agreement on the problems in Estonian mathematics education, they had quite different views on the solutions and even on the aims of mathematics education. Yet, the participants had similar expectation regarding the project. They were looking forward to share ideas with other teachers. This provided a fruitful ground for collaboration.

INTRODUCTION

Since regaining its independence 15 years ago Estonia has gone through many changes that have also affected the educational system. While natural sciences and mathematics had been emphasised in the Soviet curriculum and in the society at large, the focus has since then shifted towards other topics.

Also the attractiveness of teacher profession fell considerably. High salaries and career opportunities tempted students and educated mathematics teachers to the commercial sector instead of schools. Those remaining in the profession had to adapt to less motivated students and a series of new curricula, each with fewer hours for teaching mathematics than the previous one. There was also a concern of the mathematics education researchers that teaching was too much based on drill and practice-methods and although students' achievement was good, students' self-confidence in learning mathematics and valuing of mathematics were low (Lepik, 2005).

As the reform movement have called for new kinds of learning and teaching worldwide, the important role of teachers has become evident. An emerging topic of research interest in mathematics education seems to be professional development of mathematics teachers and more specifically in-service training. (Llinares & Krainer, 2006)

Tallinn University applied for, and was provided a grant from European Union/European Social Fund with an aim to enhance mathematics teaching in Estonia (Project "Kvaliteetne matemaatikaharidus läbirahvusvaheliselt tunnustatud õpetajakooltuse"). The project involves salary for one professor for years 2006-2008 and running costs. In the project there are six mathematics educators and two doctoral students involved on the university level. One of the main activities in this project is a professional development project with a group of mathematics teachers.

THEORETICAL FRAMEWORKS

Professional development programs have been popular among policy makers in an attempt to bring forth changes in educational systems (Jaworski, Wood & Dawson 1999).

Unfortunately, the effect of such programs has often remained small. In an evaluation of one large professional development program within mathematics education (Bobis, 2005), the aspects that were considered most effective were the practical resources and activities, the assessment process, the influence of significant people, classroom support, and the opportunity to share ideas. On the other hand, significant barriers to teachers' implementation of the program were time, resources, class management and information overload.

To understand any human behaviour, such as teaching, we need to pay attention to the interplay of cognition, motivation, and emotion (Hannula, in print; Meyer & Turner, 2002). Within mathematics education, research on teaching has significantly expanded within the last decade. However, the main focus so far has been on cognitive and emotional aspects.

One approach that has been used extensively is teachers' knowledge. It can be divided into content knowledge, pedagogical knowledge, and pedagogical content knowledge (Schulman, 1986). The emotional aspects have been addressed as a part of the vast literature on teachers' beliefs (e.g. Philippou & Christou, 2002). Beliefs include both cognitive and emotional elements and they influence one's understanding, affective reactions and actions in different situations. Teacher's mathematical beliefs can be divided into beliefs about oneself as a learner and teacher of mathematics, beliefs about mathematics and its teaching and learning, and beliefs about the social context of learning and teaching mathematics (Op 't Eynde, De Corte & Verschaffel, 2002). Motivation in general and teacher's motivation in particular has received less attention in mathematics education. This question has been addressed as an aspect of teacher beliefs (teachers' values; e.g. Bishop, 2001). Another approach to motivation has been to look at the goals teacher has (Schoenfeld, 1998).

Communities of inquiry

Studies on pedagogical reform have indicated that external initiatives have had very little impact when it comes to changing forms of teaching (Tyack & Cuban, 1995). If any real innovation in the pedagogical practice is to take place, a necessary prerequisite is that it be implemented in collaboration with teachers (Randi & Corno 1997).

One approach has been to increase teachers' collective reflection through collaborative communities of university educators and school teachers (see e.g. Llanares & Krainer, 2006). This approach is based on Wenger's (1998) idea of communities of practice. We will focus more specifically on one type of such

communities, namely *communities of inquiry*. In a community of inquiry it is intended that all participants within the project will engage reflectively in inquiry into their own practices. All participants within such a project are researchers, inquirers and generators of new knowledge within the context of their own practices and activity (Goodchild & Jaworsky, 2005). In our case, this conference paper is part of our reflection on our practices in initiating and facilitating this community. Likewise, we encourage participating teachers to inquire their practices at school. Such enhancement of teachers' reflective practices can be seen as the driving force for experiential learning and teacher change.

METHODS

The project was advertised in a journal article in the local teachers' newspaper and on mathematics teachers' mailing list as well as for participants of two in-service training courses. During the spring term 2006 altogether 34 secondary school mathematics teachers joined the project. This group included novice teachers with only two years of teaching experience as well as expert teachers who had already taken responsibility for textbook writing and in-service training. They all had a university degree in mathematics education

When we had received the initial indication of interest, we sent out e-mail to participants of the project in order to get a view of the needs and expectations the teachers hold for this kind of project. We asked them to respond via e-mail before our first meeting and express their view on the following topics:

- 1) The situation of mathematics teaching in Estonia;
- 2) Causes for learning problems in mathematics;
- 3) Their students;
- 4) Their teaching style;
- 5) Their desired way of teaching; and
- 6) Expectations and desires for the forthcoming project.

In our first meeting we also gave the participants a one-page questionnaire about their view of mathematics.

The responses were analysed by Hannula and Lepik in collaboration. We started with independently reading the responses holistically to get an overview. After that we read the responses analytically, trying to identify important topics. Through a discussion we were able to find a consensus on classification of topics under 5 general themes (results of 4 will be presented in this paper). As the next phase we identified each teacher's responses (if any) for each of these themes. To confirm the reliability of the coding scheme, we started the coding process by independently coding six teacher cases. Agreement rate was 90 percent of the 30 units of analysis (5

topics x 6 teachers), and in the remaining 3 cases a consensus was easily found through negotiation.

RESULTS

For the e-mail survey we received responses from 26 teachers, totalling altogether 11 000 words. To the one-page questionnaire we received 17 responses with names and 8 anonymous responses that we were not able to match with the other survey response.

As an overall evaluation from the responses, based on our original holistic reading we concluded that the group consisted mainly of successful teachers who are respected professionals and satisfied with their job.

The five categories that we identified in the responses were the following:

- Teachers' beliefs about aims of mathematics teaching
- Teachers' beliefs about situation in Estonian mathematics education
- Teachers' beliefs about mathematics teaching
- Teachers' expectations for the project
- Teachers' beliefs about reasons/ sources of learning difficulties

Teachers' beliefs about aims of mathematics teaching

Under the first category, we found three distinctively different views about the aims of mathematics teaching. The first view emphasized basic skills' training, usually through drill and practice type of teaching. This view is apparent in the following quote:

“The curriculum contains a lot of facts to know and skills to master that need to be learnt well.”

“I teach through putting pressure, I demand a lot of practise.”

Another view of the aims emphasized the features that characterize mathematics as an axiomatic system:

“Through proving theorems and deriving formulas pupil will learn to think mathematically”

“The goal is that students would comprehend the system of mathematics”

“I feel that mathematics in school is a goal in its own right, mathematics is for mathematics itself.”

The third view focuses on students' understanding and sees meaningful learning as the aim of mathematics teaching:

“Students need to dare share their own understanding. I aim at giving more weight to solving real life problems.”

“I try make children see mathematics not as only a subject in their exam that determines their future, but as a necessary and very interesting subject,”

Teachers' beliefs about situation in Estonian mathematics education

The teachers had well informed view of the Estonian mathematics education and they made references to the Estonian results in TIMSS 2003 study. As strengths indicated by the TIMSS results they saw the high qualification of teachers and the achievement of students, both above the international average in TIMSS. The teachers also appreciated the broad spectrum of teaching materials available and a well functioning system for developing able students.

Weaknesses that were mentioned by the respondents, and that have been also confirmed by the TIMSS study, were students' negative attitudes towards mathematics and their relatively low self-confidence in doing mathematics. Many of the teachers were also concerned about the overloaded programmes and the dominant transmission model in teaching.

Teachers' beliefs about mathematics teaching

Teachers' beliefs about mathematics teaching were focused on describing the needed changes in it. There was rather strong agreement on the problems: overloaded program, differentiation and students' motivation. However, there was less agreement of the solutions.

There was a strong view that it is important to encourage deep learning and for that purpose the overloaded curriculum should be revised. However, the teachers clearly felt unable to change the constraints of curriculum and teaching time.

“Mathematics curriculum is extensive and there are too few hours. I would teach more deeply if there was more time.”

“Had I more lessons, I would – together with my students – try to find the joy for working with mathematics.”

One specific concern of the upper secondary school was the dominant role of state exam.

“Preparation for state exams does not allow focusing on the substance of the subject.”

“More emphases ought to be put on developing the students' creativity. Now we need to deal with teaching of facts and drilling of standard tasks instead.”

If the constraints would allow, many would encourage active learning through discovery learning, projects, practical tasks and games.

“Mathematics should be ... changed more creative”

“I wish there was as little as possible of routine teaching”

Although almost all teachers wrote about differentiation, they provided quite different solutions.

A popular solution was to do differentiation through ability grouping. Some schools already had this practice while some teachers just indicated their wish for it. An alternative approach suggested by some teachers exhibiting that practice was to differentiate in class.

“The content and teaching time ... should be varied according to the different abilities of the students”

“I try to organize the lessons so that both the faster and the slower ones would find tasks of their own skill level”

Some teachers had an emphasis on weaker students, while others had an emphasis on able students

“We have problems with weaker students, they would need special treatment”

“I enjoy working in special classes with more able students”

There was apparent also a view that differentiation is not needed.

“Students are different. It's natural, not a problem”

With regard to increasing students' motivation there were three approaches suggested: Learning through inventing, Practice centred approach and Supportive class climate.

“I won't give them 'processed knowledge', but I let them find out themselves instead”

“I try to relate mathematics as much as possible with everyday situations”

“I appreciate a peaceful and open classroom climate. Students must dare to discuss.”

These approaches focus respectively on the aspects of cognition, motivation and emotion in learning.

Teachers' expectations for the project participation

Teachers seemed to have rather similar views regarding the expectations they had for the project. Most of them expressed more than one of the following expectations. Hence, the following categories should not be seen by any means contrary to each other, but as complementary aspects of teacher's views.

The participating teachers expected to gain new ideas for their teaching, especially regarding teaching methods.

“First of all I expect to get new ideas that I can immediately use in my everyday work”

Teachers also expected to partake in joint discussions and reflections.

“I expect to get good ideas and experiences from my colleagues and also an opportunity to share”

Furthermore, many were willing to participate in collaborative development of curriculum or teaching material.

“I would like to compose interactive teaching materials together with others”

“I hope that we would develop some useful teaching material together”

“I hope to participate in the process of renewing the curriculum”

What is the starting point for the project?

We have managed to involve in our project a group of successful and self-confident mathematics teachers. They have a shared view of the main problems in mathematics education and willingness to share ideas and reflect. This provides a good basis to build community of inquiry.

However, participating teachers do not share one view of solutions or even aims of mathematics teaching. In fact, while some of them emphasize drill and practice – others wish to involve more creative approaches to their teaching. Such strongly opposing views of the basic aims of mathematics education can cause friction in trying to find solutions for the identified problems.

We have already met several times with this group of teachers, although the data from these meetings has not been analysed yet. Our experience of the collaboration so far is quite positive. Teachers are willing to reflect and open for new views and ideas. Their differing views regarding solutions to the problems have so far not caused friction in discussions. This may be due to our general approach, where we frame the meetings as a space to raise questions and share own ideas to others rather than a place where solutions are being provided.

DISCUSSION

Research on participants' beliefs was able to inform opportunities and threats for building a community of inquiry. Some of this is likely to be specific to our local context and this specific group, but some of this is likely to apply for other similar efforts. At the least we can say that analysing the beliefs of participants is likely to be useful for anyone starting such a community of inquiry.

Although our data is of a specific case, it raises a general question regarding communities as this. How to build a community, when basic philosophies of teaching differ?

As the case often is in professional development programs, we have not succeeded to involve teachers who would feel a need for change in their teaching. Teachers participating in our project are professionals with well functioning teaching style. Are they ready to change? A study of Estonian language teachers' professional identity has indicated some characteristics of semi-professionality, most notably low sense of autonomy (Oder, 2007). Even in our selected group, the teachers seemed to feel lack of autonomy under the constraints of the curriculum.

Our own experience so far is that although the teachers who participate in our project are among the best professionals in the field, they are already active in developing themselves as mathematics teachers. Hence, bringing them together in the project is

likely to help them in their development through collaboration with colleagues with similar interests. However, what is the added value we gain? Should we not rather find ways to involve teachers who are less competent and less motivated to develop themselves?

We have been able to collect together a group of mainly expert teachers, many of whom are clearly seeking for possibilities for professional development and new challenges. There is much potential in this group. How could we use this resource for the development of mathematics education? Over the winter 2006-2007 we have engaged these teachers in design research, where they worked in groups, developing and testing together a variety of material for teaching percentages – a topic that was chosen together. Now each group is preparing a workshop for a national conference for mathematics teachers based on their work. Moreover the instructional material will be published on a CD. We expect the new ideas to be disseminated among teachers in Estonia, more extensive testing of the material, and more feedback over the next academic year. The group has also agreed to continue similar work with another topic - functions and the derivative.

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MATHEMATICS IS - FAVOURITE SUBJECT, BORING OR COMPULSORY

Kirsti Hoskonen

University of Helsinki

Mathematics student teachers of secondary school have written an essay about their school time experiences in mathematics. They write their memories of their school time, both good and bad experiences, the teaching, their teachers, and their own actions. Many students find it self-explanatory that students who study mathematical subjects, especially mathematics, have always liked mathematics, mathematics has been easy and their favourite subject. However, the experiences of the students are different. Some think mathematics has not always been easy, but they have coped. Some have the opinion that mathematics is only one subject among many other subjects. Some think mathematics is boring because of its easiness. Teachers are significant.

INTRODUCTION

Many people think mathematics is boring, difficult or terrifying. Teacher education students' experiences from their school time are seen to influence their view of mathematics (Pietilä 2000, Huhtala & Laine 2004). Negative experiences in mathematics affect students' encounters with mathematics and impact on the learning and teaching of mathematics. If mathematics is not relevant, it is evaded and people grow away from mathematics (Huhtala 2000). The research of the view of mathematics of the class teachers is seen to be important because their view of mathematics is formed by the memories of school time (Kaasila, 2000; Pietilä, 2002; Kaasila, Laine, & Pehkonen, 2004). In the studies concerned with class teachers, it is seen that teacher education students' views of mathematics has relevance to their avoidance of situations involving mathematics. Negative experiences are seen in students' self-esteem and their feelings against mathematics. Furthermore teachers can move their negative beliefs to their pupils (Gellert, 2000). Teacher education students who have coped well at school may not understand the situation of the slow and weak pupils (Kaasila, 2000).

Beliefs about mathematics of the class teachers are studied both in Finland and abroad (i.a. Lindgren, 1996; Kaasila, 2000; Pietilä 2002; Trujillo & Hadfield, 1999). Experiences on mathematics of mathematics teacher education students seem to be studied less. Sunnari's study (1999) deals with unforgettable experiences of school learning. Some teacher education students in her research study mathematics in the university. Karsenty (2004) has explored the nature of affective recollection of adults in regard to their experience as high school mathematics students.

Affect has been in a focus of mathematics education research, especially in mathematical problem solving. Mandler (1989, p. 3) describes affect:

The term affect has meant many things to many people, acquiring interpretations that range from “hot” to “cold”. At the hot end, affect is used coextensively with the work *emotion*, implying an intensity dimension; at the cold end, it is often used without passion, referring to preferences, likes and dislikes, and choices.

McLeod (1992) has identified three concepts for affect: beliefs, attitudes and emotions. Later, DeBellis and Goldin (1997) have added a new element, value, to the system. Goldin (2004) sees affects as a representational system that is intertwined with cognitive systems. Both systems encode important information regarding problem solving. Goldin (2002) describes emotions to be rapidly changing states of feeling, which are mild to very intense, and are usually local or embedded in context.

There are 2-18 classifications of basic emotions. Many psychologists have claimed that certain emotions are more basic than others, often for very different reasons. According to Power and Dalglish (1997) basic emotions are fear, sadness, anger, disgust and happiness. Researchers have agreed on some aspects of emotions. They are seen to be in connection with personal goals, as well as to involve physiological reactions. They are also seen to be functional, i.e. they have an important role in human coping and adaptation. (Hannula, 2006)

Huhtala (2000) has studied weak students' relations to mathematics. Their experiences are that mathematics is unpleasant, terrifying, discouraging, and irritating. They tell about anxiety, fear, and disgust. Leder and Grootenboer (2005) accept the public opinion that many students are constrained by negative attitudes and feelings about mathematics. According to the research of Kaasila (2000) the memories of class teachers can be divided into two types. Two thirds of the stories are permanency stories and the rest were stories about change with a turning point changing the story. The permanency stories are divided into five types:

1. "It was important to be the fastest in mathematics in the class" (15%)
2. "Mathematics is AHA insight" (20%)
3. "By cramming I coped" (9%)
4. "Mathematics was boring; I lost interest" (36%)
5. "I lost the thread" (20%)

The teacher education students of mathematical subjects are thought always to like mathematics and it is assumed that they always have coped well in mathematics because they are prospective teachers of mathematics. In this research, the focus is to check up on the teacher education students' experiences at the beginning of their pedagogical studies. Especially the focus is on the positive and negative experiences the students have had and the reasons for their experiences.

METHOD

The data of this research are the writings of the teacher education students in mathematical subjects at the beginning of their pedagogical studies. They are asked to

write a letter or an essay about their experiences in mathematics during their school time. It is added a recommendation to write e.g. one good and one bad experience in comprehensive school or in upper secondary school. The writing dates from the years 2004-2005 in the autumn. They are part of pedagogical studies in mathematics. Altogether 136 students have written their experiences, 77 are women of whom 37 have mathematics major. There were 59 men, of whom 34 have taken mathematics as a major. In Finland, elementary education teachers teach all the subjects for the students of six first grades of comprehensive school (7-12 years). In lower and upper secondary schools the teachers of mathematics is taught by subject teachers who have majored in mathematics, physics or chemistry.

The data is analyzed by using content analysis. In writings certain themes are found. The giving of the task has been free, so all the students have not written about the same themes. The themes in writings concern single events, teachers, themselves as mathematics learners, their own study strategies and subjects to study.

RESULTS

The teacher education teachers of mathematics are asked to tell both positive and negative experiences about school mathematics. Some teacher education students have written single events writing nothing about their relation to mathematics or their success in mathematics at all (28%, 38 students). Students remember teachers, their personality and ways to action and the atmosphere the teacher engenders into the class. The teacher can be a good example for them or a bad model of a teacher. Some contents of the mathematics courses can be easy or difficult. Students tell how they managed learning mathematics and their relations to mathematics at school time. 45 pieces of writing (33%) are stories about change. A typical story of change is that motivation vanished. The rationales are different. Many stories about change are connected to the transition phase (transition from elementary school to lower secondary school or from lower secondary school to the upper secondary school) or when the teacher is changing.

Some students write a small-scale memoir. Some of the students could not write a single negative experience. Some think that mathematics is just one subject among all the others.

Positive experiences

The majority of the stories are positive although separate occasions are not mentioned. Many students write that they have not been a genius but a little bit better than medium students. The topics have been simple enough for them. The students have been pleased to do problems, they have accepted the challenge, they are doing nicely, and they have got good marks on tests. A part of students needs parents' or teachers' assistance and then they have succeeded in it. Some have enjoyed that they have had a possibility to puzzle over the problems and to look for right solutions. Mathematics is positive when mathematics has been full of challenges, problems and

solutions, even if teachers are not very enthusiastic. A positive experience is that students themselves enjoy the action and learning.

"Generally speaking I can best remember the tremendous feeling, insight, what I got when I had managed to solve the problem in the class or at home."(s36)

Some students are interested in mathematics itself because *"the logic and regular world of mathematics is fascinating"*. It is also nice to do the homework in the lesson when the teacher has told them before the lesson is at the end.

Mathematics is a favourite subject for eight students (6%). More students do like mathematics. 18 students (12%) write like this student

"During my school time I have liked mathematics. It has been my favourite subject and I have been one of the best in my class in mathematics."(s 130)

Students find it self-explanatory that they like mathematics, because they have been good at it, done their tasks fast and succeeded in tests. All of them do not think so. Some write that they have been good or relatively good in mathematics; however mathematics has been one subject among others. At the same time, students have liked mathematics even if their marks have not been excellent.

However, not all the students write that they have liked mathematics. In many writings it is told implicitly

"Because mathematics is my main subject it is easy to guess that I have had more positive images than negative ones."(s 117)

Many students tell they have liked or always have liked mathematics, because *"learning did not require efforts and the tasks were fast done"(s 32)*. The favourite subject may require efforts but the pleasure got from it compensates all and brings satisfaction. The student has also thought to be mathematically talented or the topics have been easy. However, another student had the view of herself that she was really untalented and she did not like mathematics lessons. At the lower secondary school, the situation changed. She feels herself average and is anxious to choose the advanced course in mathematics. The relation to mathematics is changing positive and mathematics is favourite subject for

"From the first mathematics lesson onward, I was gaily surprised: I just understood what the man in front of was talking!"(s 111)

There is the impression that teachers have had a great resonance for the student's self-respect during the whole school time in mathematics.

Mathematics has been easy and it is liked in some grade. At the same time, they mention some reason contingent on the teacher. All the students have not had the same kinds of experiences. Some students write about neutral or negative experiences, some have had experiences of boredom.

Favourite subject is changing

Mathematics can be boring. Boredom can begin at the elementary school, when pupils are counting only the tasks of the mathematics book or pupils think that everything goes too slowly so that they do not need to listen to the teacher or they can learn to listen with only half an ear. Teachers can avert the boredom when they give nice extra tasks, word problems or letting the students to count problems beforehand. Pupils themselves have taken pleasure in having the competition of who calculated more tasks. In order to prevent boredom, pupils have had substitute actions like reading computing journals.

At the lower secondary school mathematics is boring, too. The students tell that motivation decreases because the tasks are too easy to require intellectual efforts. Bad, clamorous and uninterested class makes the school time bad at the lower secondary school. One student is wondering if it is because of mathematics or the myth that nobody likes mathematics. To avert the boredom, the student has coped by studying alone.

"At that stage of school going my thinking had developed so much that learning mathematics without the teacher was possible, I calculated much and read the old mathematics books of my parents. I was simply interested." (s 80)

At the upper secondary school, the teaching can be exceedingly boring when at the advanced level students are mathematically talented and the teacher is teaching the matters straight from the book copying them on the blackboard.

"So we became frustrated and made up a new program. ...I and one of my friends were extremely interested in intelligence games and brain-teasers. So we made each other brain-teasers. Another way to play for time is to have a competition which of us has calculated more problems during the lesson." (s 132)

One student has made the studying more interesting and challenging by neglecting his homework and going to the blackboard to calculate them a book in his hand. According to the student at school the teachers concentrate only on weaker and intermediate students and the talented are forgotten. Also they ache for meaningful activity from their teachers.

Although mathematic was the favourite subject, the situation has changed. Too easy problems and routine tasks do not give challenge resulting in mathematics being boring and disgusting. One student, Lisa, writes that lessons are OK and the tasks are easy. They have no homework but good marks on the tests. The situation continues at the lower secondary school and the boredom is increasing. Routine lessons lose the interest and motivation. Mathematics at the upper secondary school has nothing to do with everyday mathematics.

"For me, mathematics seemed remote from the daily life as a collection of rules and formulas which had fairly nothing to use in life. There were too many matters in lessons and the progress was fast for at the following lesson we had a new

matter. Fast progress influenced the shallow learning and deep understanding did not occur.” (s 92)

The students of mathematics can feel that mathematics is compulsory. Even good success cannot help when mathematics is seen to be compulsory. The tasks are felt to be compulsory when the focus of the studying is on the matriculation examination and the success is not enough to bring interest. Everybody has the same tasks without any more advanced ones. This student writes about the way a ”bad” teacher has taught:

”in the lessons, matters were hardly treated or progress was slow and inconsistent. Homework was not checked. If you were not able to do something, it would be our problem. Tests not always corresponded to the matters in the lessons. The lessons went listening to the teacher’s story of his tortoise”(s 104)

The student finds mathematics easy or self-evident; she became frustrated and thought that it is write-off to sit in the lessons

Negative experiences

Negative experiences seem to stick in students’ minds. Most of them have connected to teachers. Motivation seems to miss from the teachers. The teaching is either too slow or too fast, or only routine tasks were counted. Teachers have posed dislike when they behave.

”In the lower secondary school, I had a teacher who nearly killed my enthusiasm for the subject. He jeered at his pupils’ mistakes and we did not dare to express our uncertainty about the matter to study. Pupils were not too motivated and behaved unseemly. My marks in mathematics plunged and I hated to go to the lessons. Happily, math anxiety did not stay but when the teacher changed my enthusiasm awoke again. (s 121)

Students do not trust their teachers’ professional skills.

”My mathematics teacher had no authority. Studying was self-study during two years with my school friend and waiting for the break. The enthusiasm of mathematics in elementary school was changed.” (s 10)

”The matters he has taught, he posed monotonously. My questions asking for clarification and elucidation did not help because his answer was the same clause he had mentioned just before. He was not able to explain matters in other words, understandable.” (s 2)

Learning happens alone.

”I concentrated on understanding the matters and learning. I cut down on doing my homework because I rebel against my teacher’s dull system. In tests, I succeeded excellent but I got a worse mark.” (s 62)

The teacher is unprofessional and influences the student's choice between the basic and the short course.

"In the upper secondary school, I chose advanced mathematics. During my studies I have seen many kinds of teachers. The first experience was a horror to me. The teacher came into the class, opened the book and began to write on the blackboard. He wrote the whole lesson without turning once towards the pupils. When he wrote on the blackboard, his cheek was tight on the blackboard and he explained the matter. When he wrote the proofs, he favoured the utterance: ... and from this we see direct that... When I noticed that advanced mathematics was not for me I changed to the basic course." (s 15)

Mathematics itself is not negative. Students join negative experiences into their teachers and their actions. Most students tell negative experiences only about some teachers. Their relation to mathematics changes when they have another teacher. Even the pupils in the lower secondary school have missed a teacher who has authority and who can present mathematics in a coherent and meaningful way.

DISCUSSION

All the students in mathematical subjects do not remember any good or bad experiences regarding mathematics. According to Antikainen (1996), school as an institution prevents the significant memories of learning experiences. The essays can be different because the students are not precisely asked for some experiences in mathematics. Nearly one third of the writings of the students do not tell anything about the students themselves but about teachers or about individual action that have stuck in students' minds. A third part of the writings is change stories reported by Kaasila (2000). There are some stories like "I have always liked mathematics" (26 students, 19%), "Mathematics is one subject among others" (7 students, 5%), but not at all stories like "I lost the thread", "By cramming I coped" or "Mathematics is difficult and unpleasant". Instead mathematics is boring, but never all the time. Boring means for these students that mathematics is not challenging or that mathematics is not a part of everyday life. For 15% of the students, mathematics is challenging and interesting at least at some moment. The success in mathematics tests does not always assure positive experiences in mathematics. More challenging tasks are required. Goldin takes an example of a student approaching a problem in mathematics. First the experience is curiosity and perhaps frustration after some unsuccessful attempts. Frustration can either lead to satisfaction or anxiety, anger, fear, and/or despair. (Hannula et.al., 2004)

Weak students have feelings like anxiety, fear, and disgust towards mathematics. Different students bring new aspects to the relation to *mathematics*. They have different emotions, like joy and boredom. From the basic emotions of Power and Dalglish (1997) there are fear, disgust, and joy as a part of happiness. The way to encounter mathematics is quite different. Mathematics is a challenge for many of

them. Students tell how home, school society and friends have affected their experiences in mathematics. Op't Eynde has a socio-constructivist view where he sees affect is grounded on the social context and is also defined by it (Hannula et.al., 2004). Students have written about the experiences and emotions which affect with their view of mathematics. Fear and disgust are seldom the emotions these students have. Mathematics has brought them joy. Even the routine tasks can influence joy. Many students remember the feeling they have had because of a well-done test or task. Boredom is a negative experience. Some students think about the negative sides of encounter mathematics, meaningless, avoidance and estrangement.

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STUDENTS' BELIEFS AND ATTITUDES CONCERNING MATHEMATICS AND THEIR EFFECT ON MATHEMATICAL ABILITY

Kapetanas Eleftherios and Zachariades Theodosios

University of Athens, Greece

In this paper we study students' beliefs and attitudes concerning Mathematics and whether they affect their mathematical ability. The data was taken from a more general study which investigates students' beliefs and attitudes concerning Mathematics. The sample of this study was 1645 students of 10th, 11th and 12th grade. From our data three factors of beliefs and two factors of attitudes were traced. We investigate whether there are differences in students' beliefs and attitudes, concerning their social status, type of school and gender. We examine which of these five factors correlate and which of these affect students' achievement and mathematical ability too.

INTRODUCTION

There are many studies concerning students' beliefs and attitudes about mathematics. All researchers agree that these affect students' achievement in Mathematics. In Shoenfeld (1982), McLeod (1992) and Broun et al. (1988), it is verified that there is a link between students' attitudes and their achievement in mathematics. According to Cobb (1986) there is a relation between beliefs and learning of mathematics and that "students reorganize their beliefs about mathematics to resolve problems". In Schoenfeld (1989) it has been demonstrated that students' beliefs about Euclidean Geometry were a consequence of teaching. Some researchers agree that students' attitudes can be changed into more positive ones. Regna and Dalla (1992) assert that when teachers are enthusiastic in their teaching and plan activities which are accessible to students, then students' attitudes can be improved. In Dossey, Mullis & Jones (1993) it is verified that students' positive attitudes are stronger among elementary school students than among high school ones. In Kifer & Robitaille (1989) and in Philipou & Christou (2000) it is verified that students' beliefs are influenced by their social surrounding. According to Dematte et al. (1999) it seems that students' beliefs about mathematics are influenced by the educational system of their country. In Pehkonen (1995) students' beliefs from eight countries were investigated. In Christou C. & Philipou G. (1999) factorial structure of 13 years old students' beliefs among four countries (Cyprus, Finland, U.S.A., and Russia) were investigated. In this paper we investigate 10th, 11th, 12th grade students' beliefs and attitudes about mathematics and examine their correlation. We also investigate whether they influence students' performance at school and mathematical ability.

THEORETICAL BACKGROUND

As it comes from the literature, there are various opinions concerning the notion of “beliefs”. According to Goldin (1999), a belief may be “the multiply encoded cognitive configuration to which the holder attributes a high value, including associated warrants”. Cooney (1999), asserts that a belief is “a cluster of dispositions to do various things under various circumstances”, which leads to the acceptance that “different circumstances may evoke different clusters of beliefs” (Presmeg 1988). McLeod (1992) categorizes beliefs as follows: beliefs about mathematics, about self, about mathematics teaching and about the contexts in which mathematics education occurs. It is widely accepted that beliefs are the individual’s personal cognitions, theories and conceptions that one constructs for subjective reasons. Their nature is partly logical and partly emotional. They influence the individual’s behavior in mathematics. In this paper we will use the term “beliefs” in the meaning of personal judgments and views, which constitute one’s subjective knowledge, which does not need formal justification.

Another important element that affects students’ behavior about mathematics is that of attitudes. McLeod (1992) asserts that attitudes are persons’ reactions to negative or positive emotions, with medium intensity, but with sufficient stability. He accepts that “attitudes may result from the automatizing of a repeated emotional reaction to mathematics” or from “the assignment of an already existing attitude to a new but related task”. According to McLeod beliefs are more cognitive in nature than attitudes, are characterised by less intensity of response and by greater response stability than attitudes. Hannula (2002) explores four “different emotional-cognitive processes that produce an expression of an evaluation of mathematics”: the emotions (positive or negative) evoked when a student is engaged in a mathematical activity; the emotions evoked when a student is not actually engaged in a mathematical activity (for example, a questionnaire), because of previous experiences with mathematics; the emotions evoked by student’s expectations concerning the consequences of a mathematical situation; the cognitive analysis (which is often unconscious), that the student does, while evaluating the role of mathematics in the achievement of his personal goals. Hannula also accepts that “attitude is not seen as a unitary psychological construct but as a category of behavior that is produced by different evaluative processes. Students may express liking or disliking of mathematics because of emotions, expectations or values” and declared that attitudes can change under appropriate circumstances. In this study we investigate 10th, 11th, 12th grade students’ beliefs and attitudes concerning mathematics and we explore their factorial structure; we investigate whether there are differences in student’s beliefs and attitudes concerning their social status, gender and type of school; we examine whether these factors correlate and influence students’ achievement in school and their mathematical ability.

THE STUDY

Methodology

Data reported in this paper was collected by a questionnaire administered to 1645 students of 10th, 11th and 12th grade. These students were from 17 public general and 2 private general high schools and 6 technical public high schools in the district of Athens in Greece. The public general schools and the technical ones were selected by the stratified - two stages cluster sampling method, among the 317 public general schools and the 109 technical ones of this district. The private schools were selected by the simple random sampling method among the 48 private schools of the same district. We constructed a questionnaire taking into account analogous questionnaires from the literature, as in Schoenfeld, (1989), which consists of 27 questions. 15 questions (statements) concern beliefs and 8 concern attitudes about mathematics. The question Q24 concerns students' performance in mathematics at their school in the previous year. There are three more tasks, Q25, Q26 and Q27, which in this paper we call *mathtest*, in order to evaluate students' ability to understand mathematical proofs. These last three tasks were differentiated according to the students' grade (10th, 11th, 12th). Below we present one task of this type for each grade, because of lack of space. Students were asked to choose one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, that best describes what they feel or think for each one of the first 23 statements, using number 1 to declare "I don't agree at all" and number 9 to declare "I absolutely agree". We used a scale range from 1 to 9 (instead of 1 to 5), in order to give the students the opportunity to express their opinion in a more precise way.

Twenty one of the questions-statements of our questionnaire are presented in table 1 (see below in the Results). These are the ones which constitute the five factors. Two of the statements of the questionnaire were omitted, because of their low loadings in the factors, while statements Q24 and *mathtest* are presented below:

Q24. Your overall grade average in mathematics last year was:.....

Mathtest

Q25. For $\alpha, b > 0$, if $\alpha > b$, then $\alpha + 4 > b + 4$ (1). So, $\frac{(\alpha + 4)\alpha}{b} > b + 4$ (2). Thus $\frac{b + 4}{\alpha + 4} < \frac{\alpha}{b}$ (3).

Explain why relations (1), (2) and (3) hold. (This task was for 10th grade students).

Q26. Let a, b, c be real numbers such that $|a - b| \leq 5$ and $|b - c| \leq 5$. Then the following hold: $b - 5 \leq a \leq b + 5$ (1), $-b - 5 \leq -c \leq -b + 5$ (2). So, we obtain $-10 \leq a - c \leq 10$ (3). Therefore, $|a - c| \leq 10$ (4). Explain why relations (1), (2), (3) and (4) hold. (This task was for 11th grade students).

Q27. Let f be a real function, defined by $f(x) = x^3 + 1, x \in \mathbb{R}$. We observe that $f(-1) = 0$. We suppose that there is $p \in \mathbb{R}$, with $p \neq -1$, such that $f(p) = 0$. Then, if $p < -1$ it holds that $f(p) < f(-1)$ (1) and if $p > -1$, it holds that $f(p) > f(-1)$ (2). In any case there is a contradiction. Explain why the relations (1) and (2) hold and what the contradiction is. (This task was for 12th grade students).

Data analysis

Exploratory factor analysis which was applied led us to three factors F_1 , F_2 and F_3 , which concern beliefs and to two factors F_4 , F_5 which concern attitudes. These results (factors, the related items, means, standard deviations, factor loadings and Cronbach's alpha) are shown in table 1. The multivariate analysis of variance (manova) was applied in order to test if there are differences in factors F_1 , F_2 , F_3 , F_4 , F_5 , among students according to the variables: social-economic status (low, medium and high), kind of school (public general, private general, technical) and gender. We calculated Pearson correlations for these factors and variables 24 and mathtest, in order to investigate which of them and how correlate. These results are shown in tables 2 and 3 respectively, below.

RESULTS

Table 1: The five factors.

Factors	Cronbach's α	Mean	St.D.	Loadings
F_1 "Students' difficulties about mathematics"	0.743	4.275	1.73	
Q9 "Doing exercises in mathematics causes difficulties to me"				0.767
Q7 "Calculations in mathematics cause difficulties to me"				0.688
Q10 "Solving mathematical problems causes difficulties to me"				0.653
Q8 "Memorizing mathematical formulas causes difficulties to me"				0.633
Q6 "Mathematical symbols cause difficulties to me"				0.550
F_2 "Proofs' and mathematics' utility"	0.604	6.584	1.58	
Q25 "You study the proof of a theorem, because you believe that understanding of proofs can give you ideas, which will help you in problem solving"				0.665
Q4 "Mathematics which I learn in school contributes to improving my thinking"				0.634
Q24 "You study the proof of a theorem, because you believe that understanding of the proof will help you to understand the"				0.631

respective theorem”				
Q5 “Mathematics which I learn in school is useful only for those who will study in the university mathematics or technological sciences” (reversed)				-0.573
F₃ “Mathematical understanding through procedures”	0.601	5.812	1.35	
Q19 “If you are able to write down the proof of a theorem, then you believe that you have understood it”				0.751
Q20 “If you are able to express a definition, then you believe that you have understood it”				0.717
Q18 “ Studying mathematics means you learn to apply formulas and procedures”				0.575
Q21 “Anyone who wants to learn mathematics, has to memorize formulas and procedures”				0.450
F₄ “Love for mathematics”	0.735	5.642	2.23	
Q29 “You loved mathematics in junior high school”				0.869
Q28 “You loved mathematics in elementary school”				0.812
Q30 “You loved mathematics in senior high school”				0.665
F₅ “External students’ motives for studying mathematics”	0.500	5.341	1.45	
Q22 “You study the proof of a theorem, because your teacher will probably ask you, during the lesson”				0.727
Q23 “You study the proof of a theorem, because your teacher will probably ask this proof in the exams ”				0.656
Q26 “You don’t study the proof of a theorem, because your teacher will not ask				0.548

you about it, during the lesson”				
Q27 “You don’t study the proof of a theorem, because you believe that it is not necessary to learn the proof of the theorem, in order to do exercises or to solve problems”				0.455
Q17 “ Whenever you don’t manage to do an exercise or to solve a problem you ask for help, because you mind for your teacher’s good opinion”				0.389

As it is shown from table 1, Cronbach’s alpha is sufficiently high for factors F_1, F_2, F_3, F_4 , while it is poorer for factor F_5 (0.5). This result might be attributed to the fact, that technical schools’ students realize theorem’s proofs utility in an essential different way than general schools’ students do, because of their different culture and goals.

Table 2: Manova analysis with dependent variables the five factors $F_1 - F_5$ and independent variables “gender”, “kind of school”, “Social status”

Factor s	Gender				Kind of school				Social status	
	F 4.61 2	p .0 0	Means		F 5.28 2	p .0 0	Mean difference	p	F 1.433	p .08
			girl	boy						
F_1	.319	.5 7	4.2 9	4.2 5	14.7	.0 0	(1)-(3)=-1.05** (2)-(3)=-.92**	.00 .00		
F_2	3.32 8	.0 7	6.6 8	6.4 8	3.33 8	.0 3	(1)-(2)=-.46** (2)-(3)=-.59**	.01 .05		
F_3	8.70 7	.0 3	5.9 5	5.6 7	5.41 1	.0 0	(1)-(3)= -.33** (2)-(3)=.49**	.00 .00		
F_4	1.13 0	.2 8	5.5 1	5.7 6	5.68 9	.0 0	(1)-(3)= .93** (2)-(3)=1.13**	.00 .00		
F_5	3.47 5	.0 6	5.4 4	5.2 4	2.56 4	.0 7				

(1): general public school, (2): general private school (3): technical school

According to our data there are no significant statistical differences in students’ beliefs and attitudes concerning social-economic status ($F=1.433, p=0.08 > 0.05$). On the contrary, there are differences for factors F_1, F_2, F_3 and F_4 , among the students,

according to the kind of their school. Especially, there are differences between the students of public general schools and those of technical ones, as well as between the students of private general schools and those of technical ones, concerning factors F_1 , F_3 and F_4 . Especially, it emerges that the difficulties of the students of general schools are less than those of students of technical ones, as well as that the students of general schools love mathematics more than those of technical ones. It also emerges that technical school students believe that mathematical understanding is achieved mainly through procedures more strongly than those of general ones. Concerning factor F_2 , it emerges that there is significant statistical difference between the students of public and private schools, as well as between those of private schools and technical ones. It is estimated that private school students' beliefs concerning the utility of proofs and mathematics in general, are stronger than those of public and technical school ones. It also emerges that there is significant statistical difference between boys and girls, concerning factor F_3 . It is estimated that girls have a stronger belief than boys do, that mathematical understanding is achieved mainly through procedures.

We also traced correlations among the factors and variables Q24, mathtest (See table 3).

Table 3: Correlations between the factors and variables Q24, mathtest

	F_1	F_2	F_3	F_4	F_5	Q24	Mathtest
F_1	1						
F_2	-.223*	1					
F_3	.046	.155*	1				
F_4	-.434*	.343*	.039	1			
F_5	.235*	-.181*	.264*	-.164*	1		
Q24	-.277*	.155*	-.080*	.343*	-.106*	1	
Mathtest	-.389*	.203*	-.080*	.370*	-.189*	.395*	1

As it is shown from table 3 factor F_1 correlates negatively with factors F_2 , F_4 and variables Q24, mathtest and positively with factor F_5 . Especially, it seems that students with great difficulties in mathematics do not believe in proofs' and mathematics' utility in general, don't love mathematics, have low performance at school, low ability to understand proofs and study a proof mainly because they will be asked by the teacher.

Factor F_2 correlates negatively with factor F_5 and positively with factors F_3 , F_4 and the variables Q24, mathtest. That is, students who believe in the utility of proofs and mathematics don't need external motives for studying mathematics, love mathematics as well as they have high performance and ability to understand proofs. Factor F_3

correlates negatively with Q24, mathtest and positively with factor F_5 . This means that students who believe that mathematical understanding is achieved through procedures, don't have high performance at school and have difficulties to understand mathematical proofs. These students study proofs because they will be examined on these. Factor F_4 correlates negatively with factor F_5 and positively with variables Q24, mathtest. That is, students who love mathematics, study proofs not only because they will be examined, have high performance at school and ability to understand proofs. Factor F_5 correlates negatively with variables Q24, mathtest. That is, students who mainly study proofs because they will be examined on these, don't have high performance in mathematics and have difficulties to understand proofs. Variables Q24 and mathtest correlate positively; hence students who have high performance have also strong ability to understand mathematical proofs.

CONCLUSIONS

The results of this study clarify the structure of upper high school students' beliefs and attitudes concerning mathematics and the way in which mathematical performance and ability to understand proofs are influenced by them. It has been made clear, that students' beliefs and attitudes are independent from the social-economic status. This finding would probably be different if we compared students from agricultural districts of Greece with students from Athens. Philippou & Christou (2000) claim that, students' and teachers' beliefs concerning mathematics change from country to country. It seems that essentially different social surroundings, affect students' beliefs and attitudes in a different way.

Three different factors for beliefs and two different factors for attitudes were traced. Our study made clear that the variable "kind of the school" (public general, private general and public technical) influences students' beliefs and attitudes for all factors. Students of public and private general schools have less difficulty in mathematics than those of technical ones. This result is an expected consequence of the fact that technical school students' cognitive level is lower than the general school students' one. Private general school students believe more strongly in the utility of proofs and mathematics in general, than those of public general and technical ones. Technical school students believe more strongly that mathematical understanding is achieved through procedures than those of general ones, as they use only algorithms and computations to solve practical problems. Girls of all kinds of schools believe more in mathematical understanding through procedures, than boys do.

Difficulty in mathematics correlates with weak belief in the utility of proofs and mathematics, with dislike of mathematics and low mathematical performance and ability. Love for mathematics correlates positively with high performance and mathematical ability. The procedural view of studying mathematics is connected with low performance and ability in mathematics. Students with this view study proofs because they will be examined in them. Students who study proofs motivated by external reasons have low performance and ability in mathematics. These findings

agree with analogous conclusions of other researchers. Scoenfield (1985) notices that “the students’ overall academic performance, their expected mathematical performance and their sense of their mathematical ability all correlate strongly with each other” and “the better the student is, the less likely he or she is to believe that mathematics is mostly memorizing”. Kloosterman (2002) mentions that “beliefs are an important influence of motivation and motivating students is a major goal of instruction”.

The results of this research agree with the idea that beliefs are “a hidden variable in mathematics education” as well as that beliefs and attitudes influence performance and mathematical ability. Hannula (2002) found that attitudes can be changed. Therefore, one of the purposes of instruction must be the appropriate change in students’ beliefs and attitudes in order to improve students’ mathematical ability.

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THE NOTION OF CHILDREN'S PERSPECTIVES

Troels Lange

Aalborg University, Denmark

In this paper, I discuss methodological concerns relating to the notion of children's perspectives. My starting points are that children are social actors with their own ways of constructing meaning and interpreting their world, and second, that meaning is what children ascribe to their actions in the field of school mathematics learning. Meaning in this sense of the word is taken as a key notion in constituting and exploring children's perspectives. Insights into this meaning can be gained from adopting a life story approach to research that invites children to tell from their perspective. The paper ends with a methodological self reflection.

INTRODUCTION

The inclusion agenda officially manifested in the Salamanca Statement (UNESCO, 1994) invites schools - and mathematics education - to move the focus from the shortcomings of individual students to the structures, attitudes, social and pedagogical practices that hinder students' participation in the school and learning community (Booth, Ainscow, Baltzer, & Tetler, 2004). This agenda calls for a systemic reconceptualisation of low achievement in mathematics (and other school subjects) and of defective learning as a manifestation of imbalances in the system (see Lange, forthcoming). According to Magne (2001), most research in special needs education in mathematics, however, assumes either a content deviation model or a behaviour deviation model. In either case, the low achieving student is seen as deviating from a norm, that of the standard curriculum. Only a few studies deal with the complexity of the problem by considering the multiple factors involved in the creation of learning difficulties. Furthermore, children's subjectivity and experience of being in trouble with mathematics is seldom taken as a key source of insight.

Recent sociological and anthropological research in childhood generally recognizes children as actors in their own lives and not just objects of socialization (James, Jenks, & Prout, 1997; Kampmann, 2000). In their capacity as social actors, children have meaningful and interesting knowledge and experience. Their experiences and stories are as significant and valuable as those of adults are.

Children's or students' perspectives and other linguistic variations have become common terms in recent mathematics education research literature (e.g. Young-Loveridge, Sharma, Taylor, & Hawera Ngarewa, 2005). However, the notion is mostly used in an everyday sense and generally not treated as a theoretical construct. This is surprising given that ethnographic research has a long tradition for studying what the world is like for people who are different from the researcher. Discussions of methodological issues and pitfalls in this enterprise are an integral part of the tradition (Reed-Danahay, 2005), but that does not seem to be the case in mathematics

education research. Almost twenty years ago, Eisenhart (1988) pointed to the ethnographic research tradition as a valuable source of inspiration for mathematics education research because it requires researchers to scrutinise their own views and assumptions and investigate instead of taking for granted the intersubjective meanings that might constitute schools, classrooms, teaching practices, the arrangements in time and space etc.

An ethnographic, whole life approach, capable of capturing the complexity of affective issues in mathematics education, is also what McLeod (1994) called for in a review on research on affect:

They [Ivey, 1994; Ivey & Williams, 1994; Walen, 1994; Villiams & Baxter, 1993] suggest a new approach to affective issues – one that emphasizes the student as an individual with a comprehensive belief system, or world view. ... They suggest that students' affective reactions to mathematics occur within a larger framework of how students make sense of their world in general. ... Thus the students' views of mathematics can't be considered in isolation but must be analyzed in the context of an integrated approach that considers all the beliefs and motivating forces that influence the student. (McLeod, 1994, p. 644)

These approaches to methodology resonate with my current research work. In my ongoing PhD project, I focus on children's perspectives on learning difficulties in mathematics and explore how mathematics and learning it is positioned in children's life and world view; in McLeod's words, 'within the larger framework of how students make sense of their world in general'.

My notion of *children's perspectives* so far (see Lange, forthcoming), comprises *children's voices*, *experiences* and *meaning ascriptions* as constituents, and an aspiration of *contextualizing* and *theorizing* these. In this paper, I want to explore the notion further and consider how this affects methodology in regard to my PhD research. My argument shall be that the core of children's perspectives is the meaning they ascribe to the actions that they undertake when learning (or not learning) school mathematics. The argument rest on a paradigmatic choice that claims that meaning of tasks takes priority over the meaning of concepts (see Skovsmose, 2005b). Further, children's perspective being an analytical construct raises the question of the perspective in which I, the researcher, look at children's perspectives; I discuss this briefly in the end of the paper.

CHILDREN'S PERSPECTIVES

The etymological root of perspective, *spicere* from Latin, means *to look*. Central to the different meanings of *perspective* is the arrangement of objects (physical or mental) to represent their relative interrelations when 'seen' from a certain *point of view*. *Perspective* presupposes and indirectly acknowledges that there are different ways of looking at the same phenomena. Each of the different actors at school, teachers, students, parents, school leaders and authorities have their perspective on

school matters and develops knowledge from their different perspectives. This may be illustrated with an example of teachers' perspective. Højlund (2002, p. 155ff) found that in her interviews teachers stereotype children as asocial and egoistic, and generally characterise them by insufficiencies: they lack respect, manners, social sense and discipline. This picture of children is obviously neither complete nor neutral, but is derived from teachers' perspective. The function of teachers is to teach, and this determines their professional relations to children whom they see as students and as part of a class. Their definition is functional and relational and as such contains its own logic and rationality. Compared to the teacher, a child 'looks' at school matters from a different *point of view*, that is in a different perspective that may contain phenomena invisible in a teacher's perspective or differently interrelated.

A child's perspective is how the child 'looks' at 'the world'. As seeing is not a one-to-one imprint of 'the world' on the retina, but an active interpretation of the sensory impulses on part of the brain, a child's perspective is an active making sense of and ascribing meaning to – in this case – mathematics learning. That is, not only the cognitive or conceptual meaning the child ascribes to mathematical concepts but more important the meaning of teaching and learning of school mathematics in the child's life and worldview, and the meaning the child ascribes to actual and potential learning acts or other acts in the school mathematics field. Schools are socio-political settings. Hence, in order to grasp children's meaning ascriptions I need a theoretical framework that links them to the socio-political context of mathematics learning. Such a framework is the object of the next section.

Foreground and background

Ole Skovsmose connects meaning, (mathematics) learning and action by a cluster of interrelated notions: foreground, background, dispositions, intentions, meaning, action and reflection (Skovsmose, 1994; 2005a; 2005b). The main features in the network of notions are described briefly in the next few paragraphs.

The notion of *foreground* refers to

a person's interpretation of his or her learning possibilities and 'life' opportunities, in relation to what the socio-political context seems to make acceptable for and available to the person. Thus the foreground is not any simple factual given to the person; rather, it is a personally interpreted experience of future possibilities within the social and political frame within which the person acts. (Alrø, Skovsmose, & Valero, in press)

Similarly, the *background* of a person is

the person's previous experiences given his or her involvement with the cultural and socio-political context. ... [W]e consider background to be a dynamic construction in which the person is constantly giving meaning to previous experiences, some of which may have a structural character given by the person's positioning in social structures. (Alrø et al., in press)

Taken together foreground and background make up the person's *dispositions*, which “embody propensities that become manifest in actions, choices, priorities, perspectives, and practices” (Skovsmose, 2005a, p. 7). A person's dispositions are not always homogeneous and in fact can be contradictory as the person may conceptualise different foregrounds and backgrounds at different times and situations.

In order to understand a person's actions we need to consider his or her *intentions*. Hence, intentionality is taken to be a defining element of action, thereby separating action from mere activity. Intentions emerge from a person's dispositions, that is his or her background and foreground. Some forms of learning are seen as *action*, and so we can speak of intentional learning acts. Students can be invited into situations where they can be involved in processes of learning as action, but it cannot be forced upon them. In school, not all forms of learning are intentional learning acts; learning also results from forced activity, and unconscious learning is occurring. (Skovsmose, 2005a)

Meaning is an integrated aspect of acting, and something that is produced and constructed. Disposition, foreground and background, are resources for the production of meaning. All sorts of intentions emerge in children's actions in school mathematics teaching and learning situations and a variety of meanings are constructed. A child might want to please the teacher, sit next to the right person, finish tasks in time, avoid homework, be happy to solve the task, and want to play football. If children are not invited to engage in meaningful learning acts the field is not void of intentions and meanings, but left open to all sorts of other meaning productions, for instance ‘underground intentions’ (Alrø & Skovsmose, 2004). Thus, a child's interpretation of his or her previous experiences, of learning possibilities and ‘life’ opportunities, their availability and acceptability in the given socio-political context, are key resources of meaning production and hence key aspects of the child's perspective.

Looking with children

One may look *at* or look *with* children, or at least try to put oneself in their place, try to see with their eyes. Understanding children's perspectives, the logic of their meaning constructions, means looking into their foregrounds and backgrounds as major sources of information. Talking with children in interviews aimed at exploring how they make sense of and ascribe meaning to mathematics and mathematics education seems to be a way of looking with them. In this, I have two main sources of inspiration. First, life history research (Goodson & Sikes, 2001; Goodson, 2005) in which the (adult) informant ideally only is given the prompt: “Tell me about your life”. The interviewer interrupts as little as possible and only with clarifying questions, maintaining a curious, open minded, and non-interpreting state of mind, thus letting the informant's story unfold as ‘uncontaminated’ as possible by the interviewer's perspective. My informants are 10 to 12 years old; hence, the second source of inspiration is researchers with experience in conducting interviews with

children. Doverborg and Pramling Samuelsson (2000) have interviewed children from the age of three about their thoughts. Andenæs (1991) has conducted “way-of-life-interviews” with 4-5 year old children by interviewing them on locations relevant to the themes of the interview, for example their home. Researchers have found it fruitful to support the interviewing of young children with drawings, pictures, film, or stories (Kampmann, 2000). This research suggests that it is quite possible to interview children about their thoughts and meaning making and have them tell their stories. According to Andenæs there is no principal difference in doing qualitative interviews with children and adults; the challenges are the same although more acute with children: “When interviewing children, you have to put even more effort and care in the contract, in establishing a common focus of the conversation, and in motivating and create optimal conditions for the interviewee.” (Andenæs, 1991, p. 290; my translation)

It follows that the interviews should have an open, loosely structured character and take place in an atmosphere of genuine interest in order to support and stimulate children in unfolding their stories. The interview prompts and questions should be initiating, circular, supporting, and clarifying, and explore the children’s ‘world view’, learning trajectories, and connections, patterns and meaning making related to school, teaching, learning, mathematics, leisure, friends, mates, interests, etc.

An Example

Children have insights and points of view, which the other actors of the school system do not have. Quite often, their perspective is significantly different from that of adult professionals. It may for example contain a logic that differs from a rational, didactical perspective. The following extracts from an interview with two boys provide an example.

David and Dennis are 10 and 11 years old, friends and in fourth grade. At the time of the interview, the children in this grade were grouped in their mathematics classes according to level of achievement as perceived by the teachers. David is not quite aware of this criterion, but Dennis is. The extract begins with their reflections on this and continues with the story of why they are in the same group and how they managed to obtain that. [1]

- | | | |
|---|--------|---|
| 1 | David | actually, I think that the groups are given out [i.e. formed] from those who are best, I don't know ... |
| 2 | Dennis | they are |
| 3 | David | I think it is Ann [teacher], she takes the best, I think ... |
| 4 | Dennis | that is why I have gone up; started to be in the other [group] |
| | (...) | |
| 5 | Dennis | we used to have been together always |
| 6 | David | yeah |
| 7 | Dennis | and then I was going to go down |
| 8 | David | (?) |

Working Group 2

- 9 Dennis and then I made me good again because we were just chatting occasionally ...
- 10 Int and then you made – do you say that you made yourself good again?
- 11 Dennis yes, then I did my ...
- 12 Int how did you do it?
- 13 David then he did his best not to go down
- 14 Dennis then I did it again - not to go - stay there in that group, and then I went up in his [group] again
- 15 Int well, okay, how, what did you do to go to that group again?
- 16 David tried to do himself better
- 17 Dennis (?) mathematics and everything

In my interpretation, Dennis displays a strong disposition for autonomy or being in control. For instance, he explains earlier in the interview that it was his choice to repeat a class: “Once, I was fighting a lot in school, but that was because they tease me every day and therefore I did not bother to go in that class and then I repeated a class and came into his [David’s] class” In the extract, he is completely aware of the ground rules of the game, that is the criterion for forming the groups (2). He is the one who decides in which group he will be. Originally he was placed in the low set (4, 7) but then he made himself better (9, 14, 17). David supports and supplements his story (13, 16). The reason they give is friendship: they have always been together (5, 6) and want to be so; their friendship is expressed in David’s confirmation, support and taking over (6, 13, 16). It is background and foreground because it was a valuable previous experience that they want to continue into the future. They also tell a story of identity, which reflects their interpretation or perception of the socio-political context, their background: they belong to the *best* group (1-3) which consist of the *good* and *better* (9, 16). These categories are explicitly embedded in a hierarchical order expressed as *up* and *down* (4, 7, 13, 14); you are *up* if you are *best*. Alternatively, the grouping might have been conceived as a means to facilitate learning of mathematics, and thus reflecting intentions of learning mathematics on part of the children, but that possibility seems absent from their considerations.

A little later in the interview, I tried to investigate their relation to this hierarchy:

- 18 Int is it cool to be in the best group, or
- 19 David Yes, it ...
- 20 Dennis I don’t think so!
- 21 David I think it is cool because I know ...
- 22 Dennis I don’t think so!
- 23 David that I am one of the best
- 24 Int mm
- 25 Dennis I don’t think it is cool, rather cool
- 26 Int why don’t you think so?

27 Dennis because then you get more homework than they [the other group] do

Being good at mathematics has a high social valuation, and this is reflected in the children's background in two different ways. David appreciates the social status of being in the best group (19, 20) and thinks that he rightly deserves it (23). Dennis on the other hand, strongly denies that it is cool to be with the best (20, 22, 25) because he dislikes the consequence of more homework (27). This may be seen as another example of his strong valuation of autonomy in that homework may interfere with or even infringe on the social life in his free time. This interpretation is supported in a later part of the interview, where Dennis explains why practicing the multiplication tables is (the only?) good mathematics homework: you can do the tables in your head while you ride your bike from your home to your friend's home. However, the social status of belonging to the top end of the hierarchy that he expressed earlier (4, 7, 14) is a mixed blessing to him. In the conflict between social status and autonomy, Dennis seems to make a conscious compromise: he works hard enough to maintain the status mathematics provide (and stay with David as well) but no more. The social valuation of mathematics is subjectively interpreted as background and foreground, and come into play in the different dispositions of David and Dennis to engage in learning mathematics. Whereas David's need for recognition goes hand in hand with the social valuation of mathematics and adds positively to his disposition for learning mathematics, Dennis' disposition shows a conflict between status and autonomy which impacts on his engagement with learning mathematics.

The example suggests that these two children interweave the meaning of mathematics education into a fabric of friendship, belonging, expression and construction of identity, and the social practice of everyday life. In the extracts as well as in the rest of the interview, learning intentions and meaning constructions have their basis in their lives as children, their background and foreground, and are seemingly not related to mathematics as such. Their perspectives are very different from that of the curriculum. However, it would be possible for the teacher to use this information when trying to engage students in meaningful mathematics education.

SEEING PERSPECTIVES FROM PERSPECTIVES

Children are not a homogeneous group, children's foregrounds and backgrounds are different, their interpretations of the socio-political context are fluctuating, discontinuous and contradictory, their intentions and meaning constructions likewise. Hence, there is not one child perspective; *the* child perspective does not exist.

As well, a child's perspective is not a 'thing', an empirical entity that one may for example take a picture of; it is an analytical construction of the researcher. Informants do not have privileged access to the truth about their own world. The researcher's analytical account is of another order than that of the children's experiential knowledge.

However, children's perspectives as objects of the researcher's gaze, are seen from what perspective? I cannot reflect on my perspective without stepping out of it and look at it from a different point of view. The question then becomes more introspective as I consider the perspective from which I look at the perspective from which I look at children's perspectives. (This chain of perspectives on perspectives continues – we have a principally infinite regress.)

Giving voice or silencing

My PhD project may be seen as an attempt to “give voice” to an exposed group, children in difficulties with learning mathematics. However, in an endeavour of this type, one may silence in effect the voices if they are not linked to a theoretical understanding of their social and cultural context. Goodson writes:

A particular problem ... is posed by those genres which ... have sought to sponsor new voices – the world of ‘stories’, ‘narratives’ and ‘lives’. ... [A]s currently constructed these genres tend to lead us away from context and theorizing, away from the conceptualization of power.

... In the dialectical development of theories of contextualities, the possibility exists to link our ‘stories’, ‘narratives’ and ‘lives’ to wider patterns of structuration and social organization. So the focus on theories of context is, in fact, an attempt to answer the critique that listening to lives and narrating them valorizes the subjectivity of the powerless individual. In the act of ostensible ‘giving voice’, we may be ‘silencing’ in another way, silencing because, in fact, we teachers and researchers have given up the concern to ‘theorize’ context. (Goodson, 2003, p. 5)

The background-foreground ‘model’ incorporates a research interest, that of emphasizing the socio-political nature of mathematics education and learning. Hence, this choice of perspective on children's perspectives serves my attempt to avoid silencing the voices of children, because it allows theorising children's meaning constructions and agency, their perspectives, in a wider socio-political context.

That is my – present – perspective on children's perspectives.

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NOTES

1 In Denmark, children are not streamed in primary and lower secondary school. Recent legislation has allowed the formation of groups across classes and year groups for limited periods of time.

The interview was conducted in an early phase of the project when I was trying out interviewing children, and not intended to become part of my empirical material. Hence, the informants do not belong to my primary target group, children being in difficulties with mathematics. I have translated the extracts and normalized the language a little though still trying to maintain the characteristics of children's language.

In the transcript “...” marks interruption, “(...)” omission, and “(?)” short unintelligible passages.

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BELIEF CHANGE AS CONCEPTUAL CHANGE

Peter Liljedahl

Simon Fraser
University,
Canada

Katrin Rolka

University of Dortmund,
Germany

Betinna Rösken

University of Duisburg-
Essen,
Germany

The theory of conceptual change starts with an assumptions that in some cases students form misconceptions about phenomena based on lived experience, that these misconceptions stand in stark contrast to the accepted scientific theories that explain these phenomena, and that these misconceptions are robust. In this paper we examine the idea of changing beliefs in preservice elementary school teachers' vis-à-vis a theory of conceptual change. This is our first attempt at using such a framework, and as such, our work in this area is tentative. Our specific focus in this paper is to rationalize why this is a fruitful theoretical framework to use, and through the brief presentation of data, verify this fruitfulness. In so doing, we open up the possibility of more closely examining the mechanisms associated with such changes of beliefs.

INTRODUCTION

"It has become an accepted view that it is the [mathematics] teacher's subjective school related knowledge that determines for the most part what happens in the classroom" (Chapman, 2002, p. 177). One central aspect of subjective knowledge is beliefs (Op't Eynde, De Corte, & Verschaffel, 2002). In fact, Ernest (1989) suggests that beliefs are the primary regulators for mathematics teachers' professional behaviour in the classrooms. "Beliefs form the bedrock of teachers' intentions, perceptions, and interpretations of a given classroom situation and the range of actions the teacher considers in responding to it" (Chapman, 2002, p. 180). What are the implications of this for teacher education?

"Prospective elementary teachers do not come to teacher education feeling unprepared for teaching" (Feiman-Nemser et al., 1987). "Long before they enrol in their first education course or math methods course, they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (Ball, 1988). These ideas are more than just feelings or fleeting notions about mathematics and mathematics teaching. During their time as students of mathematics they first formulated, and then concretized, deep seated beliefs about mathematics and what it means to learn and teach mathematics. It is these beliefs that often form the foundation on which they will eventually build their own practice as teachers of mathematics (cf. Skott, 2001). Unfortunately, these deep seated beliefs often run counter to contemporary research on what constitutes good practice. As such, it is one of the roles of the teacher education programs to reshape these beliefs and correct misconceptions that could impede effective teaching in mathematics (Green, 1971).

In this paper we examine the idea of changing beliefs in preservice elementary school teachers' vis-à-vis a theory of conceptual change. This is our first attempt at using such a framework, and as such our work in this area is tentative. Our specific focus in this paper is to rationalize why this is a fruitful theoretical framework to use, and through the brief presentation of data, verify this fruitfulness.

TEACHERS' BELIEFS

Researchers have recently turned their attention to beliefs as a way of explaining the discordance between teachers' knowledge of mathematics and teaching capacity and their demonstrated abilities in these domains. This research has revealed that beliefs about teaching mathematics arises from teachers' experiences as learners of mathematics (c.f. Chapman, 2002; Feinman-Nemser et al., 1987; Lorti, 1975; Skott, 2001). So, a belief that teaching mathematics is 'all about telling how to do it' may come from a belief that learning mathematics is 'all about being told how to do it', which in turn may have come from personal experiences as a learner of mathematics.

Beliefs are complex constructs, and belief structures are even more so. This complexity is represented in Green's (1971) organization of beliefs "along a central-peripheral dimension that reflects psychological strength or degree of nearness to self" (Chapman, 2002, p. 179). Green (1971) distinguishes between beliefs that are primary and derived. "Primary beliefs are so basic to a person's way of operating that she cannot give a reason for holding those beliefs: they are essentially self-evident to that person" (Mewborn, 2000). Derived beliefs, on the other hand, are identifiably related to other beliefs. Green (1971) also partitions beliefs according to the psychological conviction with which an individual adheres to them. Core beliefs are strongly held and are central to a person's personality, while less strongly held beliefs are referred to as peripheral. Finally, Green distinguishes between evidential and non-evidential beliefs. Evidential beliefs are formed, and held, either on the basis of evidence or logic. Non-evidential beliefs are grounded neither in evidence nor logic but reside at a deeper, tacit level.

In general, beliefs can be referred to as "messy constructs" (Furinghetti & Pehkonen, 2002; Pajares, 1992). Some of this 'messiness' can be reduced, however, if we focus on the composition of these beliefs. Törner and Grigutsch (1994) suggest that beliefs are composed of three basic components called the toolbox aspect, system aspect and process aspect. In the "toolbox aspect", mathematics is seen as a set of rules, formulae, skills and procedures, while mathematical activity means calculating as well as using rules, procedures and formulae. In the "system aspect", mathematics is characterized by logic, rigorous proofs, exact definitions and a precise mathematical language, and doing mathematics consists of accurate proofs as well as of the use of a precise and rigorous language. In the "process aspect", mathematics is considered as a constructive process where relations between different notions and sentences play an important role. Here the mathematical activity involves creative steps, such as generating rules and formulae, thereby inventing or re-inventing the mathematics.

Besides these standard perspectives on mathematical beliefs, a further important component is the usefulness, or utility, of mathematics (Grigutsch, Raatz & Törner, 1997).

CHANGING BELIEFS

Beliefs and belief structures are complex constructs. Educational research into the professional growth of teachers in general (c.f. Zeichner, 1999), and of mathematics teachers in particular (c.f. Franke et al., 2001) tends to ignore this complexity; both in methodology and in analysis (for exception see Gates, 2006; Leatham, 2006). In particular, such research uses an objective stance to probe the belief structures of a large number of teachers, and hence, is only capable of producing generalization about changes to teachers' beliefs. Conclusions such as 'beliefs are difficult to change' and 'any changes are tenuous and fragile' (Kagan, 1992) do not say much about the nature of beliefs and why changes to them are robust or fragile. Closer observation and deeper analysis of beliefs in the context of mathematics teachers' professional growth is needed to penetrate the surface stories of the data and reveal the nuanced and situated belief structures that are often hidden, even from the possessor.

Our own research in this area does not escape this criticism. Through our work we have shown that a method that combines all three of the aforementioned interventions is very effective in producing changes to preservice teachers' beliefs about mathematics as well as the teaching and learning of mathematics (Liljedahl, P., Rolka, K., Rösken, B., in press). What this research has failed to show, however, is how and why these changes are occurring. That is, our research, like much of the aforementioned research in this area, shows that changes to beliefs have occurred, but does not show the mechanisms behind this change.

THE THEORY OF CONCEPTUAL CHANGE

The theory of conceptual change emerges out of Kuhn's (1970) interpretation of changes in scientific understanding through history. Kuhn proposes that progress in scientific understanding is not evolutionary, but rather a "series of peaceful interludes punctuated by intellectually violent revolutions", and in those revolutions "one conceptual world view is replaced by another" (p. 10). That is, progress in scientific understanding is marked more by theory replacement than theory evolution. Kuhn's ideas form the basis of the theory of conceptual change (Posner, Strike, Hewson, & Gertzog, 1982) which has been used to hypothesize about the teaching and learning of science.

The theory of conceptual change starts with an assumptions that in some cases students form misconceptions about phenomena based on lived experience, that these misconceptions stand in stark contrast to the accepted scientific theories that explain these phenomena, and that these misconceptions are robust. For example, many children believe that heavier objects fall faster. This is clearly not true. A rational

explanation as to why this belief is erroneous is unlikely to correct a child's misconceptions, however. On the one hand, it would require far too much specialized knowledge to access any of the explanations that could be given. On the other hand, it is attempting to replace understanding developed through lived experiences with an understanding developed in rational thought. In the theory of conceptual change, however, there is a mechanism by which such theory replacement can be achieved – the mechanism of 'cognitive conflict'.

Cognitive conflict works on the principle that before a new theory can be adopted the current theory needs to be rejected. Cognitive conflict is meant to create the impetus to reject the current theory. So, in the aforementioned example a simple experiment to show that objects of different mass actually fall at the same speed will likely be enough to prompt a child to reject their current understanding. This experiment will not be enough, however, for them to then adopt an understanding of the nuances of physics and logic required to arrive at a correct understanding. What is more likely to happen is that the child would develop a 'synthetic model' (Vosniadou, 2006) which can be viewed as an intermediary between their initial misconception and the scientifically correct theory. In the best case, this synthetic model can be seen as incomplete understandings rather than incorrect understandings. The mitigation of these synthetic models is achieved through further instructional methodologies derived from constructivist theories of learning.

The theory of conceptual change is not a theory that applies to learning in general. It is highly situated, applicable only in those instances where misconceptions are formed through lived experiences and in the absence of formal instruction. In such instances, the theory of conceptual change explains the phenomenon of theory rejection followed by theory replacement. The theory of conceptual change, although focusing primarily on cognitive aspects of conceptual change, is equally applicable to metaconceptual, motivational, affective, and socio-cultural factors as well (Vosniadou, 2006).

CHANGES IN BELIEFS AS CONCEPTUAL CHANGE

In this section we argue that the theory of conceptual change, as presented in the context of science education, is equally applicable to some instances of change in preservice teachers' beliefs about mathematics and the teaching and learning of mathematics. In particular, the theory of conceptual change can be used to more closely examine instances of belief replacement. In so doing, we open up the possibility of more closely examining the mechanisms associated with such changes of beliefs.

The theory of conceptual change, as the explanatory framework described above, has four primary criteria for relevance – (1) it is applicable only in those instances where misconceptions are formed through lived experiences and in the absence of formal instruction, (2) there is phenomena of theory rejection, (2) there is a phenomena of

theory replacement, and (4) there is the possibility of the formation of synthetic models. We propose that each of these criteria is equally relevant to instances of replacement of preservice teachers' beliefs about mathematics, as well as beliefs about the teaching and learning of mathematics. In the next section we demonstrate this with the brief presentation of research results. First, however, more discussion of teachers lived experiences as well as synthetic models is needed.

In the context of preservice teachers, the relevant lived experience occurs in their time as students. As learners of mathematics they have both experienced the learning of mathematics and the teaching of mathematics, and these experiences have impacted on their beliefs about the teaching and learning of mathematics (c.f. Chapman, 2002; Feinman-Nemser et al., 1987; Lorti, 1975; Skott, 2001). The question is – can these experiences be viewed as having happened outside of a context of formal instruction? Although their experiences as learners of mathematics are situated within the formal instructional setting of a classroom, the object of focus of that instruction is on mathematics content. That is, while content is explicitly dealt with within such a setting theories of learning, methodologies of teaching, and philosophical ideas about the nature of mathematics are not.

The term 'synthetic model' is a specific term reserved for the description of incomplete or incorrect scientific model. This is not an appropriate term for the context of beliefs – instead we use the term 'synthetic beliefs'. The ideas of 'incomplete' and 'incorrect' beliefs are equally inappropriate. Beliefs, unlike scientific theory, can be accumulated into belief clusters. Hence, a 'complete' belief cluster could easily be understood to be a cluster that incorporates all relevant beliefs for a given context. Such an understanding of 'complete' is incommensurate with the theory of conceptual change which is built on a principle of, in this case, belief rejection. As such, we are modifying the theory of conceptual change in general and of synthetic beliefs in particular. Instead of using demarcating characteristics such as 'complete' or 'correct' we adopt instead the use of 'inconsistent'. This choice is informed, in part, by the data that we will present. Mostly, however, this choice is made because we hypothesize that the characteristic of 'consistency' is a strong indicator of the robustness of a set of beliefs.

Incomplete synthetic beliefs, although not comprehensive, are likely sustainable. We see this in the mathematical practices of 'traditional' teachers who possess beliefs that are mostly aligned with the toolbox and/or utility aspects of mathematics and teaching and learning of mathematics. We know that such traditional teachers can consistently maintain their practice for many years – even entire careers. We hypothesize, however, that such consistency is not sustainable if there exists discordance between a teacher's belief about mathematics and their beliefs about the teaching and learning of mathematics. More research is needed in this area to confirm or refute this hypothesis.

RESEARCH INTO BELIEF CHANGES

The data for this paper comes from a research study that looked more broadly at documenting changes in preservice teachers' beliefs about mathematics and the teaching and learning of mathematics (c.f. Liljedahl, P., Rolka, K., Rösken, B., in press). In working with the data for this study we encountered instances of change that could not be explained by an evolutionary model. It was these instances that formed the impetus to produce this paper.

METHODOLOGY

Participants in this study are 39 preservice elementary school teachers enrolled in a Designs for Learning Elementary Mathematics course for which the first author was the instructor. During the course the participants were immersed into a problem solving environment. That is, problems were used as a way to introduce concepts in mathematics, mathematics teaching, and mathematics learning. This design for the course emerged out of the literature on producing changes in preservice teachers' mathematical beliefs. This included, for example challenging their beliefs (Feiman-Nemser et al., 1987), involving them as learners of mathematics (Ball 1988), or occasioning experiences with mathematical discovery (Liljedahl, 2005; Smith, Williams, & Smith, 2005). All of these methods of intervention, as well as their combination, can be viewed as attempting to incite cognitive conflict.

Throughout the course the participants kept a reflective journal in which they responded to assigned prompts. These prompts varied from invitations to think about assessment to instructions to comment on curriculum. One set of prompts, in particular, were used to assess each participant's beliefs about mathematics, and the teaching and learning of mathematics (What is mathematics? What does it mean to learn mathematics? What does it mean to teach mathematics?). These prompts were assigned in the first and final week of the course. The data for this proposal comes from the journal entries responding to these prompts.

The three authors independently coded the data according to each of the four aforementioned components of mathematical beliefs: toolbox, utility, system, and process. Discrepancies in coding were resolved as part of a recursive process of discussion-coding-discussion that the three authors engaged in. This recursive process not only led to a more stringent treatment of the data, but also led to a greater and shared understanding of the interpretive framework at hand. For the purposes of this paper, we further examined these data for instances of change that reflect the criteria of conceptual change.

RESULTS AND DISCUSSIONS

For the sake of brevity, and because our primary objective is to exemplify the viability of the theory of conceptual change for the analysis of belief replacement, we have chosen to present the results of the analysis of one participant – David – whose

journal is most representative of belief replacement. These results are organized according to the four aforementioned criteria of lived experience, belief rejection, belief replacement, and synthetic beliefs.

Lived Experience

David nicely articulates where his understanding of mathematics comes from.

When first pondering the question, "What is mathematics?" I initially thought that mathematics is about numbers and rules. It is something that you just do and will do well as long as you follow the rules or principles that were created by some magical man thousands of years ago. That is a struggling student's point of view. To be honest, I don't like math. [...] I found it so boring and so robotic. Lessons were even set up in a robotic way. The teachers would show us the principles and then we would do the exercises.

His lived experience as a student of mathematics is now informing his 'teacherly' understanding of what mathematics is. It is also informing his understanding of what it means to teach mathematics – robotic.

Belief Rejection

David finishes of his aforementioned statement with the following sentence:

I wish my initial definition could be different but this is the kind of math that I was exposed to.

David has come into the course already rejecting his beliefs about mathematics and the teaching of mathematics. From further analysis of his journal it becomes apparent that he has not yet fully let go this belief, however, because there exists no alternative for him to synthesise with. It could be said that, although not initiated through a teaching intervention, David has already experienced cognitive conflict with respect to these beliefs.

Belief Replacement

David, himself, makes the coding of some of the data easy. He self-identifies that he finds his initial belief to be inadequate. He further self-identifies that his beliefs about mathematics have changed.

However, after experiencing a couple of challenging problems and exciting classes, I have to say that my definition [of mathematics] can be summed up very simply. To me, mathematics is not about answers, it's about process. Mathematics is about exploring, investigating, representing, and explaining problems and solutions.

David also self-identifies the changes he has made in his beliefs about the learning and teaching of mathematics. His new belief is much more representative of the 'process' aspect of teaching and learning.

Learning math is about inquiry and the development of strategies. It is about using your intuition, experimenting with strategies and discussing the outcome. It is about risk taking and experimenting. To teach mathematics is to welcome all ideas that are generated and

facilitate discussion. It is about letting the students make sense of the math in their own way, not 'my way'. The teacher's role is about guiding the process, but handing the problem over to the students.

Synthetic Models

As mentioned above, if we consider David's new beliefs to be inconsistent with one another then we judge them to be synthetic beliefs. In the case of David, we coded his new beliefs about mathematics solely as representative of a process way of thinking about mathematics. At the same time, we coded his new beliefs about the teaching and learning of mathematics to be representative of both a process and a toolbox aspects of mathematics. We see these as being inconsistent, and thus we see his new beliefs as synthetic beliefs.

Treating the data more broadly reveals 25 instances of belief replacement and 5 instances of belief evolution. Of those who demonstrated belief replacement there are only two participants that are explicit about their rejection of their initial beliefs (David and Hannah). The rest are implicit in their rejection of earlier beliefs through their omissions. That is, beliefs that were coded for in the entries at the beginning of the course are absent in at the end of the course. Of the 25 students who demonstrated belief replacement (explicatively or implicitly), 16 demonstrated an inconsistency between their beliefs about mathematics and their beliefs about the teaching and learning of mathematics, and hence were coded as having developed synthetic beliefs. The other 9 participants developed internally consistent beliefs for themselves.

CONCLUSIONS

The theory of conceptual change is a powerful theory for explaining the phenomena of theory replacement when the rejected theory has been tacitly constructed through lived experiences in the absence of formal instruction. Such organically constructed theories are not too dissimilar from the beliefs which may also be tacitly constructed through lived experiences. When such beliefs are later subjected to scrutiny they too may be rejected. As such, the theory of conceptual change is an ideal framework for more closely examining and explaining the phenomenon of belief rejection.

In this paper we have attempted to construct the link between the theory of conceptual change and specific instances of change in preservice elementary school teachers' beliefs about mathematics and/or the teaching and learning of mathematics. We have done so, through an alignment of literature on the theory of conceptual change with theories of beliefs. Having established this link we then modified the theory of conceptual change slightly to more precisely fit the context of belief replacement. In doing so we extend the scope of the theory of conceptual change.

One of the measures of a framework is how effectively it can 'fit' the data. In this paper we have demonstrated that this modified framework fits the data from, at least, one participant – we have, in essence, constructed a sort of educational existence proof. Another measure of a framework is how well it can inform us of something in

the data that we could not previously see. Although this was not the focus of this paper, we did see some of this effectiveness in the means by which the framework was able to discern the difference between instances of belief evolution and belief replacement. We hope to use this framework more precisely in the future, and further hope that in doing so we will gain further insights into the context of belief change.

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CHANGES IN STUDENTS' MOTIVATIONAL BELIEFS AND PERFORMANCE IN A SELF-REGULATED MATHEMATICAL PROBLEM-SOLVING ENVIRONMENT

Andri Marcou Stephen Lerman

London South Bank University

This study focuses on the theory of self-regulated learning (SRL) and examines the changes on primary students' motivational beliefs and performance in mathematical problem solving (MPS). Students coming from 15 different classes received a seven-month teaching intervention in MPS according to the principles of the SRL theory whereas control group students from 13 other classes received the usual method of teaching. Paired samples t-test and repeated measures ANOVA and ANCOVA applied on the data collected from tests and questionnaires indicated statistically significant differences between and within groups in task-value, goal orientation beliefs and performance in MPS. The results draw attention to teaching practices for independent, intrinsically oriented and more efficacious students in MPS.

INTRODUCTION

During the last 30 years there has been abundant evidence stressing the importance of multiple affective variables in educational settings and particularly in the context of students' learning, such as motivational beliefs or self-beliefs about the reasons that encourage a student to work on a task. Motivational beliefs are frequently found in the literature to be associated with the theory of self-regulated learning (SRL) (e.g. Pintrich, 1999; McWhaw & Abrami, 2001), one of the flourishing areas of research, since it redistributes and transmits the responsibility and control from the teacher to the students and provides tools for lifelong learning (Boekaerts, 1997).

Mathematical problem solving (MPS), as an important aspect of mathematics education that demands the application of multiple skills (De Corte, Verschaffel, & Op' t Eynde, 2000), seems to be a potentially rich domain to study SRL and motivational beliefs since it requires the application of cognitive and metacognitive skills (Panaoura & Philippou, 2003). There have been many studies in the area of MPS (e.g. Schoenfeld, 1985; Verschaffel, De Corte, Lasure, Vaerenbergh, Bogaerts, & Ratinckx, 1999) as well as general studies in the area of SRL and motivational beliefs; nevertheless most of the studies approached SRL as a general aptitude of human behaviour that in a way can be associated to MPS performance and motivational beliefs (e.g. Marcou & Philippou, 2005). There is a paucity of research that theoretically incorporates in depth the fields of MPS and of SRL and motivational beliefs. A recent study of Marcou and Lerman (2006) revealed that the various aspects of different models of MPS and SRL can be combined to contribute to the emergence of a self-regulated mathematical problem solving model that can be used as a tool in primary school teaching situations. Following up that study, the aim of this study is to examine the impact of a seven month teaching intervention, which

incorporates the aforementioned model as the basic tool of teaching as well as basic principles of the theory of SRL, on primary students' motivational beliefs and performance, all related to MPS.

THEORETICAL BACKGROUND

Motivational beliefs and self-regulated learning

Although there are various approaches and models connected to the theories of motivational beliefs and SRL (Marcou & Philippou, 2005), we predicate our study on the models of Pintrich (1999) and Zimmerman (2004) since these incorporate both “skill” or cognitive and “will” or affective components of learning (McWhaw & Abrami, 2001). The “skill” component refers to the use of different SRL strategies which are assumed to have an impact on students' performance (McWhaw & Abrami, 2001). According to Pintrich (1999), such strategies are general cognitive (rehearsing, elaborating, organising), metacognitive (planning, monitoring, regulating) and resource management strategies (e.g. help-seeking). Zimmerman (2004) depicted graphically the theory of SRL as a cyclical procedure that incorporates the SRL strategies, task strategies and motivational beliefs. The “will” component refers to the notion of motivational beliefs such as self-efficacy, task value and goal orientation beliefs (Pintrich, 1999). Self-efficacy pertains to judgements of one's ability to execute certain actions; task value refers to one's beliefs about how important, interesting and useful a task is; whereas goal orientation involves students' perceptions of the reasons for engaging in a learning task (Pintrich, 1999). Such reasons can be intrinsic such as challenge, curiosity and self-improvement or can be extrinsic such as rewards, evaluation by others and competition (Pintrich, 1999). The motivational beliefs have been assumed to support and be supported by the use of the SRL strategies (McWhaw & Abrami, 2001).

Mathematical Problem Solving

Mathematical problem solving is considered one of the most difficult tasks primary students have to deal with (Verschaffel et al., 1999) since it requires the application of multiple skills (De Corte et al., 2000). Similarly to the theory of SRL, there are various approaches and theories of how to attack a problem most of which focused on dividing the procedure of MPS in separate, hierarchical steps. Some examples are the well-known four-step model of Polya (1957) and the three-stage problem solving strategy suggested by Schoenfeld (1985).

Research studies in MPS tend to apply the various models in real mathematics classrooms in order to investigate students' performance, having the belief that such models will enhance the students' ability (e.g. Verschaffel et al., 1999). For example, Schoenfeld (1985) showed that teaching the strategies of his model to college students resulted in higher performance in mathematical word problems. However, a closer look at various relevant research results may call into question the assumption that teaching MPS according to such models can lead to higher performance in MPS

as concerns primary school students. For example, the research of Verschaffel et al. (1999) showed that after teaching 5th graders how to use the strategies of their five-step model in realistic and challenging word problems the overall performance was not as high as expected. Given that those models include aspects of SRL, although not closely related to the theory itself, there may be circumstances in which the use of strategies may interfere with performance, especially when students are in primary school. McKeachie (2000; cited in Boekaerts, Pintrich and Zeidner, 2000) expresses the worry that being self-regulated can take capacity needed for basic information processing and thus lead to low performance. It seems plausible that his concerns could stand for primary school students while trying to solve the difficult for them mathematical word problems (De Corte et al., 2000). Given that very little is known about young children's SRL (Winne & Perry, 2000), primary students may possibly have difficulties in handling both MPS and SRL strategies at the same time.

THE PRESENT STUDY

The principles of the theory of SRL adjusted to MPS

The aim of this quest is to check the impact of a teaching intervention designed according to the principles of the theory of SRL on students' motivational beliefs and performance. To do that we first conducted a theoretical investigation to gather the principles of the theory of SRL that can be adjusted to a learning environment in MPS. The main principle was that students should be taught how to use certain cognitive, metacognitive and resource management strategies (Pintrich, 1999) while working on MPS. For this purpose we used the self-regulated mathematical problem solving model suggested by Marcou and Lerman (2006). This new-born model was especially developed for primary students as a tool to attack routine and process mathematical problems in a self-regulated way. It includes three stages of problem solving, similar to Polya's (1957) stages; 'reading and analysing the text', 'carrying out the plans' and 'looking back'. Each stage combines features of both SRL and MPS models since it includes all the cognitive, metacognitive, resource management (SRL aspect) and mathematics strategies (MPS aspect) that can be potentially used. For example, the cognitive-elaboration strategy, "I distinguish relevant from irrelevant data" can be used in the 'reading and analysing the text' stage, the mathematics strategy "I use the guess and check method" is located in the 'carrying out the plans stage', and the metacognitive-regulation strategy "I review my notes and the answer I found" is in the 'looking back' stage. There are strategies like the metacognitive-monitoring strategy "I try to think aloud" and the resource-management strategy "I ask for help" that are placed in all three stages. The model is represented graphically in a two-dimensional way by three rectangles which are connected with two way arrows indicating that students can oscillate between stages in order to regulate their behaviour. The model can not further be elaborated here due to space limitations.

A second principle, also adopted by Verschaffel et al. (1999), was that this external teacher regulation should be gradually phased out as students take over more and more agency of their solving process, in their process to become more self-regulated problem solvers. This could be achieved if gradually moving from the teacher regulation phase, in which the strategies of the model were taught through certain activities by whole class-discussions for which guidance was offered, to a phase in which the students start systematically using the strategies and finally to a third phase in which students are expected to use the strategies automatically and the model in a spontaneous way. Finally, a third principle was that motivational beliefs like self efficacy, task value and goal orientation should be enhanced and sustained parallel to the teaching of the SRL strategies. Teachers should provide positive feedback to their students whenever possible, like “you are very good at MPS”, or “solving problems is important because...”

Methodology

640 year 4, 5 and 6 students (ages 9 to 11) participated in the study which was carried out in Cyprus. 325 of them from 15 different classes were assigned to an experimental group whereas the rest 315 students coming from 13 other classes were set in a control group. A letter was sent to all schools in Cyprus asking for volunteer teachers. The classes of which their teachers had expressed interest in participating were included in the experimental group whereas the control group classes were selected by requesting other teachers to participate. We are aware that this difference in selection may have some limitations concerning the findings of the study.

The volunteer teachers attended a two-hour training session, during which they were introduced to the theory of SRL and the model of Marcou and Lerman (2006) and were asked to implement a series of 30 forty-minute lessons within seven months designed according to the aforementioned principles. The first 15 lessons were to be taught within three months during which the teacher had to regulate the learning process and all the strategies of the model had to be taught according to self-developed activities. For example, to teach the metacognitive-monitoring strategy “I check if the outcome I found is reasonable” the teacher could give a variety of possible answers to a problem and the students could justify whether and why these answers seem reasonable. The next ten lessons were to be implemented within two months and involved sharing the regulation of learning between the teacher and the students by a combination of both frontal teaching and group work. Finally, during the last five lessons within a month, through not frontal teaching but through group and individual work, students were expected to develop self-regulated mathematical problem solving behaviour. The volunteer teachers were also informed about the theory of motivational beliefs, its strong relation to the use of the SRL strategies and the different statements and ways they could use to enhance their students’ motivational beliefs. A 30 paged booklet was provided to each teacher which included all the details and guidelines concerning the intervention. Several visits to

the schools were carried out during the year to observe and video-tape lessons and discuss these with the teachers. It should also be noted that many of the teachers were constantly kept in contact via email with the researcher either asking for advice on how to proceed or to discuss the outcomes of an already implemented lesson. It can be said, by some initial analysis of the video and the visits to the schools, that almost all the teachers were successful in implementing the teaching the way it was requested. The teachers of the control group were not informed at all about the theory of the study and continued teaching according to the guidelines given by the national curriculum that basically focus on teaching mainly the mathematics strategies preferably through group work and investigation.

We followed a quasi-experimental design of research, a procedure that can be summarized in four steps (Robson, 2002); (1) select an experimental and a control group by means other than randomization, (2) give pre-tests to both groups, (3) the experimental group receives the teaching intervention or “treatment” whereas the control group gets no special ‘treatment, and (4) both groups are given post-tests. Therefore, to achieve the aim of the study, four research questions were formulated: (1) Is there a significant difference within each of the experimental and control groups throughout the year in their motivational beliefs (self-efficacy, task value, intrinsic and extrinsic goal orientations)? (2) Is there a significant difference within each of the experimental and control groups in their performance scores in MPS before and after the teaching intervention? (3) Is there a significant difference between experimental and control groups in their motivational beliefs? (4) Is there a significant difference between experimental and control groups in their performance in MPS?

Two isomorphic pre and post performance tests were devised with the purpose to measure students’ achievement in school word mathematical problem solving. There was an effort to apply balance of coverage of the test items by including four types of routine and process problems; one-step routine problems (e.g. ‘Nikos loves collecting stamps. This year, he added in his collection 29 new stamps. How many stamps had he had last year, if his collection includes now 87 stamps?’), two-step routine problems (e.g. ‘Mr Vasilis’ salary is £1230 per month. His wife’s salary is the half of his salary. How much money does his wife take each year?’), process problems, (e.g. ‘Demetra likes having golden fish. This morning, she found that 16 of her fish had died. From the ones left she gave half to her sister. From the rest she gave half to her friend for her birthday. Her cat ate the half of the rest fish. At the end of the day, 8 fish were left. How many fish had she had yesterday?’), and process problems with more than one answer (e.g. ‘Anthonis bought some sweets and paid £1.55 by giving coins of 5p, 10p and 20p. How many coins of each type did he give?’). Students were given 3 points in the case which they could apply both the right mathematical strategy and reach the correct answer, 2 points if they could identify and apply the mathematical strategy but could not reach a correct answer mainly due to

computational mistakes and no points if they could neither use a strategy nor reach an answer.

The Motivated Strategies for Learning Questionnaire (MSLQ), a self-report instrument designed by Pintrich, Smith, Garcia and McKeachie (1991), was modified for primary students to measure their motivational beliefs before and after the intervention at experimental and control conditions. The Likert type questionnaire (from 1 = “I disagree a lot” to 4 = “I agree a lot”) consists of 20 items divided in four sub-scales. Five of these items assess students’ self-efficacy ($\alpha=0.53$) in problem solving, in terms of ability and confidence skills (e.g. “I believe that I am good in mathematical problem solving”). The other five items measure the task-value beliefs of MPS ($\alpha=0.55$) in terms of importance, interest, and utility value (e.g. “I think solving mathematical problems is useful for me”). Intrinsic goal orientation ($\alpha=0.46$) is measured in terms of challenge, mastery, and curiosity (e.g. “I prefer working on mathematical problems that arouse my interest and curiosity, even if they are difficult to be solved”), and extrinsic goal orientation ($\alpha=0.15$) is assessed in terms of grades, evaluation by others, and competition (e.g. “I try to solve mathematical problems to show my peers that I am better than them”). We recognize that the alphas are rather low; however we accept these since we are not seeking for an accurate score in motivational beliefs but we are concentrating on possible differences in scores at different times of assigning the same questionnaire. Further information, which we will not discuss here, was collected about students’ general ability in Greek and mathematics, family background, such as educational and socio-economical status as well as parents’ country of origin.

We ran paired-samples t-test to explore any statistical differences within each of the two groups on levels of motivational beliefs and performance. For differences in motivational beliefs between the two groups a 2 (condition: experimental, control) \times 2 (time: pre-test, post-test) ANOVA, with repeated measures on the second factor was conducted. To explore differences in performance scores and taking into consideration that the two groups differed in initial performance measurements, we analysed dependent measures via condition \times time analysis of covariance (ANCOVA), with repeated measures on the second factor and task-value, intrinsic and extrinsic goal orientation as covariates. The three covariates were chosen since, as indicated in the following tables, were the only variables that appeared to have significant differences at the post-tests either between or within the two groups. Sizes of effect and power, as suggested by Kinnear and Gray (2004), are also reported to get a better idea of the statistical power of the obtained effects.

Findings

Table 1 indicates that the experimental group appeared to have significantly different goal orientation beliefs after the intervention. The score means and [standard deviations] for the intrinsic and extrinsic goal orientation beliefs altered from 3.21 [0.55] and 2.57 [0.50] respectively to 3.33 [0.50] for the intrinsic and 2.33 [0.56] for

the extrinsic goal orientation. These means differed significantly, $t(1, 256) = -3.29, p < 0.01, n^2 = .04, \text{power} = 0.91$ for intrinsic and $t(1, 258) = 6.46, p < 0.01, n^2 = 0.14, \text{power} = 1.00$ for extrinsic goal orientation. In other words, after the intervention students appeared more intrinsically and less extrinsically oriented. Furthermore, a significant change with a large effect size emerged for performance in MPS, $t(1, 291) = -15.8, p < 0.01, n^2 = 0.46, \text{power} = 1.00$. Means are shown in Table 1, where it appears that performance improved at the post-test from 1.55 [0.85] to 2.15 [0.90].

	pre-test		post-test		paired-samples <i>t</i> -test			
	M	SD	M	SD	<i>t</i> -value	<i>p</i> *	n^2	power
Experimental group								
Self-efficacy (N=255)	3.08	.51	3.07	.55	.36	.72	.00	.06
Task-value (N=246)	3.50	.47	3.56	.49	-1.88	.06	.01	.46
Intrinsic (N=257)	3.21	.55	3.33	.50	-3.29	.00**	.04	.91
Extrinsic (N=259)	2.57	.50	2.33	.56	6.46	.00**	.14	1.00
Performance (N=292)	1.55	.85	2.15	.90	-15.8	.00**	.46	1.00
Control group								
Self-efficacy (N=239)	3.07	.52	3.06	.63	0.33	.74	.00	.06
Task-value (N=231)	3.44	.54	3.45	.52	-0.38	.70	.00	.07
Intrinsic (N=248)	3.15	.56	3.21	.57	-1.62	.11	.01	.37
Extrinsic (N=251)	2.56	.51	2.39	.52	4.63	.00**	.08	1.00
Performance (N=288)	1.76	.79	2.01	.89	-6.31	.00**	.12	1.00

$p^{**} < 0.01$

Table 1: Motivational beliefs and performance at pre- and post-tests within groups

Similarly, and as expected, by the end of the school year the control group appeared to have a significantly higher performance in MPS $t(1, 287) = -6.31, p < .01, n^2 = 0.12, \text{power} = 1.00$. A decrease for control group was also observed in extrinsic goal orientation beliefs $t(1, 250) = 4.63, p < 0.01, n^2 = 0.08, \text{power} = 1.00$. Means and standard deviations can be seen in Table 1. Unexpected, especially for the experimental group, was that self-efficacy and task-value beliefs remained unchangeable during post-test for both groups. It should also be noted that the control group, contrary to the experimental, had no difference in intrinsic orientation scores between pre- and post-tests ($p = 0.15$).

As appears in Table 2 below, significant differences between groups with medium effect sizes emerged for condition (experimental, control) \times time (pre-test, post-test) in task-value, $F(1, 475) = 4.69, p < 0.05, n^2 = 0.01, \text{power} = 0.58$, intrinsic goal

orientation, $F(1, 503) = 4.84, p < 0.05, n^2 = 0.01, \text{power} = 0.59$ and performance, $F(1, 419) = 5.28, p < 0.05, n^2 = .01, \text{power} = 0.63$. In particular, as shown in Table 1, experimental group achieved higher post-test score means in all three abovementioned, task-value ($M=3.56, SD=0.49$), intrinsic goal orientation ($M=3.33, SD=0.50$) and performance scores compared to the respective scores of the control group. For the performance scores, ANCOVA analysis revealed that the means of the scores obtained by the experimental group, before and after the intervention were 1.62 [0.84] and 2.26 [0.85] respectively. For the control group though, the scores were 1.90 [0.78] at pre-test and only 2.18 [0.82] at post tests. We should note here that the interaction effect (condition \times time) reached statistical significance only in the case of performance scores $F(1, 419) = 26.77, p < 0.01, n^2 = 0.06, \text{power} = 0.99$. Unexpected was that self-efficacy beliefs appear to have no statistically significant difference between the two groups by the end of the intervention ($p = 0.79$, see Table 2).

Condition \times Time repeated measures ANOVA and ANCOVA (between groups)				
	<i>F</i> -value	<i>p</i> *	n^2	power
Self-efficacy	.74	.79	.00	.06
Task-value	4.69	.03*	.01	.58
Intrinsic goal orientation	4.84	.03*	.01	.59
Extrinsic goal orientation	.60	.44	.00	.12
Performance ¹	5.28	.02*	.01	.63

* $p < 0.05$

Table 2: Motivational beliefs and performance at pre- and post-tests between groups

DISCUSSION

The findings indicate that by the end of school year the experimental group of students appeared to consider MPS as more important, useful and interesting task compared to the control group. Similarly, they reported higher intrinsic goal orientations than the control group, meaning that they were stating to a greater extent that they engage in MPS tasks for reasons such as mastery and self-improvement and not for receiving rewards or pleasing others. Those differences were not observed before the intervention. These are enhanced by the findings of paired-samples *t*-test revealing that the experimental group is more intrinsically and less extrinsically goal oriented by the end of the year. Those findings imply that the teaching of SRL strategies in MPS possibly has an impact on students' task value and goal orientation beliefs and this certainly is not only in line with many correlational studies in the field (Pintrich, 1999; Marcou & Philippou, 2005) but it may also contribute to the establishing of an assumption that there may be a causal relationship between SRL

and motivational beliefs in the sense that the teaching of SRL strategies can promote enhancement of task value and intrinsic goal orientation beliefs. Research so far (e.g. Pintrich & Garcia, 1991; cited in McWhaw & Abrami, 2001; Pintrich, 1999) has investigated the relation under the scope of whether motivational beliefs can promote the use of SRL strategies and reported that intrinsically oriented students employ more SRL strategies than extrinsically oriented students (Pintrich & Garcia, 1991; cited in McWhaw & Abrami, 2001; Pintrich, 1999). Our study suggests that if students are taught to use SRL strategies² can be more intrinsically goal oriented and have higher task value beliefs.

However, the finding that self-efficacy beliefs appeared the same before and after the intervention was not expected for the experimental group since relevant research reports significant correlations between self-efficacy and the use of cognitive and metacognitive strategies (Pintrich, 1999; Marcou & Philippou, 2005). It was expected that, since students could by the end of the experiment employ a variety of SRL strategies to attack a problem and given that a significant increase in their performance was observed, they would start feeling more efficacious and capable in dealing with MPS tasks. It seems that the assertion of Verschaffel et al. (1999) that MPS is considered as one of the most difficult tasks primary students have to deal with is well established in children's belief system and is difficult to change.

The findings concerning students' performance imply that the teaching intervention has possibly contributed to the difference between the performance of the experimental and control groups and supports the assumption of McWhaw and Abrami (2001) that the use of SRL strategies may have an impact on performance in MPS. This implies that teaching children how to use SRL strategies does not interfere with performance as McKeachie (2000; cited in Boekaerts et al., 2000) assumes, but on the contrary, it can enhance their performance at satisfactory levels (from 1.62 to 2.26).

The results of this study suggest that teaching MPS according to the principles of SRL and incorporating the self-regulated mathematical problem solving model (Marcou & Lerman, 2006) can increase students' task value and intrinsic goal orientation beliefs and also have a positive impact on their performance. The findings should draw attention to primary school teachers to try to adopt the principles of the theory of SRL in their MPS teaching not only for purposes of increasing motivational beliefs and performance, but also for giving their students tools for lifelong, independent learning.

NOTES

1. ANCOVA analysis with task value, intrinsic and extrinsic goal orientation as covariates.
2. The use of the SRL strategies in the experimental group was measured by conducting clinical interviews. The analyses of the transcriptions show that there is a substantial increase in the frequencies of the strategies used by the end of the intervention (see also Marcou & Lerman, 2006).

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ABOUT MATHEMATICAL BELIEF SYSTEMS AWARENESS

Manuela Moscucci,

University of Siena- Italy

Although mathematics education researchers acknowledge a central role for belief systems in teaching and learning processes, the issue has not yet been sufficiently studied, because the analysis of their connections with emotions and attitude is not yet satisfactory. This paper deals with an experimentation -carried out in seven different learning places- founded on the hypothesis of the importance of a particular type of activity. Basically, it consists in making learners aware of their belief systems regarding mathematics: we may therefore talk of 'a meta-belief systems activity'. The aim of the paper is to submit to the community of mathematics education researchers both some preliminary results regarding this issue and the hypothesis of studying the issue from a theoretical viewpoint.

INTRODUCTION

Most of affect research either implicitly or explicitly highlights the interdependence of beliefs, attitudes and emotions (Hannula & Evans & Philippou & Zan, 2004), although there is much still to be investigated regarding links, interactions and implications among the three identified by McLeod (1992). As a matter of fact, beliefs, attitudes and emotions are strictly connected, because although the two systems of emotion and cognition are located in different parts of the brain, there are connections between them that allow interaction (Damasio, 1999; LeDoux, 1998) and they constitute a closely connected cognitive-emotional structure. The importance in learning of beliefs – which is the component of interest in this paper – has already been underlined by Anderson (1982), who, interpreting Piaget's theory, observes how children “reinvent” maths according to information from the environment in which they are educated: essentially the school environment. Their beliefs, which “filter” their personal approach to mathematical objects, are, therefore, also the result of received stimuli, among which teachers' beliefs regarding maths are of great importance. The aim of this paper is to link mathematics educational research on beliefs with school practice in maths teaching and maths teachers' teaching practice, according to Burkhardt and Schoenfeld's (2003) call to target affect research to “make progress on fundamental problems of practice”. At the same time, I would like to provide an example of how useful elements for theoretical research may come out of field research. I mean to give an example of productive synergy between theoretical research and practice.

THEORETICAL FRAMEWORK

Since Schoenfeld's *naive* definition of beliefs as a “mathematical world view” (Schoenfeld, 1985), the construct has been the subject of many studies. Regarding the nature of beliefs, many researchers have highlighted the affective components

(McLeod, 1989, 1992), others have emphasised the cognitive ones (Thompson, 1992), others the metacognitive ones (Kilpatrick, 1985; Schoenfeld, 1987). Nowadays there are publications available that report in detail the development of research and the latest results in the investigation of many interesting related theoretical aspects (i.e. Leder & Pehkonen & Törner, Eds. 2002). However, the definition of “belief” has yet to be clarified. Here, according to Pehkonen & Pietilä (2003), we understand “beliefs as an individual’s subjective knowledge and emotions concerning objects and their relationship, and they are based usually on his personal experience”. The coexistence of beliefs is therefore well known: Green (1971) stressed this fact as early as 1971 and Pehkonen & Törner (1996) studied the theoretical aspect in depth, introducing the concept of ‘belief systems’. Regarding the “modifiability” of beliefs, Green (1971) claims that they “are in continuous evaluation and change”, while Furinghetti & Pehkonen (in Leder & Pehkonen & Törner, 2002) underline that “beliefs are open to change”. This characteristic makes beliefs an element of great interest in the practice of teaching mathematics discussed in this paper, for at least two reasons. The first is because “belief systems form the structure of attitude about mathematics” (Pehkonen & Törner, 1996), and the second is because, according to Hart’s definition of attitudes (which is assumed here), attitudes towards maths are a consequence of emotions and beliefs regarding maths, and of behaviours (Hart, 1989). Pehkonen & Pietilä (2003), recalling McLeod’s view of beliefs as “cognitive” and “rather slowly formed”, characterize emotions, attitudes and beliefs symbolically as “hot”, “cool” and “cold” respectively. Thinking of affect as a dynamic and evolving structure, we might say that emotions are the most elastic part, while beliefs are the most rigid components, simply because they harden very slowly. However, it is precisely this feature that implies their “vulnerability”: if we find a “weak point”, it’s easier “to break off” a rigid element than an elastic one. So, in our opinion, they constitute the *weak* element of affect structure. The importance of metacognition in learning is well known, and Favell (1976), who coined the term “metacognition” in 1970, highlighted its importance in mathematics education, especially in problem solving. In maths learning, however, this element alone is not sufficient to overcome learning difficulties, as affect, with all its components, plays a determining role. About thirty years after the birth of the concept of metacognition, De Bellis and Goldin (1999) introduced the concept of meta-affect, which Goldin himself (2004) defined “the most important aspect of affect”. Recently, Schlöglmann (2005) contributed to the investigation of this concept with an interesting link to Ciompi’s concept of affect logic and neuroscientific results, proposing a successful use of meta-affect to understand learning strategies.

METHOD

Working on beliefs seems to be appropriate, as many common “myths” may represent not just a mere obstacle to the learning and teaching of maths, but an absolute hindrance, an “insurmountable wall”. Changing beliefs (when necessary, of course!) is a very hard target, but it may be a good starting point for changing students’ and mathematics teachers’ attitudes toward maths and maths teaching. On the other hand, working on the emotions that have contributed to determining a person’s relationship with mathematics seems to be even more challenging, from a practical point of view if nothing else. In fact, the manipulation of emotions requires the expertise of a psychologist. Although it would be ideal to work with a team of psychologists, this is not very realistic in schools, so more practical methods need to be found. All this is why working on beliefs seems an appropriate way of trying to rebuild a relationship with maths. To understand which are the best, or “weak”, points for the “attack” on beliefs to be changed, we ought to reflect upon all the belief systems regarding maths education. A single belief may have its source in repeated emotions (e.g. fear, frustration, anxiety), or repeated reasoning or reflections (e.g. “a particular aptitude is necessary to study maths,”), but a belief system is neither rooted entirely in emotions, nor entirely in cognition. Therefore action (towards belief systems mostly involved in maths education) with elements affecting both cognition and emotions has a higher probability of success. Knowing the positive role of awareness (Marton & Booth, 1997) in overcoming and monitoring learning difficulties, the existence of “negative” beliefs, which influence a learner’s relationship with mathematics, is treated here as a real learning difficulty. A hypothesis is hereby submitted for the attention of the community of mathematics education researchers: perhaps a new strategy, founded on acquiring awareness of personal belief systems, (which we may call meta-belief systems activity (mBSA, from now on)), can play an important role in building a productive, or positive, relationship with maths. Meta-belief systems activity is aimed at rebuilding a non-positive (or very negative) relationship with maths, or restructuring a relationship with maths that is insufficiently productive in some sense. The activity is structured in such a way that it may be used with both mathematics students and mathematics teachers. It may be used before every activity involving maths, in every kind of maths course and in every kind of course for maths teachers, whether addressed to pre-service teachers or to in-service teachers. It is an outline that needs to be adapted to the type of learners (students, teachers, or others), also bearing in mind their age. The main characteristics of mBSA are: 1) investigating the nature of learners’ relationships with maths; 2) creating an environment that allows learners to acquire awareness of their own beliefs “independently”; 3) helping the learners to build (or re-build) a productive relationship with maths through awareness of the origin, effects and dynamics of their beliefs and belief-related issues. The tools used to carry out this activity are individual, group and class interviews, written work, completing sentences, ‘discussions’ and simultaneous problem solving activities (Lester & Charles, 2003). From a theoretical viewpoint, realization of the activity requires the caution necessary in any activity that deals with awareness of

self. This issue is examined in-depth by Dehaene and Naccache (2001). Another problem needs to be considered when the activity is carried out with teachers as learners. The “mismatch between *espoused beliefs* and *beliefs-in-practice*, demonstrated by many studies on teachers' beliefs (Hoyles, 1992), confirms the results of research on problem solving (Schoenfeld, 1989): the beliefs that teachers declare are, in the end, definitely different from those that guide their solving processes and their behaviour in general” (Malara & Zan, 2002). In the activity presented here, a constructivist methodology is applied so that everybody should be able to understand by him/herself his/her own beliefs, the relationships between them, implications and, in conclusion, the nature of his/her belief systems. The operator's task (the operator is who carries out the activity) is to create a learning environment in which everybody is able to undergo a personal learning process regarding awareness of his/her belief systems. Thus, if the activity is carried out with maths teachers, they are not required to explicitly declare their own beliefs about the issue in question regarding maths, but they “discover”, step by step, their innermost beliefs, which are often subconscious, hidden to themselves. Moreover, when they begin to discern these beliefs, the simultaneous acquisition of awareness of the origins of their beliefs helps to overcome the “physiologic affect” of the same beliefs: everybody thinks his own beliefs are true! (Pehkonen & Pietilä, 2003).

META-BELIEF SYSTEMS ACTIVITY: STEP BY STEP

The following are notes and comments on the five basic steps through which mBSA is carried out.

Step1. (The operator keeps a “logbook” in which he writes privately, after each activity's meeting, his/her observations and his/her predictions regarding the quality of every learner's belief systems, according to the elements that emerged from the various activities, thus building an “a priori” analysis of every learner's processes). In this step individual interviews or class/group conversations are used to discover learners' interests and their expectations, but without referring to mathematical issues. If a learner mentions a mathematical issue, it would be better for the operator to avoid the subject and not let the learner know his/her opinion on the mathematical issue. Soon afterwards the operator has to clarify that the following activities will seem to have nothing in common with maths, but that, on the contrary, they are essential to the quality of work in maths: the learners are asked to “suspend judgement”. If the previous phase has been carried out successfully, learners' reactions will usually not be too suspicious, and such an unusual approach to maths will probably appeal to them.

Step 2. This is the phase of ‘My story with maths’. The recreation of one's own story with maths, with the explicit request to focus one's attention above all on emotions experienced during maths activities, is the first important step towards awareness. The written description has an important role in the revival of emotions linked to maths. If they are negative emotions, this is the first step towards awareness of

affective elements regarding the subject. Simultaneously, it leads learners to the “split” between negative emotion felt on a certain occasion and mathematical object dealt with on that occasion. This “separation” begins when the learners start to become aware that a certain activity/object is associated with a certain emotion (the “split” never or almost never happens completely, but what matters is that it happens in such a way that it does not provoke rejection of the activity/object in future). Such a split is achieved when learners become aware that the emotion is not an integral part of the mathematical object. The operator has to encourage them to ask: when/how/why do I feel like this?... All this is initially very difficult, but subsequently comforting and motivating. Every participant must be given all the time he needs, without limits. For this reason, in case the activity is addressed to more than one person (which it most often is), all those who have finished the work will be given recreational activities to carry out individually, in the form of simple “logic games”. The routine procedure (which is not an organizational tactic at all) is to propose simple but appealing tasks that are general “logic” problems, rather than problems involving typically mathematical subjects. These will always be used in the break between one activity and another, or to allow everyone to finish the given task. Step 1 of the activity is completed with interviews and conversations on the same subject, i.e. about learner’s experiences linked with maths. The aim of the activity in this step is, above all, to get a picture of the quality of the learners’ belief systems - beliefs that are somehow linked with maths. This picture is very important, because what learners write in this phase is not supposed to be influenced by the context at all, while in the following activity, elements that “disturb” the expression of their *naive* beliefs might be added: *naive* beliefs are valuable elements as a written proof to be critically examined by learners (at the end of the educational path) for self-assessment of their mathematical educational development. Awareness of this development will in itself constitute a very important part of mBSA.

Step 3. Now the operator has to create a learning environment in which some questions about belief categories (Schoenfeld, 1992, Leder & Pehkonen & Törner, 2002) arise spontaneously (through a talk involving everybody). We refer to learning and teaching, mathematical learning, mathematical education, the nature of mathematics and intellectual faculties. As these are all interactive and interdependent, the order of the above list is casual. Examples of questions are: A. 1) What is the role of the school in the person’s education? 2) What’s the difference between instruction and education? 3) What is the school’s focus? B. 1) What do we mean by a “student/person’s potentialities”? 2) What is the link between a “student/person’s potentialities” and intelligence? 3) What do you mean by human intelligence? And what do people usually mean? What is intelligence? C. 1) Is there a link between intelligence and maths learning? 2) Does *inborn aptitude* towards maths learning exist? What is it? 3) What is the role of maths in a person’s education? D. 1) What is maths for you? 2) Are maths and school maths the same thing? 3) What is school maths? 4) What is maths language? Or would it be better to speak of maths

languages? Each time a question is explicitly asked, learners are required to complete sentences chosen from a list of possibilities linked to the question. After the learners have completed the sentences (they can choose to complete as many as they like, or none at all), the conversation resumes from where it was interrupted. The elements of discussion are many and in this phase the operator must let learners lead the conversation and must refrain from giving any information to clarify the situation, while underlining the different opinions and comparing different elements and myths with or without scientific origin. Typical examples are: “one is born intelligent”, “one becomes intelligent”, “one is in part born intelligent and in part becomes intelligent”, “to cope with scholastic maths, one needs to be talented”, “there is an inborn aptitude for studying maths and it’s genetically transmitted (or at least transmitted through family)”, “those who are good at maths are very intelligent”, “maths is identified with arithmetical/algebraic calculus” (students often speak of mathematics and geometry, not of arithmetic and geometry!), “maths teaches you reasoning” and many others. During the mBSA experimentations it was observed that ‘the moment of crisis’ arrives sooner or later: if learners are teachers, both pre-service or in-service, it usually arises in the discussion about the role of school; if they are high school or university students, it occurs more often during discussion about human intelligence. Learners realise that their knowledge alone is not sufficient to sustain their beliefs - i.e. that they do not have proof to support their beliefs. In this case the task of the operator is to instil confidence: the learning community (learners and operator) will work to acquire the proper knowledge and tools to obtain and give themselves satisfactory answers. After this reassurance, the operator invites learners to pass onto the following phase, which will take place in the following meeting, and asks them explicitly not to research the subjects discussed, but to simply think them over. The choice of timing is very important: to leave learners with unanswered questions is useful for introspection and to gain awareness of their own relationship with maths, both affectively and as far as their beliefs are concerned.

Step 4. The operator points out to learners that up to this point the issues have been dealt with only using the learners’ “natural” knowledge about them. Now he/she asks them to closely examine the issues with him/her, so that they can acquire knowledge from the sphere of study and scientific research. The operator has to emphasise that the community is not looking for absolutely certain answers, but only answers that are coherent with today’s knowledge. For group A) questions, for instance, in Italy, a very important contribution is provided by the history of the Italian School from its foundation (second half of 19th century) to the present day (the operator may use appealing media, such as newspaper articles). As far as group B) is concerned, the work is more complex: we have to deal with educational sciences and neurosciences, spheres in which the operator has to manage difficult cultural mediation. However, there are easily accessible popular texts which can be useful to acquire basic knowledge about neurosciences (e.g. Bear & Connors & Paradiso, 2006), while internet resources, used under the operator’s guidance, can be very interesting and

useful. The operator leads the learners to discover the basic results of neurosciences that contribute to explaining today's knowledge about brain structure and functioning. This may help to banish prejudices regarding maths learning and predisposition to maths learning. This is the key phase of the activity: it is in this phase (together with the effect simultaneously produced by the problem solving activity, which is proposed as secondary, but during which the operator is in fact working intensively) that learners begin to understand the differences between their beliefs, education sciences, mathematical education, neurosciences, the nature of maths etc. and what is well known by research experts in the relative fields, such as the studies carried out over the last twenty years on intelligence and knowledge functions and, above all, results arising from neuroscience research. During the last ten years, innovative technological tools - above all, fMRI, TAC, PET- have allowed neuroscience researchers to get to know brain functioning and structure in a way that was previously impossible. For instance, Maguire's (2000) results show that having to solve a certain kind of problem as a job causes a not only functional but also structural change in neuronal areas (related to the type of problem). This result provides the opportunity to link current discussion with the issues of group D) questions. It deals with the analysis of the role of 'problem solving' in constructing mathematical thinking and improving one's mental flexibility. If the work is carried out with students or pre-service teachers, this activity and the simultaneous problem solving activity are sufficient to provide elements to "break" their natural beliefs: step by step, learners 'integrate' their knowledge, 'become aware' of their "negative" emotions (when present), 'distinguish' them from mathematical objects (or maths as a whole), and 'increase' their self-esteem in doing maths (through the problem solving activity). The synergies springing from all these elements bring about the natural replacement of beliefs that have no scientific basis. Learners thus begin to rebuild their relationship with maths. Learners' awareness that they have followed this "path" by themselves, and that the operator only offered helpful hints to facilitate the activity, is in itself a successful element. When the activity is aimed at in-service teachers (with perhaps twenty or thirty years of service), there may sometimes be another emotional/cognitive obstacle: the fear of losing their own professional self-esteem by admitting that their own beliefs are not scientifically rooted. Here, the discussion about professional ethics and professional deontology plays an important role in encouraging them to try to follow what is shared by the scientific community, even if it is at odds with their personal beliefs.

Step 5. The operator asks the learners to describe their experience throughout the activity: 'My story with mBSA' is written in the same way as 'My story with maths'. This step has exactly the same motivation as step 2. In this phase "explicit" self-assessment of the quality of the path followed is carried out and learners gain awareness of the progress they have made in their relationship with maths. The operator also describes the path taken by each learner from his/her point of view, based on the descriptions that he/she created at the beginning of the activity. The

exercise concludes with the operator and learner reading both their texts together, comparing them and discussing similarities and differences.

RESULTS AND CONCLUSIONS

The pattern of mBSA described has acquired this form after a long series of experimentations in several different contexts over about ten years. It has been carried out in the form presented here for the last seven years (ongoing), in seven different contexts with different learners: students of a vocational school, University students of Biotechnology Courses, University students of Mathematical Education Courses, in-service maths teachers (with or without a degree in mathematics), pre-service maths teachers (with or without a degree in mathematics) and two high school students in different situations. The gathering of results, which constitutes an integral part of the activity, is done by comparing the beliefs expressed during steps 1 and 2, and step 5, although it must be pointed out that with the teachers step 5 was only carried out orally. By way of an example, we report some illustrative data from the activity carried out, during three refresher courses, with maths teachers (59 in all) regarding some beliefs in particular. First, however, we highlight that neither the percentages themselves nor the quality of the beliefs taken as examples are important, but the fact that they were the source of a “non positive” relationship with mathematics and that, after mBSA, those beliefs were overcome. Before the mBSA, the learners (teachers with a degree in biological, natural, geological and similar sciences) maintained that: 1) competence in school maths means only learning the rules and knowing how to apply them (76%); 2) you need to have a particular aptitude to learn school maths (71%). After the mBSA: 1) competence in maths means only learning the rules and knowing how to apply them (none); 2) you need to have a particular aptitude to learn school maths (none). Moreover most of them (98%) thought that: 1) problem finding, posing, solving, and talking activities must be a fundamental element of all maths activities; 2) “being good at maths at school” is a goal that everyone can reach. Regarding the activity carried out with vocational school students, the head and deputy head surveyed (without the knowledge of the teachers who carried out the activity) preferences of subjects taught and maths came out in first place, which is surprising as it is normally the most feared and disliked (if not hated) subject! In conclusion, we can affirm that almost all learners who took part in the mBSA have begun to approach maths with ‘no negative’ feeling and teachers have started to focus maths teaching to promote students’ potentialities (of all their students) through maths, basing their work on meaningful maths and problem solving. The activity was unsuccessful in a few cases due to external events, such as with students who left the vocational school or teachers who deemed it too onerous to change teaching method and aims after decades of traditional teaching. We can therefore confirm that, in our opinion, the results of the activity are comforting and merit further experimentation and investigation. We attribute the positive results of the activity to three elements. The first is that ‘mBSA is not only a “juxtaposition” of elements’ (some of which are widely used, such as “problem solving” or writing ‘My

story with maths’), but ‘an organized structure that exploits the synergies springing from the single components in the best way’. The second is the ‘awareness’ that the learners have “cultivated” and that this ‘development or evolution is something they have achieved by themselves’ (the learner rather than the operator is the protagonist!). The third, regarding only teachers, is that they can see a real “working pattern” applied to themselves, and that they can use the pattern with their students.

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EFFICACY BELIEFS, PROBLEM POSING, AND MATHEMATICS ACHIEVEMENT

Aristoklis A. Nicolaou & George N. Philippou

Department of Education University of Cyprus, Nicosia

Perceived self-efficacy beliefs and problem posing are considered as two fundamental concepts in mathematical learning. In this study we examined the relation among efficacy in problem posing, problem-posing ability, and mathematics achievement. Quantitative data were collected from 176 fifth and sixth grade students, and interview data from six students selected on the basis of hierarchical cluster analysis. Perceived efficacy to construct problems was found to be a strong predictor of the ability in problem posing and of the general mathematics achievement. A strong correlation was also found between ability in problem posing and general mathematics performance. Implications are drawn about strategies for enhancing students' efficacy beliefs and problem-posing ability.

INTRODUCTION

Research on mathematics teaching has recently focused on affective variables, which were found to play an essential role that influences behaviour and learning (Bandura, 1997). The affective domain is a complex structural system consisting of four main components: emotions, attitudes, beliefs and values (Goldin, 2002). Beliefs can be defined as one's knowledge, theories and conceptions and include whatever one considers as true knowledge, although he or she cannot provide convincing evidence to support it (Pehkonen, 2001). Self-beliefs can be described as one's beliefs regarding personal characteristics and abilities and include dimensions such as self-concept, self-efficacy and self-esteem. Self-efficacy can be defined as "one's belief that he/she is able to organize and apply plans in order to achieve a certain task" (Bandura, 1997, p. 3). In this study we focus on primary students' perceived efficacy to pose problems.

THEORETICAL BACKGROUND AND AIMS

According to Bandura every individual possess a belief system that exerts control over his/her thoughts, emotions and actions. Among the various mechanisms of human agency, none is more central or pervasive than self-efficacy beliefs (Bandura & Locke, 2003; Pajares, 2000). Self-efficacy is a task-specific construct and there is a correspondence between self-efficacy beliefs and the criterial task being assessed; in contrast, self-concept is the sense of ability with respect to more global goals (Pajares, 2000; Bandura, 1986), while self-esteem is a measure of feeling proud about a certain trait, in comparison with others (Klassen, 2004). The task-specificity of efficacy beliefs implies that related studies are more illuminating when they refer to certain tasks, such as problem posing; the predictive power of self-efficacy is in this case maximized (Pajares & Schunk, 2002). On the other hand, the level of specificity

could not be unlimited; as Lent and Hackett (1987) have rightly observed, specificity and precision are often purchased at the expense of practical relevance and validity.

Research on self-efficacy has recently been accumulated providing among other things notable theoretical advances that reinforce the role attributed to this construct in Bandura's social cognitive theory. Several studies have indicated a strong correlation between mathematics self-efficacy and mathematics achievement (Klassen, 2004). It was further found that mathematics self-efficacy is a good predictor of mathematics performance irrespective of the indicators of performance (Bandura, 1986) and regardless of any other variables (Bandura & Locke, 2003). It was found that mathematics self-efficacy is a better predictor of mathematics performance than mathematics anxiety, conceptions for the usefulness of mathematics, prior involvement in mathematics, mathematics self-concept and previous mathematics performance (Klassen, 2004; Pajares & Miller, 1994). It is noteworthy that self-efficacy beliefs were even found to be a stronger predictor of performance than general mental ability (Pajares & Kranzler, 1995).

Self-efficacy beliefs have already been studied in relation to a lot of aspects of mathematics learning, such as arithmetical operations, problem solving and problem posing. Pajares and Miller (1994) asserted that efficacy in problem solving had a causal effect on students' performance. Research findings support the view that high achieving in mathematics students have higher and more accurate efficacy beliefs (Pajares & Kranzler, 1995). Efficacy beliefs towards a certain task are accurate when they correspond to what the person can actually accomplish.

Reformed mathematics education adopted the view that knowing mathematics is identified as "doing" mathematics and learning mathematics is equivalent to constructing meaning for oneself and the ability to handle non-routine problems. The development of problem posing competency is generally recognised as an important goal of mathematics teaching and learning; it lies at the heart of mathematical activity (Crespo, 2003; English, 1997). In this context, problem posing comprises a primary factor that contributes to enhancing students' ability to solve mathematical problems. Moreover, from a teaching perspective, problem posing reveals much about the understandings, skills and attitudes the problem poser brings to a given situation and thus becomes a powerful assessment tool.

Problem posing can be defined as the generation of new problems and mathematical questions, as well as the reformulation of problems within the process of solving a given problem, when a solver restates or recreates a given problem in some way or other to make it more accessible for solution. Many researchers have reported a positive relation among problem posing ability and mathematics achievement (English, 1998; Leung & Silver, 1997) as well as between problem posing and problem solving ability (English, 1998; Silver & Cai, 1996).

Despite its importance, problem posing has not yet received analogous attention from the mathematics education community. Indeed, we know relatively little about

children's ability to construct their own problems or about the extent to which these abilities are linked to mathematical competence (English, 1998). Furthermore, we are aware of no studies investigating the efficacy beliefs of primary school students towards problem posing and the relation between this construct and the ability to generate problems. A possible relationship between efficacy and ability in problem posing would enrich our knowledge about the connection among affective and cognitive factors, with obvious implications in teacher education and teaching.

The purpose of this study was to explore relationships between elementary school students' efficacy beliefs in problem posing, their problem posing ability, and their achievement in mathematics. Specifically, the aims of the present study were: a) To measure efficacy beliefs in problem posing and ability in problem posing of fifth and sixth grade students, b) to look for possible relationships between any pair of the following variables: efficacy in problem posing, ability in problem posing, and mathematics achievement, and c) examine whether efficacy in problem posing could predict ability in problem posing and mathematics achievement.

METHODOLOGY

A sample of eight intact classes was selected on the basis of purposeful cluster sampling; four urban and rural schools from the three major districts of Cyprus were first selected and then a sixth and a fifth class were randomly selected from each of those schools. We collected questionnaire data from 176 students, 87 fifth-graders and 89 sixth-graders (about 10.5 and 11.5 years of age), and from six interviews.

A four-part questionnaire, measuring efficacy beliefs in problem posing and ability in problem posing was developed on the basis of earlier studies (English, 1997, 1998; Philippou, Charalambous, & Christou, 2001). The first three parts measured efficacy beliefs towards problem posing and the fourth one measured ability in problem posing. Specifically, in the first part, students were asked to read each of the four tasks and state their sense of certainty to pose problems based on each of them, without attempting to pose any problem (see Figure 1). The second part comprised of five cartoon-type pictures and statements explaining the situation presented by each picture; the students were asked to select the picture that best expressed their efficacy beliefs in problem posing (see Figure 2). The third part consisted of 14 five-point Likert type items, reflecting efficacy in problem posing, and the fourth part of four tasks similar to those in the first part and the students were asked to pose problems based on those tasks. Specifically, in the fourth part they were asked to pose two problems on the basis of a stimulus picture that was the same as in the first part of the questionnaire, two problems that should end with a specific question, one problem that could be solved by the division operation $2 \div 3$ and one problem on the basis of a number pattern.

The questionnaires were piloted on 26 sixth grade students to detect possible weaknesses or shortcomings. After some minor language improvements, the questionnaires were administered to the sample subjects by the first author. The

students were instructed to proceed to the fourth part only after they had finished the first three parts; they were given 60 minutes to complete the questionnaire.

The Ward's method of hierarchical cluster analysis was then used on the quantitative data for the selection of subjects for the interviews. Clustering was based on the following variables: ability in problem posing, efficacy in problem posing and complexity of the problems posed. The analysis revealed that students could be clustered into six distinct groups (the Agglomeration scale showed a fairly large increase in the value of the distance measure from a six-cluster to a five-cluster solution). The means and standard deviations for the ability in problem posing, efficacy in problem posing and complexity of the problems posed, were then calculated for each of the six groups (see Table 1). From the results of Table 1, G_1 and G_2 can be considered as the relatively "high score" groups, G_3 and G_4 as the "moderate score" and G_5 and G_6 as the relatively "low score" groups. From each cluster group a student was randomly selected for interviews, in such a way that the six students came from six different classes and three of them were boys and three were girls.

The interviews were semi-structured encouraging the interviewees to answer the same questions and give explanations and clarifications where necessary. The interviews were conducted by the first author at the child's school and were tape-recorded. No time limit was set, and the interviewer was prepared to provide clarifications whenever a student seemed unable to understand something the issue. The interviews aimed at eliciting students' attitudes towards mathematics, efficacy beliefs in problem posing and ability in problem posing.

For the analysis of the interviews, students' responses were classified according to main issues and compared between them. Moreover, the responses in the questions that examined efficacy in problem posing and ability in problem posing respectively, were compared to the responses on the respective questionnaire items and the profile of the cluster group in which the student belonged.

The problems constructed by the participants in the fourth part of the questionnaire were scored as follows: in the first two tasks, one point was given for each mathematical problem constructed, in the third task two points were given for the construction of a problem and in the fourth task, one point was given for correct completion of the number pattern and one point for constructing a problem. The average score in the four tasks determined the ability in problem posing.

To determine the overall measure of efficacy in problem posing, we recoded those items of the third part of the questionnaire that were negatively stated, and then the average score of the statements of the first three parts of the questionnaire was calculated. In other words, the mean value of efficacy beliefs in problem posing was drawn on the basis of three complementary sources. Mathematics achievement was drawn on the teachers' grades for the school year 2002-2003. Though these grades

were given on a scale from 0 to 20, in the analysis we used the ordinal place of students by class, to cater for possible variance in the judgment of individual teacher.

As regards the quality of a problem, we adopted the criteria proposed by English (1997, 1998), i.e., the semantic-structure and the operational complexity of the problem. In order to determine semantic-structure complexity, the problems were classified as basic or complex (English, 1997). Basic problems were given one point, whereas complex problems were given two points. The number of operations in a problem, which was the second dimension of complexity was taken into account cumulatively; for instance in the case of a two-step problem involving one change and one comparison situation, $1+2=3$ points were given, while a three-step problem involving repeated comparison, was assigned $2+2+2$ or $3 \times 2=6$ points.

RESULTS

In general, the students' efficacy in problem posing was found to be quite high, as it can be deduced from both the questionnaire results (Mean=3.58 out of a maximum of 5) and from the analysis of the interviews. On the contrary, the students' actual ability in problem posing was found to be at a moderate level; the overall mean response was found to be 1.15 out of a maximum of 2. That was in line with children's responses in the interviews, where four children were able to construct problems in both tasks without any help, while the other two faced serious difficulties and the interviewer had to provide considerable hints in both tasks.

A significant correlation was found between efficacy about and ability in problem posing ($r=0.480$, $p=0.001$). Similarly, in the interviews, children with high efficacy in problem posing were able to construct problems without any support, whereas low efficacy children either were unable to construct problems or constructed problems after support was provided. For instance, S_2 a high efficacy student who expressed confidence in his ability to construct mathematical problems that could be solved by the operation $2 \div 3$, said:

"I think it is very easy. I feel quite sure. I believe that $2 \div 3$ is a simple operation and I would have no difficulty to construct problems that could be solved by this operation".

In line with his confidence, he was later on able to construct the following two problems that could be solved by the multiplication 7×6 , as required:

P1: *"A group of children consists of seven children and each child has six marbles. How many marbles do the children altogether have?"*

P2: *"Seven pizzas were cut into six pieces each. How many pieces of pizza were there?"*

On the contrary, S_5 who felt uncertain about her ability to construct problems that could be solved by the operation $2 \div 3$, was later on unable to pose any problem that could be solved by the multiplication 7×6 . Indeed, she didn't manage to pose any problem on her own, despite considerable help provided by the interviewer.

In order to examine whether efficacy in problem posing could predict ability in problem posing, a linear regression analysis was performed. The analysis showed that efficacy in problem posing was a good predictor of students' ability in problem posing ($R=0.480$, $F=52.097$, $p=0.001$). This was also confirmed from qualitative analysis; the level of children's perceived efficacy in problem posing expressed before they tried to pose problems matched well their actual success or failure to construct problems in each of the two tasks of the interviews. These findings are in line with previous results, where a strong correlation was found between efficacy with respect to a certain task and actual achievement in that task, and also that efficacy predicted actual achievement in that task (Bandura, 1997). The present results are also in line with earlier findings by Philippou et al. (2001) who found positive correlation among pre-service teachers' efficacy beliefs in problem posing and their ability in problem posing.

Efficacy in problem posing was positively correlated with general mathematics achievement ($\rho=0.431$, $p=0.001$). Additionally, it was found that efficacy beliefs in problem posing could predict mathematics achievement fairly well ($R=0.427$, $F=38.339$, $p=0.001$). These results are in agreement and confirm the findings of previous studies indicating that mathematics efficacy beliefs were correlated and could predict mathematics achievement (Zimmerman, Bandura, & Martinez-Pons, 1992). However, the correlation between efficacy in problem posing and mathematics achievement was slightly lower than the correlation between efficacy in problem posing and ability in problem posing. Moreover, efficacy beliefs in problem posing could predict mathematics achievement in a lower degree than they could predict problem posing ability. On the other side, a strong positive relationship ($\rho=0.566$, $p=0.000$) was found between the ability in problem posing and mathematics achievement. This finding confirms the findings of previous studies (English, 1998; Leung & Silver, 1997).

DISCUSSION

Given that ability in problem posing is influenced by related previous experiences, the problem posing performance of the participants in this study could be considered as satisfactory. According to the interviews, the subjects' prior experience in problem posing was limited; they were given this opportunity to work on problem posing tasks only a few times each month. Comparing the level of students' perceived efficacy in problem posing and actual performance in problem posing, we conclude that students have in general overestimated their competence. This finding is in agreement with the results of previous studies that revealed a tendency of the majority of students to be overoptimistic about their abilities to undertake a certain mathematical task (Pajares & Miller, 1994). This gap among the level of efficacy beliefs and the actual ability in problem posing can be attributed to social and cultural factors that encourage this tendency among Cypriot students. A regular tendency of Cypriot parents is to let their offspring to believe that they are capable to accomplish much more than they actually can.

The question concerns the magnitude of this residual. Bandura (1986) argued that one's successful functioning with respect to a certain task is best served by reasonably accurate efficacy appraisals, although the most functional efficacy judgements are those that slightly exceed what one can actually accomplish, for this overestimation may serve a motive to increase effort and persistence.

Efficacy in problem posing was found to be positively correlated with and a good predictor of the ability in problem posing. This is in agreement and reinforces the results of other studies that lead to the same conclusions (Bandura, 1997). The findings of the present study are also consistent with the results reported by Philippou et al. (2001) in Cyprus; they have found that prospective teachers' efficacy beliefs in problem posing were significantly related to their problem posing ability.

The construct efficacy in problem posing was also positively linked and a good predictor of mathematics achievement; it could predict mathematics performance fairly well. These findings again confirm similar results reported in other studies (e.g., Klassen, 2004; Bandura & Locke, 2003). Both the correlation between efficacy in problem posing and ability in problem posing, as well as the predictive power of efficacy in problem posing and ability in problem posing were slightly higher than the correlation between the respective link among efficacy in problem posing and mathematics achievement. Although the difference between the correlation coefficients is small (the difference $r-\rho=0,049$), these results seem to justify Bandura's demand for specificity, claiming that efficacy beliefs with respect to a certain task are strongly correlated and are best predictors of achievement at the same task (Bandura, 1986). The aforementioned results are also consistent with the demand for specificity in order to increase the predictive power of the construct (Pajares & Miller, 1994). The significant positive relationship that was found between the ability in problem posing and mathematics achievement could have been expected. It is consistent with the findings reported in previous studies (Leung & Silver, 1997; English, 1998). Specifically, English (1998) found that students with high achievement in mathematics were better able to generate problems.

The results of the present study reinforce the importance of efficacy beliefs in mathematics. Bandura (1997) argued that the uncertainty for one's capability can overthrow the results of his/her efforts, even if he/she is a highly competent person with respect to a certain task. Teachers must thus work towards the development of efficacy beliefs and pay attention to students' self perceptions with respect to certain tasks, since these beliefs can be an indicator of future achievement in the specific tasks (Bandura & Locke, 2003). Therefore it is argued that one's behaviour is more influenced by his/her beliefs than by his/her knowledge or ability, teachers should pay as much attention to students' perceptions about their competence in mathematics as to actual competence.

Given that efficacy beliefs towards problem posing can predict ability in problem posing, teachers must develop ways to enhance efficacy beliefs, particularly in the case they are relatively low and do not match the child's ability in problem posing.

One way is to provide children with activities in which they can succeed. Certainly, this does not imply that all problem-posing activities should be easy, but that it would be better if teaching starts with easier activities, gradually inserting more difficult activities.

In addition, efficacy in problem posing is very important due to the emphasis that is lately attributed to problem posing. Problem posing importance is shown by its positive link to mathematics achievement. Philippou et al. (2001) mentioned that pre-service teachers valued problem posing as the ultimate goal of mathematics learning. Learning how to construct mathematical tasks is considered as one of the challenges of learning and teaching mathematics (Crespo, 2003). Students' moderate ability in problem posing and the current limited opportunities given to children to get involved in problem posing activities, make the need for further inclusion of more and varied such activities in teachers' repertoire imperative.

In conclusion, the findings of the present study suggest that developing efficacy beliefs in problem posing should be an integral part of mathematics teaching and learning. It has been verified once again, this time with primary students, that efficacy beliefs constitute an important component of motivation and behaviour; the correlations found among the efficacy in problem posing, ability in problem posing and mathematics achievement suggest a possible focus for further research.

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Appendix

Figure 1: Efficacy in problem posing

The following tasks refer to your perceived ability to construct problems in various situations. Please indicate your degree of certainty on a scale from one to five, “1-not at all certain, 5-very much certain” (DO NOT TRY TO CONSTRUCT ANY PROBLEM).

1.	Construct two different mathematical problems in relation to the following picture.	1	2	3	4	5
						

2.	Construct two problems that should end with the following question: “What is the area of the field?”	1	2	3	4	5
3.	Construct a problem that could be solved by performing the operation $3 \div 4$.	1	2	3	4	5
4.	Construct a mathematical problem from the following number pattern: 2, 4, 8, 16, 32, 64, 128...	1	2	3	4	5

1= not at all certain, 2= rather certain, 3= quite certain, 4= much certain, 5= very much certain

Figure 2: Efficacy in problem posing






1.  “I am the best student in my class in problem posing” <input type="checkbox"/>	2.  “I am not the best student in problem posing, but I can construct problems without any difficulty” <input type="checkbox"/>	3.  “Problem posing? Em..., not so bad, not very well. I would say moderately well!” <input type="checkbox"/>
4. “I have some difficulties to construct mathematical problems, but occasionally I do manage to do that”  <input type="checkbox"/>	5. “I always face big difficulties when asked to construct mathematical problems. It seems that despite my efforts I can not construct mathematical problems”  <input type="checkbox"/>	

Table 1: Means and standard deviations for the cluster groups in efficacy beliefs in problem posing, ability in problem posing and complexity of the problems posed

Groups	Efficacy beliefs in problem posing ¹		Ability in problem posing ²		Complexity of the problems posed ³	
	\bar{X}	SD	\bar{X}	SD	\bar{X}	SD
Group 1 (N=20)	4.033(H)	0.510	1.515(H)	0.394	2.250(H)	1.274
Group 2 (N=31)	3.843(MH)	0.554	1.561(H)	0.399	2.006(MH)	0.724
Group 3 (N=27)	3.620(M)	0.435	1.127(M)	0.416	1.546(M)	0.897
Group 4 (N=35)	3.583(M)	0.496	1.246(M)	0.503	1.563(M)	0.792
Group 5 (N=36)	3.438(ML)	0.646	0.896(ML)	0.525	1.074(ML)	0.835
Group 6 (N=25)	3.061(L)	0.517	0.635(L)	0.535	0.818(L)	0.655

¹Maximum score = 5, Minimum = 1, ²Maximum score = 2, Minimum = 0, ³Maximum score = 5.23, Minimum = 0, L: Relatively low score, ML: Relatively moderate-low score, M: Moderate score, MH: Relatively moderate-high score, H: Relatively high score, G₁ (H, H, H), G₂ (MH, H, MH), G₃ (M, M, M), G₄ (M, M, M), G₅ (ML, ML, ML), G₆ (L, L, L).

STUDENTS' SELF REGULATION OF EMOTIONS IN MATHEMATICS LEARNING

Peter Op 't Eynde, Erik De Corte, Inge Mercken

CIP&T, University of Leuven

Over the years the concept of self-regulated learning has broadened to include motivational, volitional, and emotional components next to (meta)cognitive ones. In this paper we will present data from a survey study that specifically discuss the relevance and the functioning of students' meta-emotional knowledge and emotional regulation skills in the mathematics classroom. Results show that students know and make use of six different categories of emotional regulation strategies in stressful school situations related to mathematics learning, including active and problem focused strategies as well as more emotion focused strategies. There are, however, clear differences in the kind of strategies used by students depending on the situation confronted with, their familiarity with the stressful nature of this situation, the track level they are in, their age and gender.

THEORETICAL FRAMEWORK

Over the years the concept of self-regulated learning has broadened to include motivational, volitional, and emotional components next to (meta)cognitive ones (see (see e.g., Boekaerts, Pintrich, & Zeidner, 2000). The exact nature and role of these different components is, however, not always clearly understood. More specifically, rarely scholars have investigated students' knowledge and regulation of their emotions in the classroom, i.e. their meta-emotional knowledge and skills. Yet, this is thought to be an important determinant of the kind of emotions students experience, how they express them, and the way these emotions influence their learning behavior.

When discussing the self-regulation of cognitive processes there is a broad consensus today on what students need to have: metacognition. Metacognition is defined then as awareness and control of one's learning. It implies the integrated mastery of a sufficient body of metacognitive knowledge and skills resulting in a metacognitive awareness when engaging in learning and problem solving (see Brown, 1987; Schraw, 2001). Matthews, Schwan, Campbell, Saklofske, and Mohamed (2000), in their research, point to the distinction between metacognitive and mood awareness. They claim that there are systematic individual differences in processing information related to the person's own moods, independently of their metacognitive capabilities. They argue that people have a self-reflective meta-experience of mood (as distinct from their metacognitive experience) which includes the awareness of the mood state as well as the action to change mood through strategy use (see also Mayer, Salovey, Gomberg-Kaufman, & Blaney, 1991).

In a similar fashion one can talk about a *meta-emotional experience* which includes the awareness of the emotion as well as the action to control and regulate it. It refers to phenomenological feelings, thoughts and activities related to the experience of an emotion and its regulation. Gottman, Katz, & Hooven (1996) have introduced the notion meta-emotion in parallel with meta-cognition (see also, Goldin, 2002; Gumora & Arsenio, 2002) . A person's meta-emotional system consists of meta-emotional knowledge and skills. It is a well-organized and structured system of concepts, thoughts, metaphors, feelings and philosophies about one's own and others' emotions.

Meta-emotional knowledge refers to:

- Knowledge about the recognition and identification of emotions of oneself and of other people
- Knowledge about the antecedents and consequences of emotions
- Knowledge about the expression of emotions of oneself and of other people

Meta-emotional skills refer to:

- The knowledge of strategies that can be used to control and regulate emotions (e.g., coping strategies) and the competence to consciously and effectively use them.

Where students' emotions emerge from goal-oriented person-environment interactions determined by students' knowledge and belief systems related to a particular domain (see also Carver & Scheier, 2000; Pekrun, 2000), it becomes clear that to be able to become fully aware of these emotions and to consciously regulate them students need to develop an equally elaborated meta-emotional system of knowledge, beliefs and regulation skills related to *emotions* within that particular domain. Both systems in close interaction will determine the kind of emotions students' will consciously experience and be able to reflectively act upon (see Fig 1).

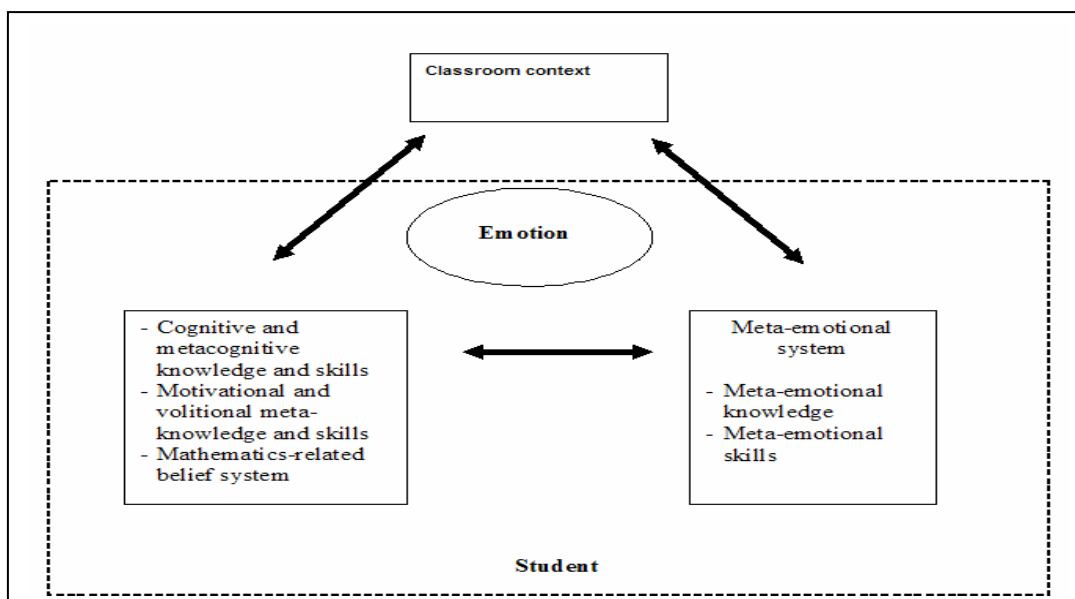


Figure 1: The role of the meta-emotional system in the emergence of emotions

STUDENTS' USE OF META-EMOTIONAL SKILLS IN MATHEMATICS LEARNING

Research design and methodology

We conducted a survey study focused on the coping behavior of 393 Flemish second (age 14) and fourth year (age 16) secondary school students in different stressful, mathematical school situations. We analyzed how students' coping behavior is related to the specific kind of stressful school situation they are confronted with and their familiarity with it. In addition, the relations between their coping behavior, on the one hand, and their gender, age, educational track (general, technical, or vocational education), motivation for mathematics, and achievement in mathematics, on the other hand, were investigated.

To assess the kind of regulation strategies students employ when managing their emotions we developed a Flemish version of Carver, Weintraub, and Scheier's (1989) COPE-questionnaire. This multidimensional coping inventory incorporates 15 conceptually distinct scales and, thus, represents a wide variety of coping strategies. Students were asked to indicate on a 4-point Likert scale (i.e. never, rarely, sometimes, often) the emotional regulation strategies they draw on to manage their emotions in three different stressful situations related to mathematics learning. Students' familiarity with these stressful situations was determined by asking the adolescents how much they already experienced these respective situations. Additionally, four questions were asked to determine students' general motivation for mathematics. Students' achievement in mathematics was established by referring to their last exam scores.

An exploratory factor analysis was performed to identify the different categories of emotional regulation strategies used by students. Next, variance analysis clarified the relations between the strategies used and the three different mathematical school situations, students' familiarity with the stressful nature of these situations, students' track and achievement level, age and gender.

Results

By means of an exploratory factor analysis we first traced the categories of emotional regulation strategies underlying the 60 items of the translated version of the COPE questionnaire when used in a Flemish school context. Six reliable factors were retained explaining 80%, 77%, and 76% of the variance in coping strategies used respectively in a mathematics test situation, homework situation, and lesson situation. The six factors were: 'active coping', 'social-emotional coping', 'humor and acceptance', 'abandoning and negation', 'religion', 'alcohol and drug use'.

Although, in this study, we were not able to exactly replicate the structure that characterized the original COPE questionnaire (15 scales/factors), the factors found are very similar. Some are constituted by exactly the same items (e.g., alcohol and drug use), other are a combination of items that originally loaded on two different

factors but in our analysis appear to be part of one more general factor (e.g., active coping, humor and acceptance). In summary, notwithstanding that the internal structure of our questionnaire is slightly different from the original one, the translated version seems to be a reliable instrument to measure students' use of coping strategies in Flemish mathematical school contexts.

General use of coping strategies

Our data indicate that, in general, students do use coping strategies from time to time. But, the means mentioned in Table 1 are never above 2 (i.e., I sometimes use this strategy) which illustrates that students do not systematically use coping strategies in stressful mathematical school situations.

Factor	Test situation	Home work situation [HW]	Lesson situation	Overall
1. Active coping	1.74	1.77	1.76	1.76
2. Humor and acceptance	1.45	1.41	1.39	1.42
3. Social-emotional coping	1.15	1.12	1.06	1.11
4. Abandoning and negation	0.98	1.04	1.00	1.09
5. Religion	0.70	0.65	0.61	0.65
6. Alcohol and drug use	0.16	0.15	0.16	0.16
Overall	1.03	1.03	1.00	1,02

Table 1: Factor means in general and by situation (max. score 3)

Based on the frequency with which students make use of the six different categories of coping strategies, three clusters can be distinguished. The first cluster includes the factors 'active coping' and 'humor and acceptance', which are most frequently used by adolescents with mean scores ranging from 1.39 to 1.77 on a scale of 0 (never used) to 3 (often used). The factors 'social-emotional coping' and 'abandoning and negation' encompass the second cluster (range from 0.98 to 1.15). Coping strategies like 'religion' and 'alcohol and drug use' are rarely reported by secondary school students. The mean scores of coping strategies belonging to this cluster range from 0.15 to 0.70.

We did not find any significant relation between the type of stressful situation and the kind of coping strategies used. Apparently students use the same coping strategies independently of the specific mathematical school situation. Given the fact that 'active coping' and 'humor and acceptance' (the more adapted and effective strategies) are most frequently used in all situations, there is reason for optimism. However, one should not overlook the fact that even for those strategies the scores are rather moderate (mean between 1.39 and 1.77). This implies that students indicate

that in quite a few cases they do not consciously regulate their emotions in these stressful situations, with possibly negative impact on their learning behavior.

Although, in general, students use active coping strategies that allow them to remain focused on the task and solve the problem, there are clear differences between students regarding the kind of coping strategies they use. In the next section we will more specifically address these differences based on the results of the variance analyses.

Familiarity with the stressful situation and the use of coping strategies

	Familiarity with the stressful nature of the respective situations											
	Never			Rarely			Sometimes			Often		
	Test	HW	Lesson	Test	HW	Lesson	Test	HW	Lesson	Test	HW	Lesson
1. Active coping	1.67	1.47	1.58	1.79	1.81	1.74	1.77	1.87	1.87	1.54	1.58	1.73
2. Humor and acceptance	1.26	1.53	1.49	1.29	1.38	1.29	1.48	1.35	1.33	1.78	1.58	1.64
3. Social-emotional coping	1.12	0.79	1.08	1.15	1.13	0.96	1.16	1.17	1.16	1.10	1.23	1.08
4. Abandoning and negation	0.74	0.80	1.11	0.85	0.96	0.96	1.02	1.10	1.04	1.24	1.28	1.11
5. Religion	0.80	0.49	0.55	0.72	0.61	0.60	0.72	0.71	0.63	0.51	0.68	0.55
6. Alcohol and drug use	0.16	0.11	0.28	0.11	0.15	0.11	0.06	0.10	0.12	0.55	0.35	0.28

There is a clear relationship between students’ familiarity with the stressful nature of these situations and their coping behavior (see Table 2).

Table 2: Factor means of students by familiarity with the stressful nature of the situation

These results indicate that students who often find themselves in stressful situations use other strategies to manage their emotions than students for whom these situations usually are not perceived as very stressful. The former use significantly more abandoning and negation ($p < 0.01$ for test and homework situation, $p < 0.05$ for lesson situation), humor and acceptance ($p < 0.01$ for test and lesson situation) en alcohol and drug use ($p < 0.01$ for test situation, $p < 0.05$ for homework situation) en less active coping ($p < 0.05$ for test situation, $p < 0.01$ for homework situation)

The finding that the more students are familiar with a stressful situation the more they use less adequate coping strategies like abandoning and negation, indicates that students do not spontaneously learn to tackle stressful situations in an effective way. More likely, they tend to end up in a negative spiral where the use of inadequate coping strategies result in only experiencing more stress.

Track level and the use of coping strategies

Factor	General			Technical			Vocational		
	Test	HW	Lesson	Test	HW	Lesson	Test	HW	Lesson
1. Active coping	1.81	1.79	1.79	1.80	1.80	1.80	1.58	1.52	1.45
2. Humor and acceptance	1.62	1.48	1.44	1.38	1.41	1.40	1.38	1.15	1.18
3. Social-emotional coping	1.19	1.13	1.06	1.22	1.12	1.05	1.01	1.11	1.10
4. Abandoning and negation	0.85	0.97	0.90	1.04	1.09	1.06	1.04	1.13	1.13
5. Religion	0.77	0.67	0.61	0.71	0.60	0.58	0.61	0.81	0.74
6. Alcohol and drug use	0.17	0.21	0.20	0.10	0.10	0.13	0.22	0.18	0.18
Overall	1.07	1.04	1.00	1.04	1.02	1.00	0.97	0.98	0.96
Over all situations	1.04			1.02			0.97		

Table 3 indicates that there are clear differences between students' coping behavior related to the educational track they are following.

Table 3: Factor means by track level and situation

When confronted with stressful situations students following general education tend to use less 'abandoning and negation' compared to students' from the technical and vocational tracks ($p < 0.01$ in all three situations). Moreover, in test situations they also use more 'humor and acceptance' ($p < 0.01$) than other students. Vocational students, on the other hand, employ significantly less 'active' and 'social-emotional coping' to manage their behavior in all three stressful situations ($p < 0.01$). More generally, our data indicate that there is a tendency for students from lower educational tracks to use less and less effective coping strategies compared to students from higher educational tracks (general > technical > vocational). When we take into account that especially students in vocational education are known to be the one's experiencing most stress and negative emotions at school, the limited and not very effective use of coping strategies makes these students even more vulnerable for stress (see e.g., Rijavec & Brdar, 2002).

Mathematics achievement and the use of coping strategies

A bit contrary to our expectations, we hardly did find any relation between students' mathematics achievement level and their use of coping strategies. The data only pointed to a negative relation between the use of 'alcohol and drugs' in stressful lesson situations and students' achievement level ($p < 0.01$). Nevertheless, although not significant, we can observe that the higher the achievement level the more

students tend to use ‘active coping’ while the opposite is true for ‘social-emotional coping’ and ‘abandoning and negation’.

The absence of a very clear relationship between students’ achievement level and the use of coping strategies might be due to the fact that, on the one hand, high achieving students generally do encounter less negative stress and are less aware of the active coping and problem-oriented strategies they use to regulate their general behavior as well as emotions (see also Op ‘t Eynde, De Corte, & Mercken, 2004). In a sense they tackle their emotional stress (unaware) in the flow of solving the problem. On the other hand, low achieving students, are less pronounced in the kind of coping strategies they use. They seem to try out and employ different kind of strategies which does not allow for a very clear relationship between certain strategies and achievement level.

Motivation for mathematics and the use of coping strategies

As shown in Table 4, students’ motivation for mathematics is clearly related to the way in which they regulate their emotions in school situations. The use of ‘active coping’ is positively correlated with students motivation while ‘humor and acceptance’, ‘abandoning and negation’, and ‘alcohol and drug use’ have a negative relation with students’ motivation.

	Motivation		
	Test	HW	Lesson
Active coping	0.23**	0.18**	0.17**
Humor and acceptance	-0.13*	-0.13*	-0.13*
Abandoning and negation	-0.20**	-0.12*	-0.13*
Alcohol and drug use	-0.23**	-0.12*	-0.13*

Table 4: correlations between students’ motivation and coping strategies

Note: * $p < 0.05$; ** $p < 0.01$

Using multiple regression analysis we found that employing ‘action coping’ strategies stimulates students’ motivation while relying on ‘humor and acceptance’, ‘abandoning and negation’, and ‘alcohol and drug use’ has a detrimental effect on their motivation. Although, this (linear) analysis explains how students’ use of coping strategies influences their motivation, a more complex model in which strategy use and motivation mutually influence each other might be a more adequate representation of the interactions between those variables in school contexts (see e.g., De Anda & Bradley, 1997).

Age and the use of coping strategies

Our findings indicate that the use of certain coping strategies is to some extent age-related. Students from the second grade of secondary education use significantly more social-emotional coping and religion ($p < 0.01$ in all situations), while older

students rely relatively more on humor and acceptance and alcohol and drugs to regulate their emotions ($p < 0.01$ in all situations).

Gender and the use of coping strategies

Table 5 shows that in all situation girls use significantly more socio-emotional coping than boys ($p < 0.01$ in all situations). More generally, with the exception of ‘abandoning and negation’ and ‘alcohol and drug use’ girls have higher scores regarding the use of the different coping strategies than boys (although the differences are not always significant).

Factor	Girls			Boys		
	Test	Home work	Lesson	Test	Home work	Lesson
1. Active coping	1.74	1.82	1.78	1.74	1.72	1.75
2. Humor and acceptance	1.50	1.47	1.46	1.40	1.36	1.34
3. Social-emotional coping	1.31	1.29	1.26	0.98	0.98	0.89
4. Abandoning and negation	0.98	1.04	0.98	0.98	1.05	1.02
5. Religion	0.73	0.71	0.66	0.67	0.59	0.56
6. Alcohol and drug use	0.13	0.14	0.14	0.19	0.17	0.18

Tabel 5: Factor means by gender and situation

This seems to indicate that overall girls demonstrate more coping behavior than boys. Indeed, they use more active coping strategies as well as more social-emotional coping. Apparently, girls aim for a double goal when regulating their behavior in stressful mathematical school situations (see also Boekaerts, 2002). On the one hand, like boys, they are focused on tackling the problem or situation. On the other hand, and (significantly) different from boys, they deliberately address the emotions they are experiencing. They further analyze their feelings or talk about how they feel with their friends etc.

CONCLUSIONS

Students clearly know and make use of several strategies to regulate their emotions in stressful mathematical school situations, but not really in a systematic way. Although students mostly use active coping strategies that allow them to remain focused on the task and solve the problem, the variance analyses indicate that students of the lower tracks as well as low motivated students and students who are frequently confronted with stressful situations typically use less adequate coping strategies. The finding that the more students are familiar with a stressful situation the more they use less adequate coping strategies like abandoning and denial, indicates that students do not spontaneously learn to tackle stressful situations in an effective way. More likely,

they tend to end up in a negative spiral in which low motivation and the use of inadequate coping strategies result in experiencing more stressful situations with only one way out... abandoning and denial. Since students in vocational tracks in general have been found to be the less motivated compared to students in other tracks, they seem to be extremely vulnerable for stressful situations and for ending up in this negative spiral.

Whereas experiencing negative or stressful emotions does not necessarily have to be detrimental for learning and problem solving, for many students it appears to be the case. Consequently it is important to find ways of organizing the classroom context in general and instruction in particular in such a way that students either experience less negative emotions or know how to deal with them in an effective task-focused way. In other words, it is important to equip students with the necessary skills and strategies to (self)regulate their emotions in effective ways.

Schools and teachers should stimulate students to acquire the necessary strategies and skills to self-regulate their emotions. Gottman, Katz, and Hooven (1997) refer in this respect to classroom environments that enable students to master the necessary meta-emotional knowledge and skills. Such a classroom context is characterized by a teacher who is functioning as an emotional coach and organizes classroom interactions accordingly. Such a teacher allows his students to self-regulate their emotions as much as possible. In situations where they not yet possess the knowledge and skills to do it effectively on their own, he/she will 'scaffold' them. He/she will not neglect negative emotions but address them and point out adaptable ways to regulate them. This emotional coaching style expresses itself in daily classroom interaction and practices, rather than in isolated sessions. Gottman et al. (1997) identify three key dimensions of (emotionally) coaching educational environments: (1) a lack of derogation; (2) warm interpersonal relations; and (3) a focus on cognitive as well as emotion scaffolding-praising. This view is in line with Meyer and Turner's (2002) findings on emotions in the classroom. They point out that teachers' affective responses are important both at the academic or cognitive level and at the interpersonal level. Indeed,

an instructional context with low affect as a feature of student-teacher interactions appeared to be similar in students' perceptions to contexts characterized by more negative affect. (Meyer & Turner, 2002, p. 111)

More research is needed that addresses the affective dimensions of learning from a learner's as well as from a teacher's perspective. Only in that way a research-based body of knowledge can become available that allows instructional designers and teachers to develop powerful learning environments that adequately address the self regulation of emotions as an important component of students' learning.

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THE IMPACT OF RECENT METACOGNITIVE EXPERIENCES ON PRESERVICE TEACHERS' SELF-REPRESENTATION IN MATHEMATICS AND ITS TEACHING

Areti Panaoura, Department of Pre-primary Education, Frederick Institute of Technology, Cyprus, pre.pm@fit.ac.cy

The present study aims to highlight the importance for learning and teaching of one of the facets of metacognition: metacognitive experiences. It examines their impact on preservice teachers' of pre-primary education self-representation in relation to their performance on mathematics. The stability of their self-efficacy beliefs about learning and teaching of mathematics, as an indication of self-image was examined after the impact of specific and recent metacognitive experiences. Results indicated that intense or repeated metacognitive experiences influence students' self-representation and self-efficacy beliefs. A model is proposed in order to outline the interrelations, which are developed between experiences and self-representation.

SELF-REPRESENTATION IN RELATION TO RELATED CONCEPTS

Research on mathematics teaching and learning has recently moved away from purely cognitive variables. Metacognition and many of its dimensions such as self-representation, self-awareness, self-evaluation and self-regulation have been receiving increased attention in cognitive psychology and mathematics education (Kramarski & Mevarech, 2003). We use the term metacognition referring to the awareness and monitoring of one's own cognitive system. The relation of metacognition with learning was first posited by Flavell (1979) and, since then, there is growing research evidence that qualifies this relationship. Metacognition has two main and distinct components: metacognitive knowledge and self-regulation. Metacognitive knowledge has come to refer to aspects of students' theory of mind, theory of self, theory of learning and learning environments" (Boekaerts, 1997, p. 165), while self-regulation refers to the processes that coordinate cognition. The present study concentrated on self-representation, as an important metacognitive ability. We consider self-representation as a part of metacognitive knowledge which concerns, acquired knowledge and beliefs about the nature of oneself and other people as cognitive processes.

According to Demetriou and Kazi (2001) "self-representation refers to how the individual perceives himself/herself in regard to a given disposition, style, type of activity or dimension of ability" (p.33). We consider self-representation to be a wider term encompassing meanings that are normally included in related terms such as self-awareness, self-image, self-evaluation and self-efficacy beliefs. According to Newen and Vogeley (2003), representations can be classificatory, compositional, recursive,

meta-representational and iterative meta-representational. The fourth and the fifth categories are connected with the concept of self-representation. The fourth form of representation, meta-representation, is involved in the consciousness of other minds. To have this capacity a human must have a representational structure that enables him/her to account for propositional attitudes by representing the propositional content, the attitude and the subject of the attitude as different elements. The fifth form, iterative meta-representation, is an adequate representation of second order belief ascriptions.

As Demetriou and Panaoura (2006) claim, self-representation is an integral part of directive-executive function of the human mind. That is, the very process of setting goals, planning his/her attainment, monitoring action goals and the plans, and regulating real or mental action requires a system that can remember and review and therefore know itself. Demetriou and Kazi (2001) study self-representation of cognitive performance from the age of 3 years to maturity. They suggest that self-representation develops in “recycling fashion”. Within each phase of development self-representation about the relevant mental operations is very low and inaccurate at the beginning and it tends increasing and becoming more accurate with development until the end of the phase. Self-representation about the cognitive processes gradually takes off with the development of a new phase. This pattern of change in self-representation indicates that the thinker needs time and experience to acquire knowledge and sensitivity to the condition of the processes of the new phase.

Preservice teachers of pre-primary, primary and secondary education have various beliefs about themselves as learners and teachers of mathematics. We believe that a part of the general self-representation is consisted of students’ self-efficacy beliefs about themselves and about the teaching of mathematics. The construct of self-efficacy beliefs was introduced during the 1970s and developed mainly along the lines of Bandura’s social cognitive theory. Bandura (1997) defines self-efficacy as one’s beliefs about his or her ability to organize and execute tasks to achieve specific goals. He assumed four sources of efficacy information: mastery experience, vicarious experience, social persuasion and psychological and emotional arousal. Mastery or enactive experiences are considered the most powerful source of efficacy information. The critical element, which contributes to the development of these beliefs, is the information that the individual gets about his / her ability (Charalambous & Philippou, 2003). That information constitutes what we call “metacognitive experiences”. Those experiences are present in working memory, specific in scope and they can be affectively charged in the case of metacognitive feelings (Efklides, 2006). At the same time there are experiences at the long-term memory, which are parts of the metacognitive knowledge about a person’s own abilities. Metacognitive feelings inform the person about a feature of cognitive processing, but they do it in an experiential way, that is, in the form of a feeling of knowing or confidence. Students also make metacognitive judgments about

the demands of cognitive processing such as how much time or effort is needed for the processing of a task, or whether the outcome produced is correct (Efklides, 2002). Lack of confidence or low confidence makes the person hesitant to further pursue a goal. High feeling of confidence, on the other hand, makes one more decisive but at the same time less critical of one's decisions. Self-efficacy theory predicts that students work harder on a learning task when they judge themselves as capable (Mayer, 1998). Very important for metacognitive experiences is the feeling of difficulty. If the feeling of difficulty is high and associated with negative affect, the person quits the task.

According to Pietila (2003) a good mathematics teacher needs sufficient knowledge of mathematics, sufficient knowledge of mathematics teaching and learning, pedagogical knowledge and positive self-image for learning and teaching mathematics. In respect to those ideas, the present study investigated preservice teachers' of pre-primary education self-representation for themselves as learners of mathematics and as teachers of mathematics.

Purpose of the present study

The present study examined the impact of the recent metacognitive experiences on students' self-representation for the learning of mathematics and its teaching. In respect to this main purpose there were two objectives. The first one was to investigate whether students were aware of their cognitive processes when they were doing mathematics and whether they were accurate in their self-representation of strengths and weakness in mathematics in relation to their performance. The teachers' view of themselves as mathematics learners is important because it influences the way they will teach mathematics. The second objective was to investigate the influence of the recent and relevant experiences on their self-efficacy beliefs about their ability to learn and teach mathematics. We believe that the accuracy of the metacognitive experiences and the respective consequences on students' self-representation is very important because it has a bearing on the efficiency of the control decisions in learning situations with respect to effort allocation, time investment, strategy use and teaching procedures which students follow.

METHODOLOGY

The sample: Data were collected from 60 preservice pre-primary teachers, attending a course for the teaching of mathematics at a Department of Pre-Primary Education. They were at the third year of their studies, and all of them were females.

Procedure: A questionnaire was developed measuring students' self-representation in mathematics and the teaching of mathematics. This instrument consisted of three main parts: The first part comprised of 20 Likert type items, of five points (1=never,

2=seldom, 3=sometimes, 4=often, 5=always). The responses to this part of the questionnaire constituted an image of students' self-representation referring to how they perceived themselves in regard to mathematics and the teaching of mathematics. The second part was consisted of a mathematical problem for which students had, firstly, to evaluate its difficulty before planning their solution, then they had to solve it and finally they had to express their confidence for their solution (see Appendix). We considered the feeling of difficulty and the feeling of self-confidence as direct metacognitive experiences derived from the long-term memory if there are expressed before problem solving and there are derived from the working memory if there are expressed after the solution of a problem. As task processing goes on initial feeling of difficulty rating change, because they get updated depending on processing. Additionally the judgment of solution correctness focuses on the quality of the answer, while the feeling of confidence monitors how the person reached the answer. At the third part of the questionnaire students had to evaluate their ability in solving geometrical tasks and in teaching geometry at the pre-primary education before and after the solution of five geometrical tasks (see Appendix). The purpose of the last part of the questionnaire was to investigate the impact of specific and recent experiences of solving mathematical tasks on students' self-representation and self-efficacy beliefs about a domain of mathematics, such as geometry and its teaching. The unit of geometry was selected because it is one of the five main units for the teaching of mathematics at pre-primary education (NCTM, 2000).

RESULTS

In order to examine the accuracy of students' self-representation and the impact of recent experiences on their self-representation, we first needed to examine whether students' responses to the questionnaire reflected the constructs that the first part of the instrument was designed to tap. A principal factor analysis with varimax rotation was used to create the factor structure of the 20 items included in the questionnaire, by using SPSS. This analysis was used to "reduce a set of observed variables into a relatively small number of components that account for most of the observed variance (Marcoulides & Hershberger, 1997, p. 164). In order to give each factor a clear and distinct meaning for both theoretical interpretation and practical implication, the orthogonal varimax method of rotation was used to minimize the number of variables that have high loadings on more than one factor. A factor loading with absolute value greater than 0.49 was considered sufficiently high to assume a strong relationship between a variable and a factor. The final 20 items were resulted in six factors with eigenvalues greater than 1, explaining 63.77% of the total variance. The factor loadings of the statements on the factors are presented at Table 1. The reliability of the whole questionnaire was very high. Specifically, the Cronbach's α was 0.86.

Table 1: Factor loading of the factors

Item	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
I like solving geometrical tasks.	.859					
I am very good in mathematics.	.626					
I understand better a mathematical concept when the teacher presents specific examples.		.784				
I feel stress when I have not enough time to solve a problem.		.732				
I can easily compare two problems and find their similarities.		.646				
When I try to remember something that teacher said, I “draw” in my mind his/her picture while saying it.		.552				
If it is necessary I change easily the solution plan for a problem while I am trying to solve it.		.549				
In order to explain an idea to my classmates I use examples.		.495				
I prefer solving problems that present the data with diagrams or tables.			.853			
In order to explain a solution to my friend I use a picture or a diagram.			.844			
In order to explain an idea to my classmates I use a picture or a diagram.			.541			
I can easily make a puzzle.				.768		
I can easily imagine the picture which is on a deflated balloon.				.773		
I can easily compare two pictures in order to find their differences.				.801		
I can easily complete the relations at a geometric pattern.					.765	
When I try to explain a solution to my friend I usually use words.					.632	
When I try to solve a problem I organize the data at a table.					.594	
While I am solving a problem I can use my hands for doing other things.						.839
I can understand better a mathematical concept when I use my own examples.						.795
I do not forget easily what I learn in mathematics.						.543
Eigenvalues	7.773	3.134	2.305	1.802	1.453	1.145
Percentage of variance explained	24.165%	10.838%	9.018%	7.017%	6.631%	6.101%
Cumulative percentage of explained variance	24.165%	35.003%	44.021%	51.038%	57.669%	63.77%

After a content analysis of the six factors, according to the results of the exploratory factor analysis, the six factors were classified into two main groups. The first group of factors (GF1) was consisted of the first, the second and the sixth factors, expressing the students' general self-representation. The items of the first factor represented students' positive self-image about mathematics, while the items of the second factor expressed their self-awareness about their cognitive procedures. Finally the items of the sixth factor expressed their self-awareness about learning mathematics. The second group of factors (GF2) was consisted of the remained three factors (the third, forth and fifth), indicating students' self-representation about spatial abilities and procedures. The items of the third factor represented their preference for the use of representations, the items of the forth factor expressed students' self-representation about the spatial ability. Finally the fifth factor expressed the use of diagrams, tables and shapes. Those results indicated that students' responses to the 20 items of the first part of the questionnaire presented their general self-representation about their abilities in mathematics and their self-representation on a specific domain of mathematics, the geometry.

Then students were clustered into three groups according to their mathematical performance at the geometrical tasks of the second part of the questionnaire. The first group (G1) consisted of 14 (23.3%) students with the lowest performance, the second group (G2) consisted of 19 (48.3%) students with medium performance and the third group (G3) consisted of 27 (45%) students with the highest performance. The performance of the three clusters of students was examined in respect to their behavior towards the two main above-mentioned metacognitive groups of factors about their self-representation. The comparison of the means by using ANOVA indicated statistically significant differences in the case of the GF1, their general self-representation ($F_{(2,59)}=4.944$, $p<0.01$). The difference, as expected, was between the G1 and G3, indicating that students with higher performance on the tasks were at the same time students with higher general self-representation about cognitive processes and procedures ($\bar{X}_1=2.833$ and $\bar{X}_3=3.38$). There were not statistical significant differences in the case of the GF2. Those results were the first indications that students had accurate self-representation about their cognitive abilities in mathematics in relation to their mathematical performance. Nevertheless they were not accurate regarding their self-representation about their specific spatial abilities that were related with the domain of geometry.

According to the results of the descriptive analysis for the second part of the questionnaire, 37.8% of the students assessed the problem as very easy, 32.4% of them assessed it as easy and only 29.8% assessed it as difficult or very difficult. Crosstabs analysis indicated that 28.6% of the students who evaluated the problem as very easy solved it wrongly and 30.4% of those who evaluated it as easy solved it wrongly. At the

same time 35% of the students who solved the problem wrongly indicated very high self-confidence for their solution and 30% of them indicated high self-confidence. It is important to note that only 4% of the students indicated low self-confidence about their solution. At the same time there were statistically significant differences among the three groups of students according to their performance on the geometrical tasks in respect to their self-confidence in solving the specific problem ($F_{(2,59)}=7.335$, $p<0.01$, $\bar{X}_1=3.50$, $\bar{X}_2=4.32$ and $\bar{X}_3=4.54$). As it is obvious students tended to overestimate their performance before and after the solution of the specific mathematical problem. As we have mentioned the feeling of difficulty before the solution of the problem was a direct metacognitive experience derived from the long-term memory, while the feeling of self-confidence was a metacognitive experience derived from the working memory. From those results it was important that even though many students solved the problem wrongly, they expressed high confidence for their solution, not realizing their mistakes and believing that they were right. This was an indication that the recent experience was not so intense in order to be at the same time stressful and a factor that could replace the previous metacognitive experience or knowledge.

The purpose of the last part of the questionnaire was to investigate the impact of specific and recent experiences on students' self-representation and self-efficacy about a specific domain of mathematics and its teaching. There was a statistical significant and strong correlation between their reactions regarding their ability to solve geometrical tasks before and after the solution of the tasks ($r=.622$, $p<0.001$). Nevertheless their self-evaluation about solving geometrical tasks was decreased significantly ($t=6.429$, $p<0.01$) after the solution of the problem solving tasks ($\bar{X}_{\text{before}}=3.39$ $SD=0.73$, $\bar{X}_{\text{after}}=2.81$ $SD=0.76$). Students with the highest performance on the tasks (G3) had significantly higher self-confidence about their abilities on geometrical tasks than those with the lowest performance on the tasks (G1) ($F_{(2,59)}=15.285$, $p<0.01$) only after the problem solving procedure and not at the initial measurement ($F_{(2,59)}=2.159$, $p=0.125$). The decrease of students' beliefs in their ability to solve geometrical tasks was highest in the case of the G1 ($\bar{X}_{1\text{before}}=3.14$, $\bar{X}_{2\text{before}}=3.26$, $\bar{X}_{3\text{before}}=3.59$, $\bar{X}_{1\text{after}}=2.15$, $\bar{X}_{2\text{after}}=2.61$, $\bar{X}_{3\text{after}}=3.25$). The recent and intense experiences for solving the geometrical tasks seemed to affect students' self-confidence for their mathematical abilities in geometry.

Regarding students' self-confidence for the teaching of geometry at the pre-primary education, there was a statistical important correlation between their responses before and after the solution of the tasks ($r=.773$, $p<0.001$) and there was not a statistically important decrease at their self-confidence ($\bar{X}_{\text{before}}=2.71$ $SD=0.88$, $\bar{X}_{\text{after}}=2.54$ $SD=0.78$). There were not statistically significant differences among the three groups of students according to their performance and their self-confidence on teaching geometry, before or after the solution of the geometrical tasks.

DISCUSSION

The starting point for the present study was that a good teacher needs positive self-image for learning and teaching of mathematics, high self-confidence about his/her abilities and precise self-representation about his/her strengths and limitations. All those information constitute a part of the metacognitive knowledge. Results indicated that recent metacognitive experiences, which are intense or repeated influence students' self-representation in mathematics and its teaching. Metacognitive experiences are products of complex inferential processes that inform the person about features of cognitive processing.

Although there was a high correlation between students' self-representation and their mathematical performance, results indicated that in many cases, especially in the case of the students with low performance, their self-representation was not accurate. They tended to overestimate their abilities. There are various possible reasons why a metacognitive judgment is not accurate. The first one is that metacognitive experiences are based on nonconscious, heuristic, inferential processes that make use of various cues, which regard the task and its presentation or the fluency of processing (Efklides, 2006). Results confirmed that mastery or enactive experiences are considered the most powerful source of efficacy information (Bandura, 1997). If we accept that self-representation develops in recycling fashion (Demetriou & Kazi, 2001), results at the third part of the questionnaire indicated that students need time and intense or maybe repeated metacognitive experiences to acquire the metacognitive knowledge and sensitivity to the condition of a new situation.

The model which is presented below (Figure 1) expressed the complicated interrelations between self-representation, as a specific dimension of metacognition, and students' metacognitive experiences at mathematics. Students' metacognitive experiences create specific negative or positive feelings. When those feelings are too strong or the experiences are repeated, students create specific image about their abilities regarding the learning of mathematics and its teaching. The stability of this image "constructs" students' metacognitive knowledge about their abilities regarding specific tasks. On the other hand when students encounter a difficulty at a problem solving situation they react in respect to their metacognitive knowledge in general and their self-representation about the specific abilities in particular.

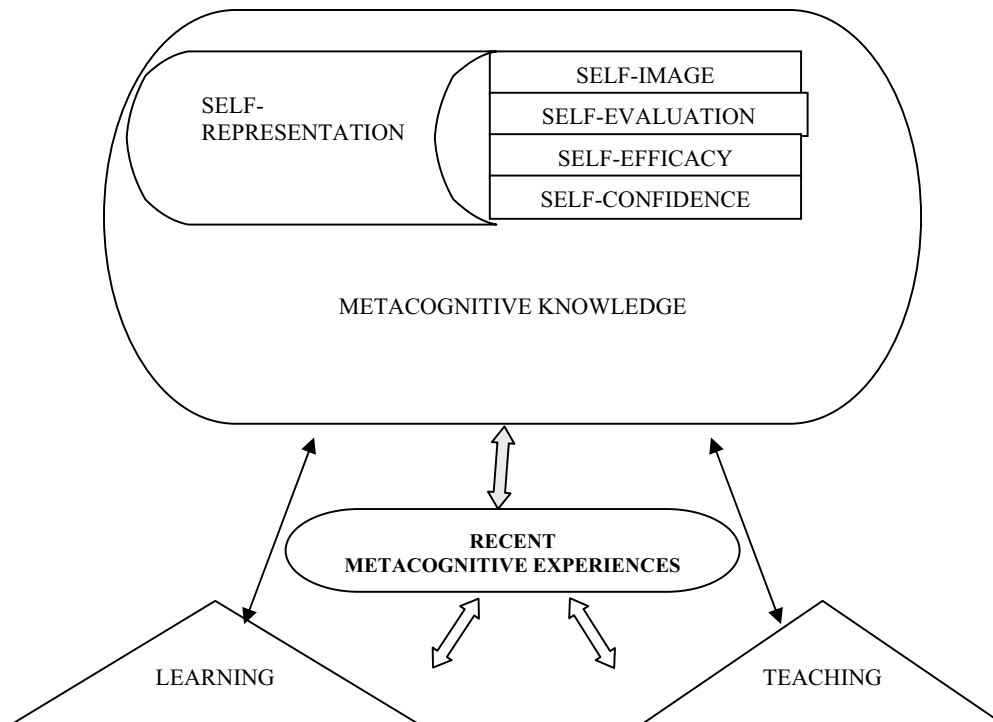


Figure 1: Complicate metacognitive interrelations of a pre-service teacher at the learning and teaching procedure.

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Appendix

The mathematical problem (similar problems at the textbooks of mathematics for primary education are expected to be solved by using the strategy of "logical reasoning") at the second part of the questionnaire was:

Michael has four books on his night table. The story of Peter Pan is below a story of Jules Verne. The book of Mythology is between two other books. There is no other book below the book of comics. Which is the arrangement of the books on the table?

The geometrical tasks at the third part of the questionnaire were:

- Draw a square with a perimeter of 8cm.
- How does the perimeter of a square change if its side is doubled?
- How does the area of a square change if its side is doubled?
- How does the perimeter of a rectangular change if one of its sides is doubled?
- How does the area of a rectangular change if one of its sides is doubled?

IS MOTIVATION ANALOGOUS TO COGNITION?

Marilena Pantziara, Demetra Pitta-Pantazi and George Philippou

Department of Education –University of Cyprus

This paper presents some preliminary results of a larger study that investigates the relationship between students' conceptual understanding of fractions, students' motivation and social context (teachers' practices in the mathematics classroom and students' socio-economic status). Data was collected from 299 sixth grade students through a questionnaire comprised of four scales measuring fear of failure, interest self-efficacy beliefs and achievement goals, and a test measuring students' understanding of fractions. The results of the study suggest that students in the upper levels of conceptual understanding of fractions were characterised by less fear of failure and more mastery goals and self-efficacy than students in the lower levels of conceptual understanding.

Introduction

The relationship between cognition and affect has recently attracted increased interest on the part of mathematics educators, particularly in the search for causal relationship between affect and achievement in mathematics (Young, 1997). This is due to the fact that the mathematical activity is marked out by a strong interaction between cognitive and emotional aspect (Di Martino & Zan, 2001). Additionally, mathematics educators (Sfard, 1991; Gray & Tall, 1994; Dubinsky, 1991) who have developed learning theories of students' concept formation refer also to affective factors.

In an attempt to describe students' mathematical thinking, that is how learners construct mathematical ideas, a number of theories have been developed (e.g. Sfard, 1991; Gray & Tall, 1994; Dubinsky, 1991). These theories aim to identify certain characteristics of students' levels of cognitive development. Even though these theories seem to consider the role of affect in different levels of cognitive development, none of them has so far associated affective characteristics that students may have when they reach different levels of concept formation.

The present study aims to probe the various affective-motivational constructs of students who belong in different levels of cognitive development of the fraction concept. The wider study investigates relationships among external factors (teachers' behaviour in the classroom) and internal factors (students' fear of failure and self-efficacy) that may contribute to students' concept formation and motivation. In the next section we consider the basic concepts and define the research questions.

Theoretical background and aims

Motivation

The three basic elements of human mind, which function as an integral unit, are emotion, cognition and motivation (Hannula et. al., 2005). Motivation refers to causes that get individuals moving (energization). According to Bandura's

sociocognitive theory students' motivation is a construct that is built out of individual learning activities and experiences, and that varies from one situation or context to another (Bandura, 1997). Therefore one of the critical influences on students' choice of cognitive strategies is their motivation to learn. Four motivational factors that have been consistently related to cognitive strategy used in the learning situations are achievement goals, self-efficacy, interest and the motive to avoid failure.

Achievement goal theory has become one of the major theoretical approaches to the focus on how students are motivated along with several other social cognitive approaches to motivation. This theoretical approach is concerned with students' reasons, purposes, or goals for achieving in school (Elliot & Church, 1997). The term goals refer to the fundamental reasons for which students take part in a given learning activity (Dweck & Legget, 1988). Two distinct goals have been emphasized in the literature, namely mastery goals that focus on learning and understanding, and performance goals that focus on the demonstration of competence. Recently, there has been a theoretical and empirical differentiation between performance-approach goals, where students focus on how to outperform others, and performance-avoidance goals, where students aim to avoid looking inferior or incompetent in relation to others (Elliot & Church, 1997).

The importance of goals comes from the assumption that they lead to levels of motivation and dimensions of information processing that are variable in terms of achievement. According to several authors, the adoption of mastery goals leads students to become more persevering in the face of obstacles and to achieve a higher level of cognitive commitment, which translates into a broader use of cognitive and metacognitive strategies (Dweck & Leggett, 1988). On the other hand, empirical studies show that performance goals entail less perseverance and a superficial cognitive commitment, i.e. the use of 'surface' learning strategies such as copying, repeating and memorizing (Dweck & Leggett, 1988). Yet, several research has revealed no relationship between performance goals and cognitive strategy use (Pintrich & De Groot, 1990; Young, 1997).

Self-efficacy beliefs are the beliefs in one's capabilities to organize and execute the courses of action required to manage prospective situations. Bandura (1997) characterized self-efficacy as being both a product of students' interactions with the world and an influence on the nature and quality of those interactions. In the first case, students' cognitive interpretations of success and failures influence subsequent self-efficacy beliefs and in the latter, students' self-efficacy beliefs influence their effort, persistence and the cognitive resources they bring to bear in their attempt to interact with the world. Moreover, it is found that mastery goals predict students' interest in mathematics and therefore interest is positively associated to achievement, while the motive to avoid failure orients students towards failure, and therefore it is hypothesized to prompt the adoption of performance goals, which in turn determine students' achievement in a negative way (Elliot & Church, 1997).

Theories of learning

Researchers suggest that learners develop mathematical ideas in variable ways, mainly procedurally or conceptually. Naturally, this difference in developing the new ideas leads also to different outcomes. For instance, conceptual learners are able to captivate sophisticated mathematical thinking, while procedural learners have troubles to manage complicated conceptual structures. There are a number of process-object theories (Sfard, 1991; Gray & Tall, 1994; Dubinsky, 1991). In particular, the procedural knowledge (Gray & Tall, 1994) or operational knowledge (Sfard, 1991) focuses on routine manipulation of objects which are represented either by concrete materials, spoken words, written symbols or mental images. On the other hand, the conceptual knowledge (Gray & Tall, 1994) or structural knowledge (Sfard, 1991) is the knowledge that is rich in relationships.

Even though there are slight differences between these theories, the broad sweep of the theories is similar. They begin with actions of known objects which are practised to become routinized step-by-step procedures, seen as a whole as processes, then conceived as entities in themselves that can be operated on at a higher level to give a further construction (Pegg & Tall, 2005).

Specifically, according to Sfard's theory of reification (1991) three levels can be distinguished in the process of concept formation; these levels correspond to three degrees of structuralization (conceptualization): interiorization, condensation and reification. When the learner is at the stage of interiorization s/he gets acquainted with the processes, which will eventually give, rise to a new concept; these processes are operations performed on lower-level mathematical objects. The learner becomes gradually skilled at performing these processes.

At the second stage of concept development, which is called condensation, the learner becomes more and more capable of thinking about a given process as a whole without feeling an urge to go into details. Condensation should be regarded as the stage where processes defining the concept become more concise for the learner and the learner becomes increasingly capable of dealing with alternate forms of the concept (Sfard, 1991).

The third stage, which is called reification, entails that a learner is able to conceive of a concept as a 'fully-fledged object' (Sfard, 1991, p.19); the various representations of the concept are unified in the learners' reified construct and the construct is no longer dependent upon a process. At this stage a new concept is officially born. The student at this stage is able to attribute meaning and significance to the construct by understanding the conceptual category in which it belongs. The reified concept is now ready to be used as an input in higher-order processes that can lead to even more powerful constructs.

As far as it concerns fractions, in Sfards' theory the fraction ($\frac{3}{4}$) is structurally a pair of integers (a member of a specially defined set of pairs) and operationally the result

of division of integers. Furthermore, there is an extensive literature on pupils' interpretation of the fraction as a multifaceted construct as part-whole, ratio, quotient, operator and measure and the difficulties students face concerning the concept of fraction (Lamon, 1999) because of its different constructs.

Although there are numerous studies investigating the relationship between motivational constructs and students' achievement, to the best of our knowledge, none of these studies has so far investigated the relationship among motivational constructs and the level of students' cognitive development concerning fractions. In this respect the purpose of this study was:

- To test the validity of the measures for the six factors: fear of failure, self-efficacy, interest, mastery goals, performance-approach goals and performance-avoidance goals, in a specific social context.
- To construct and test the validity of a test measuring students' cognitive development in fractions.
- To investigate differences in motivational constructs of students in different levels of cognitive development concerning fractions.

Method

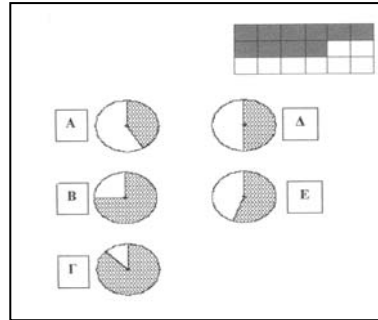
Participants were 299 sixth grade students, 135 males and 164 females from 16 intact classes of an economically homogeneous school district. All participants completed a questionnaire comprised of four scales measuring: a) achievement goals (mastery, performance-approach and performance-avoidance), b) self-efficacy beliefs and c) interest and d) fear of failure. Specifically, the questionnaire comprised of 35 Likert-type 5-point items (1- indicating strong disagreement and 5 strong agreement). The five-item subscale measuring mastery goals, as well as the five-item measuring performance goals and the four-item measuring performance-avoidance were adopted from the Patterns of Adaptive Learning Scales (PALS) (Midgley et al., 2000); respective specimen items in each of the three subscales are, "one of my goals in mathematics is to learn as much as I can" (Mastery goal), "one of my goals is to show other students that I'm good at mathematics" (Performance-approach goal), and "It's important to me that I don't look stupid in mathematics class" (Performance-avoidance goal). The five items measuring self-efficacy were adopted also from PALS; a specimen item is "I'm certain I can master the skills taught in mathematics this year". We used Elliot and Church (1997) seven-item scale to measure students' interest in mathematics; a specimen item is, "I found mathematics interesting". Finally, students' fear of failure was assessed using nine items adopted from the Herman's fear of failure measure (Elliot & Church, 1997); a specimen item is "I often avoid a task because I am afraid that I will make mistakes".

For the evaluation of students' levels of cognitive development, a three-dimensional test was also administered, each dimension corresponding to one level of conceptual understanding- interiorization, condensation and reification- proposed by Sfard

(1991). The tasks comprising the test were adopted from published research and they assessed students' understanding of fraction as part of a whole, as measurement, equivalent fractions, fraction comparison (Hannula, 2003; Lamon, 1999) and addition of fractions with common and non common denominators (Lamon, 1999). The tasks were developed further in order to correspond to the characteristics of each of Sfards' conceptual levels. An example of the three tasks assessing equivalence of fractions in the three different levels follows: a) Interiorization: Fill in the blank: $\frac{1}{3} = \frac{\quad}{12}$

b) Condensation:

Find which of the five circles is shaded in approximately the same fraction as the one represented in the rectangle. Explain your answer.



c) Reification: Three friends ordered three pizzas of the same size and shape. George ate $\frac{4}{16}$ of his pizza. Andreas ate $\frac{3}{12}$ of his pizza. Costas ate X pieces of his pizza. If the three friends ate the same amount of pizza, write how many pieces might be Costas' pizza using variable X.

Results

With respect to the first aim, students' responses were subjected to exploratory factor analysis, which resulted in a six-factor solution, explaining 54.80% of the total variance. All loadings were high and statistically significant, ranging from .45 to .86. The six factors corresponded to students' motivational constructs as were described in the questionnaire. This finding supports the construct validity of the questionnaire used to collect data on pupils' motives, goals and outcomes. Factor scores for each dimension were estimated by calculating the average of the items that comprised each factor. Table 1 presents the mean scores, standard deviations, and Cronbach's alpha coefficients for each of the six factors.

Factors	Mean (1-5)	SD	Cronbach's a
Mastery goals	4.52	.46	.71
Performance approach goals	3.08	.93	.80
Performance avoidance goals	2.85	.93	.51
Self-Efficacy	4.09	.62	.71
Interest	3.85	.89	.89
Fear of failure	2.20	.78	.66

Table 1: Means, Standard Deviations and Cronbach's alpha coefficients of the six factors identified by exploratory factor analysis.

The Cronbach's alphas were quite high (ranging from .66 to .89) for six of the factors, while poorer (.51) for performance-avoidance goals. This result might be partially attributed to the cultural difference between USA, where the scale was developed and Cypriot students or to variable samples' age; in the present study participants were just above 11 years of age, while in other studies the samples consisted of college students.

Concerning the second aim of the study, Rasch analysis was used on the whole sample to specify the hierarchy of fraction items difficulty. The Rasch model is appropriate for the specification of this scale because it enables the researcher to test the extent to which the data meets the requirement that both students' performances on the items of the fraction test and the difficulties of the items form a stable sequence (within probabilistic constraints) along a continuum (Bond & Fox, 2001). Thus, specifying an individual's position on this continuum provides information about this individual's probability of success on items below (high probability) and above (low probability) this position. At the same time, specifying an item's position on the scale provides exact information about the individuals who can succeed (those scoring higher than this item's position) or fail (those scoring lower than this item's position) on the scale. Figure 1 illustrates the scale for the 21 items of the fraction test with item difficulties and student measures calibrated on the same scale. Clearly, these 21 items have a good fit to the measurement model, indicating an agreement among the 299 students located at different positions on the scale, across all 21 items. Moreover, the items of the test are well targeted against the students' measures since students' scores range from -2.98 to 3.73 logits and item difficulties range from -4.03 to 3.90 logits. It appears that almost all students answered correctly items at the easy end of the scale (i.e. 8, 6, 3), activities that belong to the interiorization phase of the test. On the other hand, the items at the hard end of the scale (i.e. 21, 22, 23) were activities of the reification level and answered correctly only by those students who had high achievement in the fraction test.

As shown in Figure 1 only activities 10 (B2) and 15 (B7) did not correspond to the levels they were created for in the first phase of the study. The activity B2, which was easier than the level it was created to assess, required students to present the fraction $\frac{3}{7}$ in a rectangle divided in 14 equal parts. This may be due to the great amount of similar activities that students are involved with during fraction teaching or due to the fact that the part-whole subconstruct is fundamental and precedes the development of other subconstructs. Activity B7 that was more difficult than the level was created to assess, asked students to compare two fractions using two different ways. This may be due to the traditional way the fractions are taught in schools which is based on operational rather than conceptual understanding and which is also eliminated to only one way for solving problems with fractions.

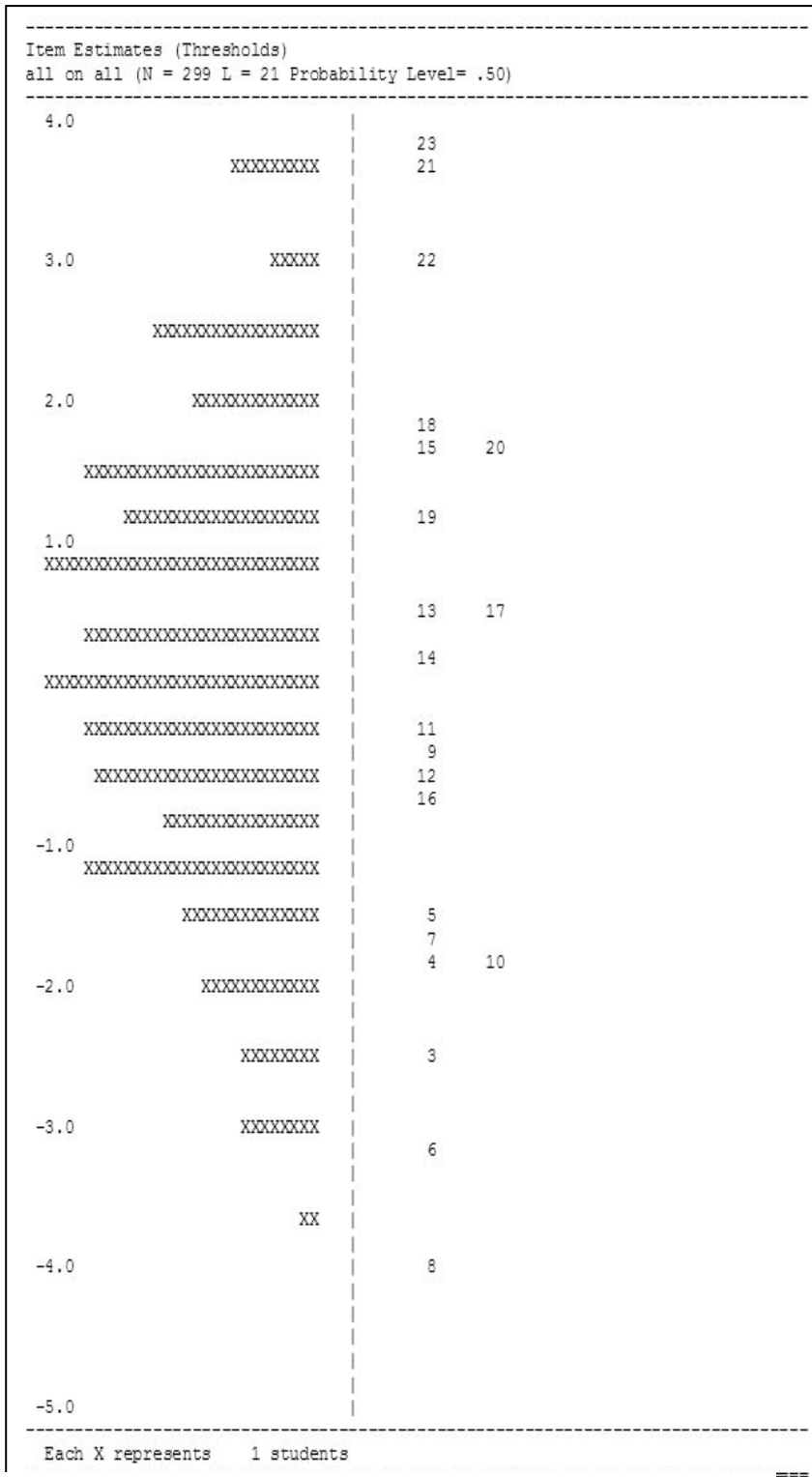


Figure 1: Scale for the fraction test. (N=299. Each X represents 1 student)

In regard to the third aim of the study, Cluster analysis was used to investigate if the various items were systematically grouped into levels of difficulty that may be taken to stand for the three levels represented in the study. As such, the procedure for detecting pattern clustering in measurement designs developed by Marcoulides & Drezner (1999) was used. This procedure enables the division of the observed measurements into constituent groups (or clusters) so that the members of any one group are similar to each other, according to selected criterion that stands for difficulty. Applying this method to separate the 21 items on the basis of their difficulty that emerged from the Rasch model, the analysis showed that the tasks were cluster into 9 subcategories of the 3 levels described in this study (table 2).

From Table 2 it can be concluded that the first four sublevels correspond to the Interiorization level except from activity 10.

The sublevels 5 to 6 correspond to the Condensation level while the

sublevels 7, 8 and 9 correspond to the Reification level except from activity 15.

Clusters	Rasch	Activities
1	-4.00 to -3.12	8
2	-3.12 to -2.52	6
3	-2.52 to -1.86	3
4	-1.86 to -1.48	4, 10, 7, 5
5	-1.48 to -0.23	16, 12, 9, 11
6	-0.23 to 0.60	14, 13
7	0.60 to 1.16	19, 17
8	1.16 to 1.84	15, 20, 18
9	1.84 to ...	22, 23, 21

Table 2: The 21 items cluster into 9 subcategories.

In order to investigate if there were any differences in motivational constructs of students who belonged to the three levels of concept formation, students were placed in groups according to their ability as described by the Rasch model and the Cluster analysis. Specifically, 33 students were placed in the Interiorization level, 126 students in the Condensation level and two other levels were created for students in the Reification level, including those who did some of the activities in this level namely 19, 17, 15, 20 and 18 (73 students) and the students who succeeded in almost all the activities in this level which included 22, 23, and 21 (67 students).

Anova analysis using LSD (Least significant difference) was performed for the investigation of difference in the means concerning students' motivational constructs. Students in the four levels differed significantly in their fear of failure ($F=9,106$ $p<0,000$), their self-efficacy ($F=4,442$, $p=0,005$), moderately in their mastery goals ($F=2,452$, $p=0,06$) while there wasn't any significant difference in students' interest on mathematics. The LSD method revealed that in terms of mastery goals the mean performance of students who belonged in the Interiorization level was significantly lower from the mean of the students who belonged in the Reification level, lower level (mean dif=0,20, $p=0,035$) and upper level (mean dif=0,25, $p=0,011$). Students in the condensation level declared slightly higher mastery goals than students in the Interiorization level. As far as it concerns fear of failure, students in the Reification level (upper level) had lower fear of failure than those students in the Interiorization (mean dif=0,48 $p=0,003$) and Condensation level (mean dif=0,55, $p<0,000$). All the same, students in the Reification level were found to have significantly higher self-efficacy beliefs than students in the Interiorization (mean dif =0,31, $p=0,016$) and the Condensation level (mean dif =0,31, $p=0,001$).

Conclusion

The present study contributes to the ongoing discussion about the relationship between students' motivation and achievement. This study did not focus on the causal relationship between students' motivation and achievement (Young, 1997). Instead it investigated the motivation of students in groups of different cognitive levels.

The findings of the factor analysis in conjunction to results of other studies (Elliot & Church, 1997) did not support the trichotomous conceptualisation of the structure of goals: approach-avoidance-achievement. Two goals (mastery and performance) were identified. This may be partially due to cultural differences between environments, and also to the difference in participants' age, since participants in the present study were sixth graders while in the other studies were college students.

The findings support the results of other studies (Dweck & Leggett, 1988; Pintrich & De Groot, 1990) about students who adopt the mastery goals but from a different perspective. Instead of focusing of the causal relationship between achievement goals to mathematical achievement, the findings of the study revealed that students in the Reification level had higher mastery goals than the students in the Interiorization and the Condensation level which can be translated as students in the Reification level who use broader cognitive strategies, adopt the mastery goals.

As far as self-efficacy is concerned, the findings support the results of other studies (Pintrich & De Groot, 1990) again from a different perspective. Students in the Reification level who have higher cognitive resources in mathematics have also higher self-efficacy beliefs than students who belong in the other two levels.

The motive to avoid failure is lower in students at the Reification level and higher in students at the other two levels. These results are in accord to the results of other studies (Elliot & Church, 1997) who found that fear of failure is a motive that is related to low achievement in mathematics.

Unlike the results of other studies (Elliot & Church, 1997) students who belonged in different levels of cognitive development did not differ in their interest on mathematics. Students' interest on mathematics may be affected by other factors like teachers practices in the classroom.

In conclusion, students with higher cognitive abilities are characterized by mastery goals, self-efficacy beliefs and low fear of failure. How these findings are implemented in schools and what is the role of the teacher in this relationship between cognition and motivation are questions that this on-going study will investigate further.

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IDENTIFYING DIMENSIONS OF STUDENTS' VIEW OF MATHEMATICS

Bettina Rösken^{1,2}, Markku Hannula^{2,3}, Erkki Pehkonen², Raimo Kaasila⁴, Anu Laine²

University of Duisburg-Essen¹, University of Helsinki², Tallinn University³
and University of Lapland⁴

Students' mathematics-related beliefs are a decisive parameter for engagement and success in school. In our paper, we primarily focus on the systematic character of beliefs and we are interested in dimensions describing such a view of mathematics. By means of exploratory factor analysis we obtained seven dimensions structuring this construct. Reliable scales describe these dimensions and we analyzed one of them in detail by considering effects of course choice (general or advanced courses). We further examined relations between the dimensions and what structure they generate; thereby a core of three high correlating dimensions could be identified. Participants in our study were 1436 randomized chosen students of secondary school, grade 11, from overall Finland.

INTRODUCTION

The study of students' mathematical beliefs has received much attention in recent years. A lot of results have been presented examining beliefs for different groups (students, teachers) under diverse conditions. These studies are in most cases descriptive, for example reporting typical beliefs held by students (e.g. Ma & Kishor, 1997). Some of the studies compare student beliefs in different countries (e.g. Pehkonen, 1994) or according to background variables such as gender (e.g. Frost, Hyde and Fennema, 1994). Furthermore, most of the studies of beliefs have been carried out with a separate focus on cognitive, motivational or affective aspects and only few contributions address explicitly belief systems (Op 't Eynde & De Corte, 2003). Green (1971) introduced this term and although the importance of the systematic nature of beliefs is widely acknowledged (e.g. Schoenfeld, 1985), there is a clear lack of studies elaborating on belief systems.

We, however, focus explicitly on studying the structure of students' mathematical beliefs, and to emphasize this, we use the term *view of mathematics*. This term was originally introduced by Schoenfeld (1985) and later adapted by others (Pehkonen, 1995; Pehkonen & Törner, 1996). We are interested in students' view of mathematics as result of their experiences as learners of mathematics. Particularly, we will explore what dimensions describe this view, how they are related and what structure they generate. Hitherto, we have explored the relational structure of teacher students' view of mathematics (Hannula, Kaasila, Laine & Pehkonen, 2005, 2006). This study led to eight scales describing these students' view of mathematics, and particularly three dimensions that were closely related. Now, we have used a modified questionnaire to collect and analyze data from a sample of secondary school students.

2. THEORETICAL FRAMEWORK

In the literature, beliefs have been described as a *messy construct* with different meanings and accentuations (Pajares, 1992). The term belief has not yet been clearly defined (Furinghetti & Pehkonen, 2002). However, there is some consensus that mathematical beliefs are considered as personal philosophies or conceptions about the nature of mathematics and its teaching and learning (Thompson, 1992). Following Schoenfeld (1998, p. 19), beliefs can be interpreted as “mental constructs that represent the codification of people’s experiences and understandings”.

Beliefs cannot be regarded in isolation; they must always be seen as part of a belief system (Green, 1971). These beliefs systems can be characterized by three dimensions as there are quasi-logicalness, psychological centrality, and cluster structure. A quasi-logical order of beliefs refers to primary beliefs, which are beliefs a person uses as reason for other ones, and derivative beliefs. Psychological centrality considers the strength by which beliefs are held, whether they are central resp. core beliefs and cluster structure points to the fact that beliefs are held in clusters around specific situations and contexts, more or less isolated from each other. Op ’t Eynde, De Corte and Verschaffel (2002) as well, consider explicitly the structure of beliefs about mathematics but with a different focus. They provide a framework of students’ mathematics-related beliefs that is based on a review of research on this construct. Constitutive dimensions are object (mathematics education), self, and context (class), which further lead to several sub-categories, for example *mathematics as a subject*, *self-efficacy* or *social norm*. This framework brings together results from beliefs research focusing separately on each of the dimensions. Although this framework was mainly confirmed by analysis of our teacher student data, we also found one emotional scale concerning students’ liking of mathematics (Hannula et al., 2006).

3. METHODOLOGY

3.1 Instrument and Participants

The *view of mathematics* indicator has been developed in 2003 as part of the research project “Elementary teachers’ mathematics” financed by the Academy of Finland (project #8201695). It has been applied to and tested on a sample of student teachers and was slightly modified for the present sample. That is, items addressing specifically aspects of teaching mathematics were removed. More information about the development of the instrument can be found e.g. in Hannula et al. (2006). The statements in the questionnaire are grouped around the following topics: (1) Experiences as mathematics learner (A1-A29), (2) Image of oneself as a mathematics learner (B1-B16), and (3) View of mathematics and its teaching and learning (C1-C12). The students were asked to respond on a Likert scale (5 point, agree to disagree) to statements such as *I have worked hard to do mathematics*, *My family has encouraged me to study mathematics*, *I can get good grades in mathematics* or *I would have needed a*

better teacher. The participants in our study came from fifty randomly chosen schools from overall Finland, including classes for both, advanced and general mathematics. The respondents were in their second year course for mathematics in grade 11. Altogether 1436 students filled in the questionnaire and gave it back.

3.3 The process of factor analysis

Since our aim was to explore the field of structuring students' view of mathematics we employed exploratory factor analysis instead of confirmatory one. We used maximum likelihood factor analysis with oblique rotation for determining useful and statistically robust dimensions regarding this construct. As one advantage, this extraction method allows making inferences from sample to population (Kline, 1994); the sample of 1436 students is therefore large and adequate enough. Maximum likelihood analysis requires multivariate normality and we found all distributions of the measured variables fulfilling this request. Further, we chose to use an oblique rotation. The dimensions of the construct cannot be regarded independently from each other. In that case, an oblique rotation will lead to a better estimate of factors because it derives factor loadings based on the assumption that they are correlated (Fabrigar, Wegener & MacCallum, 1999); as rotation method we employed direct oblim rotation. We carefully tested different factor solutions, for a detailed description see Rösken, Hannula, Pehkonen, Kaasila and Laine (in preparation). We finally chose a seven-factor solution that accounts for 59 % of variance and provides factors with excellent internal consistency reliability.

4. RESULTS

In the following we will present several results of our data analysis. First, we will provide an overview on the dimensions we found by factor analysis, second we give a detailed analysis of one of the dimensions. For the other dimensions see Rösken et al. (in preparation).

4.1 Dimensions of students' view of mathematics

Factor analysis led us to seven dimensions describing students' view of mathematics. Table 1 shows the factors, the related items as well as the factor loadings and Cronbachs' alpha.

Table 1. The seven dimensions of students' view of mathematics.

Seven-factor solution	Loadings
F1 Competence (Cronbach' s alpha = .91)	
B 8 I am no good in math	-.77
B 6 I am not the type to do well in math	-.70
B 3 Math has been my worst subject	-.43

A15	I have made it well in mathematics	.43
B 4	Math is hard for me	-.40
F2 Effort (Cronbach' s alpha = .83)		
B13	I am hard-working by nature	.86
B12	I have not worked hard enough	-.85
A 4	I have worked hard to learn mathematics	.64
B15	I always prepare myself carefully for exams	.62
B11	My attitude is wrong	-.55
B16	It is important for me to get good grades in mathematics	.30
F3 Teacher Quality (Cronbach' s alpha = .81)		
A27	I would have needed a better teacher	-.75
A 3	The teacher has not been able to explain the things we were studying	-.73
A21	My teacher has not inspired me to study mathematics	-.59
A26	My teacher has been a positive example	.57
A 6	The teacher has not explained what for we need the things we were learning	-.47
A 5	I have not understood teacher's explanations	-.45
A24	The teacher has hurried ahead	-.43
C10	If the teacher is too good in mathematics he or she can not explain clearly	-.41
F4 Family Encouragement (Cronbach' s alpha = .80)		
A17	The importance of competence in mathematics has been emphasized at my home	.83
A23	My family has encouraged me to study mathematics	.83
A18	The example of my parent(s) has had a positive influence on my motivation.	.67
F5 Enjoyment of Mathematics (Cronbach' s alpha = .91)		
A 7	It has been boring to study mathematics	-.71
A 8	Doing exercises has been pleasant	.70
C 1	Mathematics is a mechanical and boring subject	-.70
A13	To study mathematics has been something of a core	-.65

A25	I have enjoyed pondering mathematical exercises	.62
A12	Mathematics has been my favorite subject	.60
A22	Mathematics has been the most unpleasant part of studying	-.57
F6 Difficulty of Mathematics (Cronbach' s alpha = .82)		
C 4	Mathematics is difficult	.56
C 2	Learning mathematics requires a lot of effort	.560
A11	Mathematics has been difficult in high school	.53
F7 Confidence (Cronbach' s alpha = .87)		
B 2	I can get good grades in math	.81
B 1	I am sure that I can learn math	.80
B10	I know I can do well in math	.70
B 5	I think I could handle more difficult math	.38
B 9	I am sure I could do advanced work in math	.37

We obtained seven dimensions for students' view of themselves as learners of mathematics. Three factors relate to personal beliefs since a clear self-relation aspect regarding *competence* (F1), *effort* (F2) and *confidence* (F7) can be found. Thereby factor (F1) describes a more static view on abilities and competencies concerning doing mathematics while (F7) stands for a more dynamic one mentioning student's expectations about their future success. Two factors relate primarily to social context variables, namely *teacher quality* (F3) and *family encouragement* (F4), one to more emotional expressions concerning *enjoyment of mathematics* (F5) and one to mathematics as a subject; that is, difficulty of mathematics (F6).

4.2 The dimensions within students' view of mathematics

We further analyzed the dimensions themselves. Therefore, we defined subscales for each factor as being an unweighted sum of all of a student's score on any item loading on the factor¹. As there are different numbers of students participating from advanced courses (883 students) and general courses (430 students) we analyzed the dimensions within students' view of mathematics with regard to course choice. In this paper, however, we limit ourselves to only presenting results for the factor *compe-*

¹ Each dimension is described by a different amount of items therefore the absolute values vary accordingly to this. By a process of linear transformation we obtained a common scale for all factors varying from 0 to 50 whereby the range is from 0 - 10 for fully disagreement to 40 - 50 for fully agreement.

tence (F1); for more information we refer to Rösken et al. (in preparation). This first factor consists of 5 items with internal reliability² of .91. For the students of general courses the central tendency lies in the neutral range (mean = 24.33, std. error of mean = 0.7). Roughly 43% of those students regard themselves as not good in mathematics, 20% are undecided while 37% feel that they are good in mathematics. The frequency distribution is close to normal (figure 2). The standard deviation (14.31) indicates a high dispersion of scores from the mean.

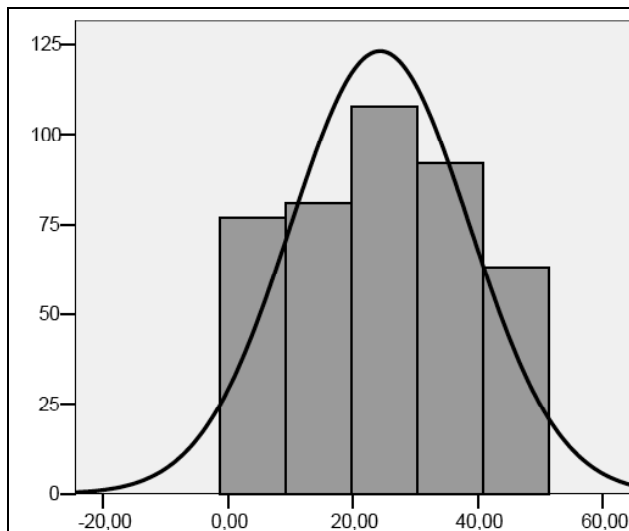


Figure 2: The dimension *competence* (F1) for general course students (score range from 0 - 10 for fully disagreement to 40 - 50 for fully agreement).

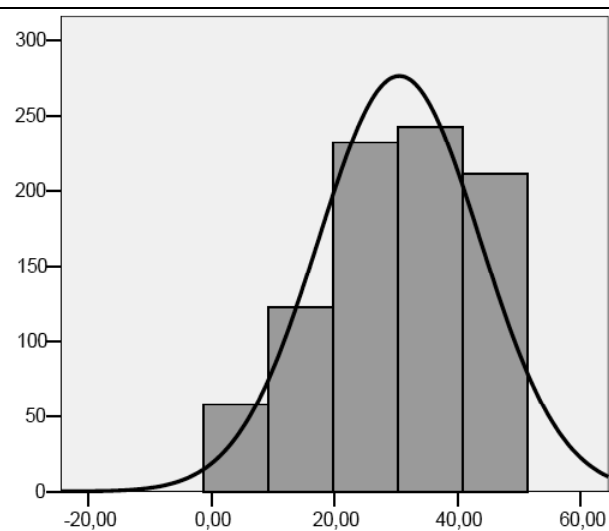


Figure 3: The dimension *competence* (F1) for advanced course students (score range from 0 - 10 for fully disagreement to 40 - 50 for fully agreement).

The situation for the students taking advanced courses is rather different. Here, central tendency for the factor *confidence* is located in the range of agreement (mean 30.51, std. error of mean .45, std. deviation 13.12). The histogram (figure 3) shows how the values are distributed along the dimension *confidence* (F1). Remarkably, more than half of advanced course students (52%) consider themselves as good in mathematics, only 25% as not good and 23% are undecided. The location parameters indicate that the distribution is slightly platykurtic (-.74).

² For group statements internal reliability is required to be higher than .7 (Kline, 1994).

5. DISCUSSION OF THE RESULTS

5.1 An *average* view of students with regard to course selection

Comparing the distributions for the dimensions as well as arithmetic means leads to an *average* students' view of mathematics. As the majority of students comes from advanced courses we analyzed for both groups mean scores separately (table 4).

Table 4: Group statistics, means and std. deviation for the factors with regard to course selection (score range is from 0 - 10 for fully disagree to 40 - 50 for fully agree).

Dimension	Course Selection	Mean (Std.Error)	Std. Deviation
<i>Competence</i> (F1)	General	24.33 (.70)	14.32
	Advanced	30.51 (.45)	13.12
<i>Effort</i> (F2)	General	20.91 (.53)	10.93
	Advanced	24.58 (.38)	11.17
<i>Teacher Quality</i> (F3)	General	26.51 (.44)	8.98
	Advanced	29.22 (.30)	8.86
<i>Family Encouragement</i> (F4)	General	18.57 (.53)	10.96
	Advanced	25.02 (.43)	12.68
<i>Enjoyment of Mathematics</i> (F5)	General	20.91 (.55)	11.14
	Advanced	28.22 (.38)	11.10
<i>Difficulty of Mathematics</i> (F6)	General	30.04 (.57)	11.74
	Advanced	32.25 (.39)	11.62
<i>Confidence</i> (F7)	General	27.48 (.54)	11.07
	Advanced	31.80 (.35)	10.23

The dimensions are differently relevant for students taking general or advanced courses. Students studying mathematics in general courses were undecided regarding their *competence* in mathematics (F1), their *confidence* (F7) and their *teacher quality* (F3). Further, they chose the neutral position close to disagreement for the dimensions *effort* (F2) and *enjoyment of mathematics* (F5) and they do not feel encouraged by their family (F4). The dimension *difficulty of mathematics* (F6) receives the highest agreement.

Advanced course students express great self-confidence, they feel able and competent to do mathematics (F1) and believe that they will be successful in the future (F7). Simultaneously, they experience mathematics as difficult (F6). All other means are located in the center of the scale; thereby the dimensions *teacher quality* (F3) and *enjoyment of mathematics* (F5) are close to agreement. A t-test indicated that the scores on the subscales are significantly different for students of general or advanced courses. Mean differences are large for *enjoyment of mathematics* (F5), *ability* (F1) and *encouragement by family* (F4). Those in advanced courses enjoy more to do

mathematics (F5), believe more in their competence (F1) and feel more encouraged by family (F4). However, both groups perceive the subject equally difficult (F6).

5.2 Relations between the dimensions

In the sections before we initially structured students view of mathematics. Furthermore, we are interested in the relations between dimensions, particularly in structural features generated by the obtained factors. We therefore calculated Pearson correlations³ for the factors F1 to F7 (table 5). Effect of course selection was checked for the correlations as well and could be ruled out.

Table 5: Correlations between the dimensions.

	F1	F2	F3	F4	F5	F6	F7
F1 <i>Competence</i>	1						
F2 <i>Effort</i>	.385**	1					
F3 <i>Teacher Quality</i>	.381**	.237**	1				
F4 <i>Family Encouragement</i>	.084**	.138**	.093**	1			
F5 <i>Enjoyment of Math</i>	.668**	.438**	.511**	.176**	1		
F6 <i>Difficulty of Math</i>	-.700**	-.226**	-.370**	.023	-.538**	1	
F7 <i>Confidence</i>	.791**	.250**	.348**	.119**	.577**	-.626**	1

Table 5 shows that nearly all dimensions correlate statistically significantly with each other. However, the strength of the correlation varies from *little if any* (size of $r = .00$ to $.29$) to *high* ($.70$ to $.89$). Further, the correlation matrix indicates that three of them (*competence* (F1), *difficulty of mathematics* (F6) and *confidence* (F7)) are more closely related because of a high correlation. That is, the more students feel competent to do mathematics the less they experience mathematics as difficult. These dimensions are crucial for view of mathematics and even form an obstacle for students when they do not feel able to do mathematics and simultaneously experience mathematics more and more difficult. In terms of Green (1971) we identify these three dimensions as held with greatest psychological strength and their spatial location, when using his metaphor of concentric circles, as based in a core circle. There is also moderate correlation between *enjoyment of mathematics* (F5) and this core; the correlations of the remaining factors are weak.

³ Correlation Matrix: ** Correlation is significant at the .01 level (2-tailed).

6. CONCLUSIONS

The present study supports previous research on mathematics-related beliefs structure. In our analysis we found evidence for an extended conceptualization of beliefs as a system. By means of factor analysis we determined useful and statistically robust dimensions of students view of mathematics, which could be assigned to all main categories and several subcategories of the framework provided by Op 't Eynde et al. (2002). However, the factor *enjoyment of mathematics* (F5) consists of more emotional expressions and is not covered by one of these categories. The dimensions are of different relevance for students whereby a clear effect of course selection (general mathematics or advanced mathematics) could be found.

When we studied the relations between dimensions, we were able to identify a core of beliefs (Green, 1970) with a large value of covariance. This core consists of the dimensions *confidence* (F1), *difficulty of mathematics* (F6) and *competence* (F7). Additionally, there is a moderate correlation of *enjoyment of mathematics* (F5) to this core. A student with a positive view believes him or herself to be competent to do mathematics, to be successful in the future, experiences mathematics as easy, and further he or she also enjoys mathematics. Significantly more students with such a positive view could be found among the ones taking advanced courses. Considering the correlations between dimensions we additionally employed a second order analysis to test whether the correlations among the first-order factors could be accounted for in terms of second-order factors. This analysis led to a first factor that confirms what we called the core of beliefs; we did not find a second factor that could be explained theoretically plausible.

Our findings support the model for describing students' view of mathematics found when analyzing data of elementary teacher students (Hannula et al., 2005, 2006). For identical items in both populations we found the same factor structure and reliability analysis confirmed excellent internal consistency of factors. We are now able to compare both samples by means of a valid and reliable instrument.

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STUDENT ERRORS IN TASK-SOLVING PROCESSES

Wolfgang Schlöglmann
 Institut für Didaktik der Mathematik
 Universität Linz
 A – 4040 Linz
 Austria

Errors are a permanent companion to human thought and action. Particularly in mathematics, student exercise or problem-solving processes often contain errors. While these errors may originate in misconceptions, we also identify errors that we call "slips". To explain the emergence of slips in processes of solving mathematical exercises or problems, we use the concepts of working memory and workspace.

Description of the problem

In his paper “Aspects of the nature and the state of research in mathematics education” (Niss, 1999), Mogens Niss underlines the dual nature of the didactics of mathematics – its descriptive aspect (what is the case?) and its explanatory aspect (why is it so?). In this paper, we address the problem of errors in task- and problem-solving processes by listing two tasks given to students undertaking a first-year university analysis course, together with a small selection of common, erroneous solutions.¹ The aim of the paper is the theoretical understanding of why errors emerge in student problem-solving processes.

The two tasks presented to the students, together with the erroneous solutions, are as follows.

Task 1: Sketch the following subset of the complex plane: $S = \{z \in \mathbb{C} : |2z - 9| \leq |z + 6i|\}$.

Erroneous solution:

$$|2z - 9| \leq |z + 6i|$$

$$|2(x + iy) - 9|^2 \leq |x + iy + 6i|^2$$

$$|2x + 2iy - 9|^2 \leq |x + iy + 6i|^2$$

$$(2x - 9)^2 + 2y^2 \leq (x - 6)^2 + y^2$$

$$2x^2 + 9^2 + 2y^2 \leq x^2 + 6^2 + y^2$$

$$x^2 - 3^2 + y^2 \leq 0$$

$$x^2 + y^2 - 9 \leq 0$$

Task 2: Given $f = \frac{y}{x^2 + y^2}$, calculate $\frac{\partial f}{\partial x}$.

First erroneous solution: $\frac{\partial f}{\partial x} = -\frac{1}{x^2}$

¹ I thank my colleague Ewald Lindner for helping me find appropriate examples to demonstrate the various kinds of error.

Second erroneous solution: $f = \frac{y}{x^2 + y^2} = \frac{1}{x^2 + y} \Rightarrow \frac{\partial f}{\partial x} = -\frac{1}{y^2}$

The erroneous solutions in these two tasks reveal two distinct kinds of error. In the first task, the student appears to grasp the main concept – which is distance in the complex plane – but commits a mistake when squaring the expressions in parentheses: the student correctly applies a memorized "trick" when squaring the modulus expressions, but then erroneously applies the same trick to the expressions in parentheses. This is a very common mistake (Malle, 1993); nevertheless it is a careless one and we should certainly expect a university mathematics student to be able to take the square of a given expression if given just that problem to solve in isolation.

On the other hand, the solution process for the second task reveals that this student's conception of partial differentiation is lacking and that for this reason the student is unable to complete this task correctly.

Thus we ought to distinguish between two kinds of error:

- 1) Errors that are based on the incorrect application of a formula, or simple mistakes in a calculation. These errors would not appear if the student were to use the formula or do the calculation in isolation rather than as a steppingstone in a more complex calculation. Such errors are often called “careless mistakes”, or “slips”.
- 2) Errors whose origin is a misconception or inadequate understanding of the mathematical problem.

It is often difficult to decide which category an error belongs to. In many cases one needs more information about the problem solver, his or her learning history, performance or mark in mathematics and so on.

In the present paper, we try to understand why slips occur. In earlier papers, as a first step (Schlögmann, 2004, 2005), we analyzed this problem from the perspective of routine processes and the role of attention in these processes. It seems that this explanation needs to be extended to take into consideration a deeper understanding of problem-solving processes. In the present paper, therefore, we use the concepts of “working memory” and “workspace” to understand the problem of shared attention and the influence of affect on the problem-solving process.

The concept of “working memory”

This section is based on papers and books by Baddely (2003a,b), LeDoux (1998, 2002) and Stern et al. (2005).

In cognitive processes such as a conversation, thinking about a problem, carrying out a calculation in one's head or solving a problem on paper, we can identify subprocesses that are necessary for the process's success. First, the relevant information must be identified and at least temporarily stored. Furthermore, this information must be available for manipulation, and for interpretation, in the light of

knowledge or experiences stored in the long-term memory. The results of these processes should be suitable for both communication in verbal or written form as well as for storage in the long-term memory. Therefore a cognitive process needs brain systems that allow short-term storage and manipulation of (aural or visual) information; that can access the long-term memory; and that are able to communicate the results. All these processes require overall control and monitoring.

In 1974, Baddely and Hitch proposed a three-component model of working memory; their model was subsequently extended to include a fourth component and is now referred to as a multi-component-model. The extended model is widely used today as a viable model for studying and analyzing cognitive processes.

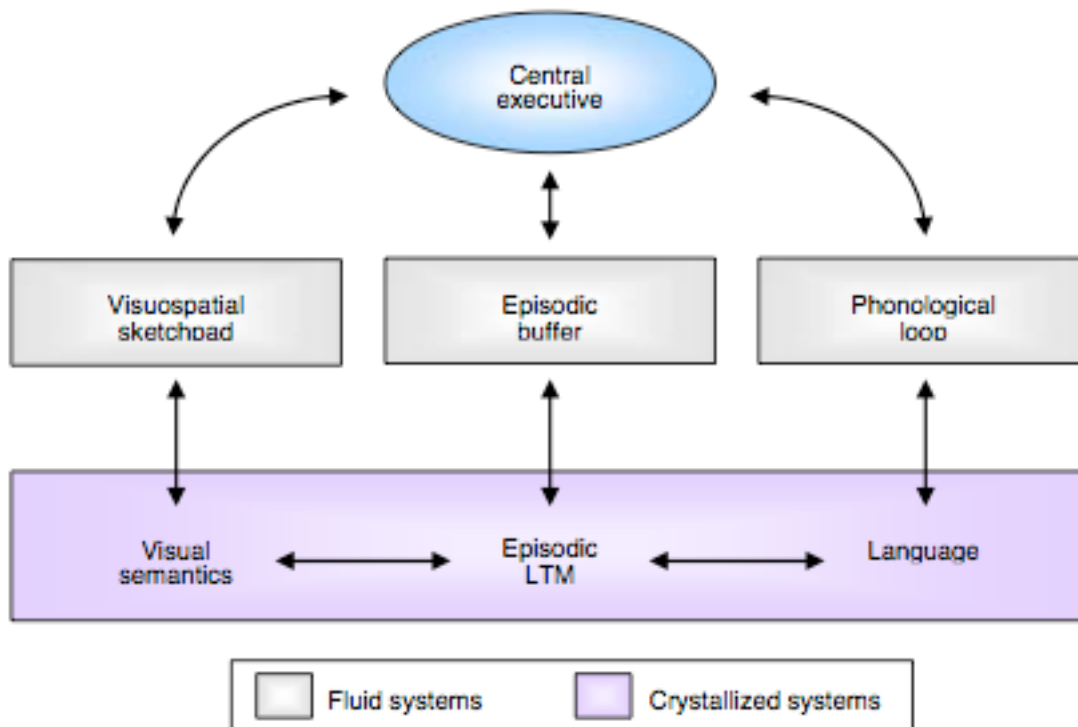
The multi-component-model of the working memory consists of two storage systems – the phonological loop and the visuospatial sketchpad (these are also referred to as “slave systems” because they cannot work independently); the central executive which acts as the overall control and executive system; and the episodic buffer.

The phonological loop is specialized for temporarily storing verbal and acoustic information. The visuospatial sketchpad is equivalent to the phonological loop but stores visual and spatial information. Both the phonological loop and the visuospatial sketchpad have a limited capacity for information storage.

Regarding the central executive, Baddeley writes: *“The central executive is the most important but least understood component of the working memory”* (Baddeley 2003a, 835). The reason for the difficulty lies in the complexity of the concept of working memory: not only does it function to provide temporary storage (as did the old concept of short-term memory), but also information must be manipulated within it. This means not only the relevant content, but also the aim of the cognitive process, must be present within it. Furthermore, all mental operations must be controlled with respect to their efficiency and usefulness with respect to the aim of the cognitive process. These conditions require that attention must be concentrated selectively on the relevant and manipulated content independent of whether the content originates from inside or from outside the central executive.

The central executive together with both slave systems – the phonological loop and visuospatial sketchpad – formed the classical three-component model of the working memory proposed in 1974. However, problems with this model lay in addressing the interaction of the short-term memory with the long-term memory; in its inability to describe aggregation of information, called “chunking”; in its not allowing the phonological and visuospatial subsystems to interact; and in its not offering mechanisms for the role of the working memory in conscious awareness. “To account for these and other issues, a fourth component was proposed – the episodic buffer. This is assumed to be a limited capacity store that binds together information to form integrated episodes. It is assumed to be attentionally controlled by the executive and to be accessible to conscious awareness. Its multi-dimensional coding allows different systems to be integrated, and conscious awareness provides a convenient binding and retrieval process.” (Baddeley, 2003a, 836)

Baddely condensed the ideas of the concept of the multi-component model of working memory into a single figure (Baddeley 2003a, 835), which is presented below.



In this figure, the systems on the lower level represent long-term – or “crystallized” – knowledge, and the episodic buffer provides an interface between the working memory and the long-term memory subsystems.

Furthermore, the buffer has an important function within the model, particularly with respect to awareness. In this regard, Baddeley writes: “*The buffer is therefore regarded as a crucial feature of the capacity of working memory to act as a global workspace that is accessed to conscious awareness.*” (Baddeley, 2003a, 836)

This concept of workspace and its significance to awareness was pointed out by S. Dehaene and L. Naccache (2001) and is founded on three major empirical observations: “(1) a considerable amount of processing is possible without consciousness; (2) attention is a prerequisite of consciousness; and (3) consciousness is required for some specific cognitive tasks, including those that require durable information maintenance, novel combinations of operations, or spontaneous generation of intentional behavior.” (Dehaene and Naccache 2001; 1)

The workspace proposed by Dehaene and Naccache is not an isolated region in the brain, but a distributed neural system with long-distance connectivity that can potentially interconnect multiple specialized brain areas. These areas do not automatically exchange information: they do so only if a particular task requires such communication. “*The global workspace thus provides a common ‘communication protocol’ through which a particularly large potential for the combination of multiple*

input, output, and internal systems becomes available.” (Dehaene and Naccache 2001; 13)

It is now of interest to ask: Which modular systems participate in the workspace? For Dehaene and Naccache, at least five systems must participate in the workspace: “perceptual circuits that inform about the present state of the environment; motor circuits that allow the preparation and controlled execution of actions; long-term memory circuits that can reinstate past workspace states; evaluation circuits that attribute them a valence in relation to previous experience; and attentional or top-down circuits that selectively gate the focus of interest... In particular, connections to the motor and language systems allow any workspace content to be described verbally or non-verbally (‘reportability’).” (Dehaene and Naccache 2001; 14)

A central element of the workspace concept is attention because “*top-down amplification is the mechanism by which modular processes can be temporarily mobilized and made available to the global workspace, and therefore to consciousness*” (Dehaene and Naccache 2001; 14).

Neuroscientific results on error detection and compensation

The multi-component model of working memory contains the central executive as “the most important but least understood component of the working memory” (Baddeley 2003a, 835). According to the model, the central executive has to monitor and control the execution of a cognitive process for both efficiency and correctness. But “*a fundamental characteristic of human cognition is its fallibility*” (Gehring et al. 1993, 385). Therefore cognitive theories often also include concepts of error monitoring. These theories distinguish two types of errors: slips, which are the incorrect execution of an appropriate motor program; and mistakes, which involve the selection of inappropriate intentions based, for instance, on faulty knowledge (Dehaene, Posner and Tucker 1994, 304; Posner and Dirigolamo 2000, 628).

In his book “Number Sense”, Dehaene (1997, 223 – 227) describes an experiment that hints at error correction. In this experiment, subjects were presented with either Arabic digits or number words. The subjects were required to press one key with one hand if the target was larger than 5 and another key with the other hand if it was smaller than 5. The event-related potentials were recorded by 64 electrodes spread out on the scalp. Special software enabled the reconstruction, frame by frame, of the evolution of the surface potentials in the various conditions of the experiment.

The measurement of the event-related potential started when the number appeared before the subject’s eyes. The electric potential remained close to zero for several tens of milliseconds. At around 100 milliseconds, a positive potential appeared that indicated the activation of visual areas. At this stage no differences were apparent between Arabic digits and number words. Divergence could be seen at between 100 and 150 milliseconds if other halves of the brain were activated (for example, words in the left hemisphere and digits in both hemispheres). At around 190 milliseconds, the first indication appeared that the size of the number was being encoded.

(Interestingly, the comparison process led to a distance effect, namely those digits closer to 5 – and therefore more difficult to evaluate – generated an electrical potential with an higher amplitude.) At about 330 milliseconds, the programming and execution of the manual response began. The subject's response occurred on average after 400 milliseconds.

However, this was by no means always the end of the process. If the individual made a mistake – for instance, incorrectly anticipating the number – immediately afterwards a negative electrical signal of great intensity suddenly appeared at the electrodes at the front of the skull. It is important to bear in mind that no such signal was found following a correct response. Furthermore, this error-related negativity appeared in such a short time frame (70 milliseconds) after the incorrect manual response that it cannot be a consequence of sensorial or proprioceptive information: it must have come directly from the detection of the incorrect reaction inside the brain. If we consider that the cognitive task is very easy to solve, the error can be classified as a slip.

Other interesting, related observations have subsequently been made. First, tests indicate that the intensity of the error-related negative signal is related to the importance of the error to the individual (Gehring et al. 1993, 389). Second, error-related negativity is only observed after slips and not after mistakes (Dehaene, Posner and Tucker 1994, 304; Badgaiyan and Posner 1998, 256). Third, the system that is responsible for error detection and compensation is located in the anterior cingulate cortex (Carter et al. 1998), a region that plays a prominent role in the executive control of cognition and shows “*activity during tasks that engage selective attention, working memory, language generation, and controlled information processing*” (Carter et al. 1998, 747).

Errors and Affect

With regards to the concepts of working memory and workspace, we find two crucial features that have a great influence on the handling of cognitive processes by the brain:

1. All components of the working memory – the storage systems, the episodic buffer, as well as the central executive – are restricted in their capacity.
2. One must keep in mind that attention is a crucial prerequisite for all conscious processes, and therefore for cognitive processes, too.
3. In our view, attention is also a crucial prerequisite for a “correctly functioning error-detection system”, in the sense that attention must be strongly directed at the task as well as at the solution process of each step of the task. This is born out by the fact that the intensity of the error-related negative signal depends on the importance ascribed by the individual to solving problems flawlessly. The importance of an aim is strongly related to the attention that is directed towards the process. But if an individual also concentrates on other aims such as the speed of the solution process, the error-related negative signal is not as strong.

In the following, we will argue that lack of attention is associated with the emergence of errors – in particular, slips – and invoking the concepts of working memory and workspace, we shall consider the influence of affect on attention as well as on the storage capacity of the components of the working memory.

In order to consider errors that in colloquial language are often referred to as “slips”, one must have a clear view regarding the role of attention during a task or problem-solving process. A slip is characterized as an error that an individual would not make if they carried out some task in isolation, committing the error for reasons such as time pressure or lack of attention. Such errors are familiar to anybody; for instance, when writing some text by hand. In this case, the motor execution of writing is behind our thoughts, and we sometimes make an error in the execution of writing by substituting a letter from the thoughts with a letter of the word that we want to write. Our error-detection system often works correctly and directs our attention to the incorrectly written word. Indeed, in neuropsychology, slips are defined as the incorrect execution of an appropriate motor program (Dehaene, Posner and Tucker 1994, 304; Posner and Dirigolamo 2000, 628).

Regarding the first mathematical task (Task 1) given to analysis students, we recognize that an algebraic formula (or memorized "trick") is applied incorrectly at a crucial step. At this level, the use of the algebraic formula must be seen as a routine process, and routine processes are usually characterized by the low level of attention necessary to run the process (Roth 2001, 229). It seems that routine processes in mathematical problem-solving processes require a higher level of attention than other routine processes such as walking or cycling. If we analyze the process of solving complex mathematical tasks, on the one hand the subject's attention must be directed at the goal of the entire process – which is to produce a correct sequence of steps – and on the other hand the subject must also focus on the correct application of, for instance, a routine formula at certain steps. That means the student's attention is divided between the heuristic and metacognitive parts of the solution process, and is directed at both the process in its entirety as well as at the cognitive processes concerned with the requirements of a special step within the process. If in such a situation the switch of attention between the process in its entirety and the demands of a single step within the process is not carried out correctly, the solution process is susceptible to errors such as slips. Moreover, the error-detection system is often unaware of slips because the attention necessary for the correct operation of the system is unavailable.

Up to now we have discussed the problem of slips and identified divided attention as a crucial element. We now extend the discussion to address the influence of affect on the perpetration of slips. The influence of affect on learning, thinking and acting is discussed in many research papers (see for instance McLeod 1992, McLeod and McLeod 2002, Goldin 2002, Furinghetti and Pehkonen 2000, Evans, Hannula, Philippou and Zan 2004, Evans 2000, Wedege and Evans 2006). Hannula (1998) developed a model for the dynamics of affect by distinguishing between latent

representations and overt representations. Latent representations are cognitive or affective representations that may be potentially activated, whereas overt representations are activated during learning, thinking or acting processes ("at the moment"). In the context of the working memory and workspace model, overt representations are representations that are occurring right now in the slave systems, in the episodic buffer, as well as partly in the central executive if we also take into consideration the planning and execution of a cognitive process. Moreover, however, the workspace has access to systems in the background that are not activated at the moment but can be activated if this were necessary for the continuation of the process. (In her doctoral thesis, Malmivuori (2001) describes the dynamic intersection of affect, cognition and social environment in the regulation of the individual learning process, and gives a perspective on the process of construction of systems that could be activated in a learning process, developing their effect on learning and thinking.)

All the concepts described or cited in the above discussion give important insights into the long-term processes and cognitive and affective representations that determine the dynamics of learning processes; nevertheless, we need more insight into the effect of emotions during problem solving in order to explain the emergence of errors, particularly slips. We know that fear can lead to learning blockades and in general increases the possibility of failure and low performance in test situations (von Aster 2005, 30; Evans, 2000). This could be a consequence of the activation of brain and body systems through emotions, since humans are in many cases aware of this activation, and *"You can't have a conscious emotional feeling of being afraid without aspects of the emotional experience being represented in the working memory"* (LeDoux 1998, 296). This implies that emotional feelings are represented in the working memory and workspace and, furthermore, that they are strongly linked to attentiveness (Matthews and Wells 1999, 171; LeDoux 1998, 289). In the light of the limited capacity of the working memory, we can imagine that there might not be enough capacity for the facts that are necessary to support the cognitive process; and that attention during the cognitive solution process is directed more towards emotions and less towards the cognitive and heuristic process components. It is understandable that in such a situation, the number of errors, particularly slips, increases. Bearing in mind that many students have represented negative attitudes toward mathematics in their memory and feel fear during mathematics lessons, students must fight against the consequences of their feelings. In a bid to control the negative emotions, such reactions (see Goldin's concept of meta-affect (Goldin 2002)) occupy capacity and divert attention from learning or problem-solving processes, making successful learning less likely and decreasing the level of performance in tests.

We conclude with a few comments on the problem of the "inverted U" in the relationship between anxiety and performance (Evans 2000, 47). The "inverted U" refers to the empirical observation that a low level of anxiety leads to better performance, while a performance decrease begins at a certain higher level of

anxiety. We know that emotion and attentiveness are intimately linked, and that sufficient attention is an important precondition for successful cognitive processes. Therefore a low level of emotional arousal could lead to the activation of attention, as well as the motivation, to overcome a test situation successfully (Goldin 2002, 63): it might aid successful problem solving. An interesting possibility would be to explore whether anxiety scales working with cognitive beliefs and attitudes can distinguish between anxiety as described by Goldin (2002, 63) and real anxiety.

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INFLUENCE OF DIDACTICAL GAMES ON PUPILS' ATTITUDES TOWARDS MATHEMATICS AND PROCESS OF ITS TEACHING

Peter Vankúš

Faculty of Mathematics, Physics and Informatics, Comenius University, Bratislava

Our article speaks about research on influence of didactical games on pupils' attitudes towards mathematics and its teaching. The aim of the research was to study how used didactical games influenced pupils' attitudes and knowledge from taught theme. Research was experimental, realized on the sample of 103 pupils, 11–12 years old. In the article we describe used didactical games, methodology of research and its results. Conclusions of the research are important for the use of the didactical games in teachers' practice.

INTRODUCTION

In studies that analysed efficacy of teaching is highlighted necessity of active pupils' work on mathematics lessons. Used activities should attract pupils' attention and motivate them. Also they should positively influence pupils' emotions, attitudes and beliefs linked with mathematics and its teaching and develop various needed skills and competencies. In compliance with this is attention of some researchers focused on didactical games as a promising educational method fulfilling aforementioned criteria. These researchers study psychological effects of didactical games, their influence on pupils' mathematical knowledge and also changes in many other factors in the environment of classroom (see Brooker, 2000; Vankúš, 2005). There are collections of didactical games directly appointed for the use on mathematics lessons based on aforementioned work of researchers and also on creativity of some teachers. Also educational math software use didactical games as essential part (Slavíčková, 2006). For the need of school practice are therefore needed research studies dealing with influences of specifically chosen didactical games on educational environment.

In this article we present our research on influence of some didactical games on pupils' attitudes towards mathematics and process of its teaching. The research had experimental character and was conducted in two phases. The first phase took place in school year 2002/2003, the second in school year 2004/2005. In both phases experimental samples were two classes of the fifth grade of primary school, one of them was experimental, the second one control class. Together 103 pupils took part in our experiment, in the first phase experimental class had 25 pupils, control class had 26 pupils and in the second phase both classes had 26 pupils. In the experimental classes we taught with integration of didactical games into mathematics education, in control classes without them. Research questions were:

- Q1 Would used didactical games positively influence pupils' attitudes towards mathematics and its teaching?
- Q2 How would didactical games affect pupils' knowledge from mathematical theme taught during experiment?

Answers on these questions were sought by methodology that is described in the next part of this article.

DESCRIPTION OF RESEARCH

First of all we clarify notion "didactical game". For the need of our research we consider as didactical game any educational activity performed by pupils that develops them and brings them pleasure and happiness. The main differences between normal meaning of children's game and between the didactical game are:

- Children's game is totally free, in didactical game all pupils have to participate.
- Didactical game is used to realize chosen educational goals; the main aim of children's game is just fun and pleasure.
- Didactical game has its external management (teacher).

Very comprehensive description of didactical game gives this quotation (Průcha, Walterová and Mareš, 1998, p. 48, translated by author):

Didactical game: Analogy of spontaneous children's activity, which realize (for children not every time evidently) educational goals. Can take place in classroom, sport-hall, playground, or in the nature. Each game has its rules, needs continuous management and final assessment. It is suitable for single child either for group of children. Teacher has various roles: from main coordinator to an onlooker. Its advantage is motivational factor: it raises interest, makes higher children's involvement in teaching activities, and encourages children's creativity, spontaneity, co-operation and also competitiveness. Children can use their knowledge, abilities and experience. Some didactical games approach to model situations from real life.

There were researches dealing with didactical games used in the teaching of mathematics (Bright, Harvey and Wheeler 1985; Randel, Morris, Wetzel and Whitehill 1992; Pulos and Sneider 1994; Brooker 2000; Vankúš 2006 etc.) When we summarize results of these researches we can make conclusion that proper didactical games increase efficacy of mathematics educational process. The most important is increase of pupils' inner motivation and also some improvement of pupils' mathematical knowledge. Very important are also positive changes in pupils' attitudes towards mathematics those are the theme of our article.

Now we describe didactical games used during our experiment. As was already said experiment took place in the fifth grade of elementary school (pupils 11–12 years

old). Taught theme was Area of square and rectangle. During this theme children learn:

- Determine area of object in square grid (see fig. 1).

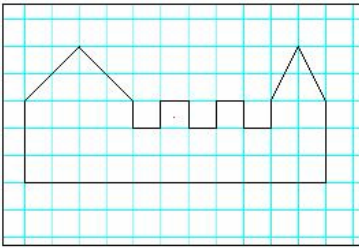


Fig. 1 Task was to determine area of castle in square grid

- Units of area measurement and their conversion.
- Area of square and rectangle (also formulae).
- Solving of tasks using aforementioned knowledge.

Used didactical games were Circles (area of object in square grid), Domino (units of area measurement and their conversion), Cipher (area of square and rectangle) and Bingo (solving of complex tasks using knowledge about area of square and rectangle).

Because of lack of space we give just short characteristic of each game. The Circles was game for teams of pupils (4–6 pupils each team). Task was to determine area of objects in square grid, printed on circles of paper. For each correct result team got certain number of points, it varied from difficultness of object. Team with the biggest number of points won, but each team got some reward.

The Domino was game for two players, similar to standard Domino game. But dominoes were covered with values of area with different units, so players put together dominoes with the same value of area although it was displayed in different units (see fig. 2).

1 m ²	500 cm ²	5 dm ²	2 cm ²	200 mm ²	10 dm ²	0,1 m ²	3 cm ²	1 000 000 m ²	0,000 3 m ²
0,04 dm ²	100 dm ²	4 cm ²	2 dm ²	0,02 m ²	1 ha	100 a	0,000 002 m ²	2 mm ²	1 km ²
1 m ²	4 cm ²	0,04 dm ²	1 000 000 m ²	1 km ²	3 cm ²	0,000 3 m ²	200 mm ²	100 a	2 cm ²
500 cm ²	100 dm ²	5 dm ²	0,02 m ²	2 dm ²	0,1 m ²	10 dm ²	0,000 002 m ²	2 mm ²	1 ha
0,000 002 m ²	100 dm ²	1 m ²	0,04 dm ²	4 cm ²	1 ha	100 a	2 cm ²	2 mm ²	3 cm ²
0,000 3 m ²	10 dm ²	0,1 m ²	2 dm ²	0,02 m ²	1 000 000 m ²	1 km ²	500 cm ²	200 mm ²	5 dm ²
1 000 000 m ²	100 dm ²	1 m ²	2 mm ²	0,000 002 m ²	500 cm ²	5 dm ²	100 a	1 ha	2 cm ²
1 km ²	4 cm ²	0,04 dm ²	0,000 3 m ²	3 cm ²	0,01 m ²	10 dm ²	2 dm ²	0,02 m ²	200 mm ²

Fig. 2 Dominoes used in game

The Cipher was game were cooperated pairs of pupils. They received list of twenty-three simple tasks and paper with coded text. The tasks were focused on the use of formulae for computing area of square and rectangle; each task had different result (different number value or unit of area measurement). One letter of alphabet was written after each task. When pupils solved all tasks they began decoding of the text. It represented substitution of value of area measurement for the letter of alphabet that was written by the task of which was given value the result. The task of game was to decode text that was a riddle and to give right answer to this riddle. Pupils were rewarded for every right decoded letter and also for right answer to the riddle.

The Bingo was game similar to standard Bingo game. Pupils worked in pairs. They received paper with Bingo card, dimension 3 x 3. They filled it with numbers from one to twelve. Each pair then received list of 12 tasks focused on using of knowledge from taught theme. After a task was solved and teacher checked the result pupils could mark the cell where was written the number of this task. For each of patterns: straight line across, straight line down, diagonal and coverall pupils received some points. The task of game was to receive the most points (to solve all mathematical tasks).

To find out answers to our research questions we used these methods:

M1 Questionnaire constructed to measure pupils' attitudes towards mathematics and process of its teaching (appendix 1). This questionnaire is based on questionnaire used by R. F. Mager (Mager, 1984) and concept of attitudes as defined in Ruffell et al., 1998. More about used concept of attitudes see in Schlöglmann, 2003; Zan and Martino, 2003. The questionnaire was used twice, at the beginning of experimental teaching and at the end. Results of questionnaires were compared, thus we could say how the using of didactical games influenced studied indicators.

M2 Test to measure knowledge from taught theme. Test was created by standard procedure (see Turek, 1996). It contains 14 tasks dealing with knowledge and computing skill from taught theme. Results of test were compared to results of control class where pupils were taught the same theme but without didactical games. (Classes were similar in number of pupils, in both classes taught the same teacher.)

M3 To observe pupils' reactions on didactical games we studied pupils' behaviour during whole experiment and performed some interviews.

RESULTS OF EXPERIMENT

On the basis of used research methods we can assumed following.

There was increase in questionnaires' score in experimental classes (see fig. 3)

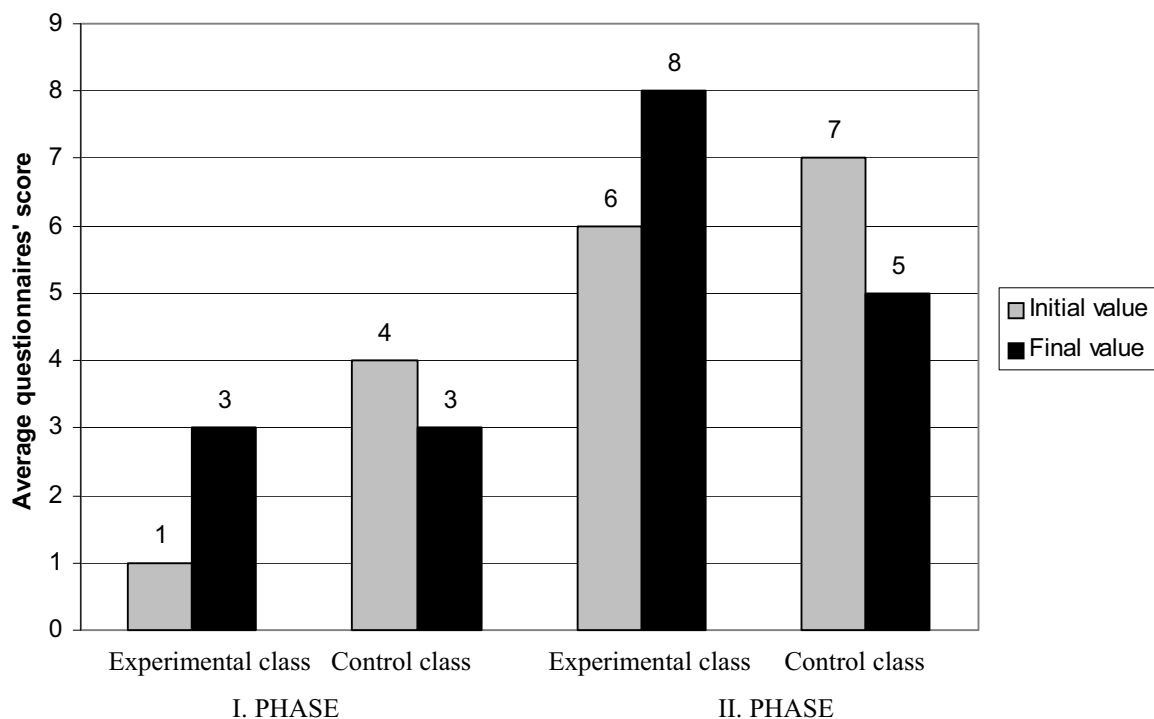


Fig. 3 Average score of questionnaires

The increase is significant especially when we consider that in the control classes we observed decrease in questionnaires' score. This decrease was likely due to difficulty of taught theme for pupils. In the control classes pupils regarded this theme as not so interesting. But in the experimental classes where we used didactical games students after experiment had according to results of questionnaire better attitudes towards mathematics and its teaching. By the means of statistical analysis we analysed differences in the initial values and final values of the average scores in the items of questionnaire (by the use of paired 2 tailed Student t-test) and we found out that these differences are statistically significant (in the first phase $\alpha = 0.05$, in the second phase $\alpha = 0.005$)

The knowledge from theme taught during experiment we compared on the basis of test results (see fig. 4 and fig. 5).

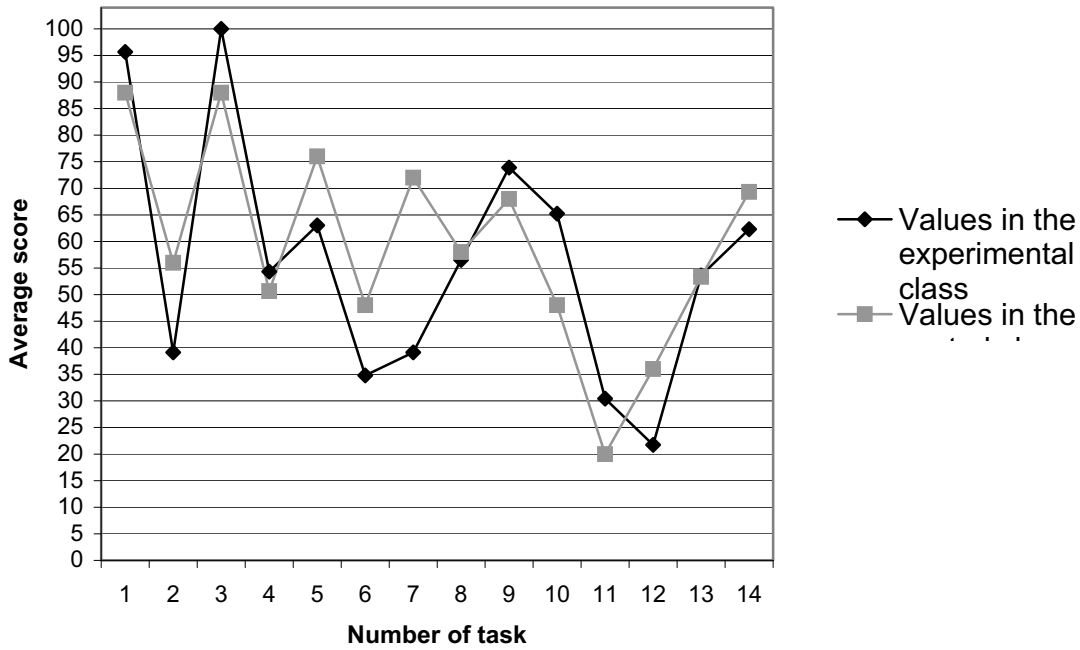


Fig. 4 Average proportional score in the tasks of the test in the first phase of the experiment

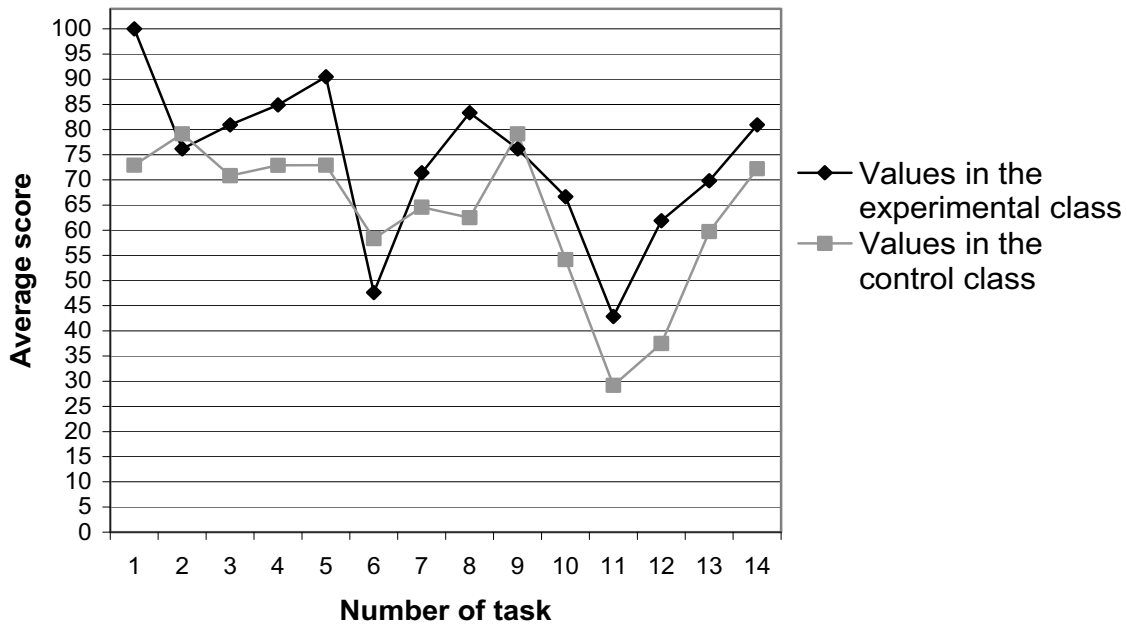


Fig. 5 Average proportional score in the tasks of the test in the second phase of the experiment

Although average proportional score in the tasks varied used statistical test (Student's 2 tailed unpaired t-test) showed that the differences in the results of test in the experimental and the control class are in both phases statistically insignificant. Using of our didactical games did not change pupils' knowledge in comparison with mathematics teaching without these didactical games.

CONCLUSION

This article refers about research on influence of some didactical games on pupils' attitudes towards mathematics and its teaching. Results of used research methods showed that there was really increase in the score of questionnaire that measured pupils' affection for mathematics. The knowledge from mathematics theme taught during experiment did not worsen in comparison with classes where we did not use the didactical games. So we can assume that our didactical games are proper for the use in schools practice.

Results of our research are limited by the number of pupils those participated and also extant of mathematics theme where we used the didactical games. So for the future we are planning enlarge our research and thus make it statistically more relevant. We have arranged cooperation with group working on mathematics-related beliefs questionnaire (MRBQ) developed at the Catholic University of Leuven (Op 't Eynde and De Corte, 2003; Diego-Mantecón, Andrews and Op 't Eynde, 2007) so we are going used this tool to measure students attitudes towards mathematics and its teaching. We are also working on tools in order to measure changes in pupils' generic skills. On the basis of these tools we can study complex asset of didactical games for teaching and so judge efficacy of teaching mathematics with the didactical games.

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APPENDIX N. 1:

Questionnaire

Dear pupils, in these questions you have possibility to express your attitudes towards mathematics and its teaching. Questionnaire is anonymous, you don't subscribe yourselves. Your answers will be used as a part of research. Please, express openly your opinions.

Thank you very much for filling in the questionnaire.

1) Imagine that you are teacher. Which of following subjects would you like to teach the most?

- a) Languages
- b) Geography
- c) Mathematics
- d) Physics
- e) Natural science
- f) Other (write which):

Score: For marked answer c) 2 points, other answers for 0 points.

2) Subject mathematics is for you

- a) Very interesting
- b) Interesting
- c) Sometimes interesting, sometimes not interesting
- d) Uninteresting
- e) Very uninteresting

Score: For answer a) 2 points, b) 1 point, c) 0 points, d) -1 point, e) -2 points.

3) Encircle every word from following that describes your attitudes towards mathematics.

- | | | |
|----------------|--------------|-----------------|
| a) Interesting | b) Boring | c) Worthless |
| d) Monotonous | e) Useful | f) Entertaining |
| g) Easy | h) Important | i) Useless |
| j) Valuable | k) Difficult | l) Unimportant |

Score: For answers a), e), f), g), h), j) 1 point; for answers b), c), d), i), k), l) -1 point.

4) Can you remember some activities linked with mathematics that you like?

a) Yes

(Write them).....

b) No

Score: For each liked activity 1 point.

5) Which mark from mathematics did you have on your last school report?

a) 1

b) 2

c) 3

d) 4

e) 5

6) You look forward to having mathematics lesson:

a) Always

b) Often

c) Sometimes

d) Rarely

e) Never

Score: For answer a) 2 points, b) 1 point, c) 0 points, d) -1 point, e) -2 points.

INTRINSIC AND EXTRINSIC MOTIVATION VERSUS SOCIAL AND INSTRUMENTAL RATIONALE FOR LEARNING MATHEMATICS

Kjersti Wæge

Norwegian University of Science and Technology

This paper presents and discusses the relation between two different concepts of motivation for learning mathematics: intrinsic and extrinsic motivation as defined in Self Determination Theory and Mellin-Olsen's concept of rationale for learning mathematics. When presenting the two frameworks the author gives examples from her own study in mathematics education. Within Self Determination Theory one suggests that extrinsic motivation varies considerably in its relative autonomy and thus can either reflect external control or true self-regulation. Their detailed description of different forms of motivation makes it possible to discuss the I- and S-rationale for learning mathematics in relation to extrinsic and intrinsic motivation.

INTRODUCTION

Theories of motivation are created to help us explain, predict and influence behaviour. Within psychology, the research field of motivation is enormous. One important approach to motivation has been to distinguish between intrinsic and extrinsic motivation (Deci & Ryan, 1985). In mathematics education there has not been done much work on motivation to date (See Evans & Wedege, 2004; Hannula, 2004b), and only a few researchers have distinguished between intrinsic and extrinsic motivation. Holden (2003) makes a distinction between intrinsic, extrinsic and contextual motivation. She suggests that the students' motivation always is governed by some kind of "rewards". According to her, students who are extrinsically motivated engage in tasks to obtain extrinsic rewards, such as praise and positive feedback from the teacher. The students' intrinsic motivation is governed by intrinsic rewards, which concern developing understanding, feeling powerful and enjoying the task. Students who are contextually motivated are doing something to obtain contextual rewards, such as acknowledgement from peer students, working with challenging tasks and seeing the usefulness of the task. Goodchild (2001) relates extrinsic and intrinsic motivation with ego and task orientation and with performance and learning goals. According to him a student is extrinsically motivated when he is doing something because it leads to an outcome external to the task, such as gaining approval or proving self-worth. A student is intrinsically motivated when he considers the task to have a value for its own sake; he is engaging in the task in order to understand. Evans and Wedege (2004) consider people's motivation and resistance to learn mathematics as interrelated phenomena. They present and discuss a number of meanings of these two terms as used in mathematics education and adult

education. In Middleton and Spanias' (1999) review of research in the area of motivation in mathematics education, intrinsic motivation is defined as the student's desire to engage in learning for its own sake. According to them extrinsically motivated students are doing something to obtain rewards, such as good grades or approval, or to avoid punishment. In Hannula's dissertation his approach to motivation involves needs and goals, rather than intrinsic and extrinsic motivation (Hannula, 2004a).

My aim in this paper is to discuss and make conclusions about the relation between the concepts of intrinsic and extrinsic motivation as defined by Self Determination Theory and rationales for learning mathematics as defined by Mellin-Olsen. First I will give a short presentation of the self-determination view of intrinsic motivation. This perspective is one of the most comprehensive and empirically supported theories of motivation available today (Pintrich & Schunk, 2002, p. 257). Second I describe in detail their model of differing types of extrinsic motivation. Third I present Mellin-Olsen's (1987) concept of rationale for learning mathematics. When presenting the theories I will give examples from my own study in mathematics education (upper secondary level). Finally I will discuss if the concepts of intrinsic and extrinsic motivation as defined in Self Determination Theory can be directly translated to Mellin-Olsen's S- and I-rationale as is usually done in Norway.

MOTIVATION IN SELF DETERMINATION THEORY

Most contemporary theories of motivation assume that people engage in activities to the extent that they believe the behaviours will lead to desired goals or outcomes (Deci & Ryan, 2000). Within Self-determination one is concerned about the goals of the behaviour and what energizes this behaviour. Self Determination Theory (SDT) is founded on three assumptions. The first assumption is that human beings have an innate tendency to integrate. *Integrating* means to forge interconnections among aspects of one owns psyches as well as with other individuals and groups in one's social world:

...all individuals have natural, innate, and constructive tendencies to develop an even more elaborated and unified sense of self. (Ryan & Deci, 2002, p. 5)

Individual's tendency to integrating involves both inner organisation and holistic self-regulation and integration of oneself with others. This assumption of active, integrative tendencies in development is not unique to SDT. However, specific to this theory is that this evolved integrative tendency cannot be taken for granted. The second assumption in SDT is that social-contextual factors may facilitate and enable the integration tendency, or they may undermine this fundamental process of the human nature:

...SDT posits that there are clear and specifiable social-contextual factors that support this innate tendency, and that there are other specifiable factors that thwart or hinder this fundamental process of the human nature. (Ryan & Deci, 2002, p. 5)

In other words, according to SDT, there is a dialectic relationship between an active organism and a dynamic environment (social context), such that the environment acts on the individual, and is shaped by the individual. Within SDT this is called an *organismic dialectic*. The third assumption is that human beings have three basic psychological needs, the need for competence, autonomy and relatedness. According to SDT, the three basic needs provide the basis to categorizing social-contextual factors as supportive versus antagonistic to the integrative process (Deci & Ryan, 2000; Ryan & Deci, 2000a, 2002). Within Self Determination Theory, competence autonomy and relatedness are defined in the following way:

Competence refers to feeling effective in one's ongoing interactions with the social environment and experiencing opportunities to exercise and express one's capacities. (Ryan & Deci, 2002, p. 7, *my italic*)

Relatedness refers to feeling connected to others, to caring for and being cared for by others, to having a sense of belongingness both with other individuals and with one's community. (Ryan & Deci, 2002, p. 7, *my italic*)

Autonomy refers to being the perceived origin or source of one's own behaviour. (Ryan & Deci, 2002, p. 8, *my italic*)

According to this definition, competence is not an attained skill, but it is a felt sense of confidence and effectiveness (effectance) in action. Relatedness reflects the human tendency to connect with and be integral to others. It is a felt sense of being with others in secure communion or unity. When individuals are autonomous they experience themselves as volitional initiators of their own actions. According to SDT, the students' motivation will be maximized within social contexts that provide the students the opportunity to satisfy their basic psychological needs for competence, autonomy and relatedness (Ryan & Deci, 2002).

Internal and external perceived locus of causality are two other important concepts within SDT, and they relate to the need for autonomy. If people perceive themselves as the origin of the behaviour, they have an internal perceived locus of causality. If people believe they are engaging in behaviour to achieve rewards, or because of external constraints they have an external perceived locus of causality (Deci, 1975).

Intrinsic motivation in Self Determination Theory

Self Determination Theory, as many other motivational theories, distinguishes between intrinsic and extrinsic motivation. A person is intrinsically motivated if he is doing an activity because it is inherently interesting or enjoyable. If a person is doing something because it leads to a separable outcome, he is extrinsically motivated (Ryan & Deci, 2000a). Within SDT, *intrinsic motivation* is defined in the following way:

Intrinsic motivation is defined as the doing of an activity for its inherent satisfactions rather than for some separable consequence. (Ryan & Deci, 2000a, p. 56)

Intrinsic motivation reflects the inherent tendency of human nature to engage in activities that are novel and challenging and results in learning and development.

From birth onward, humans, in their healthiest states, are active, inquisitive, curious, and playful creatures, displaying a ubiquitous readiness to learn and explore, and they do not require extraneous incentives to do so. This natural motivational tendency is a critical element in cognitive, social, and physical development because it is through acting on one's inherent interests that one grows in knowledge and skills. (Ryan & Deci, 2000a, p. 56)

According to SDT, if active engagement or intrinsically motivated behaviours are to be maintained, they require satisfaction of the needs for competence, autonomy and relatedness. Research studies within SDT indicate strong links between intrinsic motivation and satisfaction of the needs for competence and autonomy. Results from studies further indicate that relatedness typically plays a more distant role in relation to intrinsic motivation than do competence and autonomy (Ryan & Deci, 2000b, 2002).

Different types of extrinsic motivation

As mentioned earlier, extrinsic motivation refers to the performance of an activity in order to maintain some separable outcome. According to Self Determination Theory, extrinsic motivation can vary greatly in its relative autonomy. *Internalisation* and *integration* are important concepts in describing the different types of extrinsic motivation, and they are defined in the following way:

Internalisation is the process of taking in a value or regulation, and integration is the process by which the individuals more fully transform the regulation into their own so that it will emanate from their sense of self. (Ryan & Deci, 2000a, p. 60)

According to SDT, internalisation is a natural process where the individual tries to transform social practices, values or regulations into personally endorsed values and self-regulation (Deci & Ryan, 2000; Ryan & Deci, 2000a). I interpret regulation to be what regulates, orients or determines behaviour, or in other words, what causes behaviour. SDT assumes the following:

...if external prompts are used by significant others or salient reference groups to encourage people to do an uninteresting activity – an activity for which they are not intrinsically motivated – the individuals will tend to internalize the activity's initially external regulation. That is, people will tend to take in the regulation and integrate it with their sense of self. (Ryan & Deci, 2002, p. 15)

When the internalisation process functions optimally, the individual will fully accept the regulations as his or her own. The regulations will be fully integrated in the self, and through this process the individual will become both self-regulated and socially integrated. However, when the internalisation process is forestalled, the regulations may remain external or be only partially internalised (Deci & Ryan, 2000). In SDT internalisation is seen as a continuum. It describes how people's motivation varies

from amotivation to self-regulated motivation. How autonomous an individual is when acting depends on what extent the regulation of the extrinsically motivated behaviour is internalised. Regulations that are internalised to a small extent provide basis for more controlled forms of motivation. Regulations that are more fully internalised provide basis for more autonomous forms of motivation. SDT have identified four types of extrinsic motivation; external regulation, introjected regulation, identified regulation and integrated regulation (Deci & Ryan, 2000; Ryan & Deci, 2000a, 2002). The four types of motivation are organised in a taxonomy which reflects their differing degree of autonomy. Figure 1 shows the different types of motivation, arranged from left to right in terms of the extent to which the motivation is autonomous or self-determined.

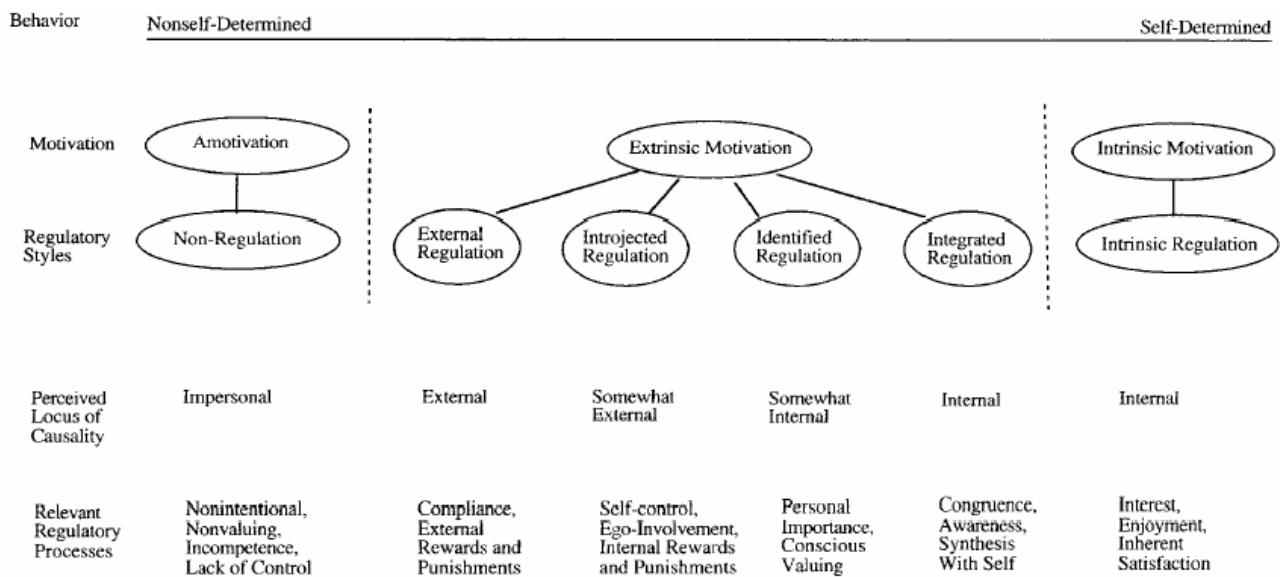


Figure 1: The self-determination continuum showing types of motivation with their regulatory styles, loci of causality, and corresponding processes (Ryan & Deci, 2000b, p. 72).

At the left end of the continuum in figure 1 is *amotivation*. When people are amotivated they lack an intention to act, and either they do not act at all or they act passively. This happens when they are not valuing the activity or the outcomes it would yield, or when they are not feeling competent to do it (Ryan & Deci, 2000a, 2000b, 2002). At the right end of the continuum is *intrinsic motivation*. Intrinsically motivated behaviours are the prototype of autonomous or self-determined behaviour, because these behaviours are interesting and enjoyable and are performed volitionally.

External regulation

External regulation is the least autonomous type of motivation. It is the classic case of extrinsic motivation in which people's behaviour is externally regulated by, for example, tangible rewards or threats about punishment. According to SDT, these regulations are considered controlling, and they have an external perceived locus of

causality (Deci & Ryan, 2000; Ryan & Deci, 2000a, 2002). Edith, in my study, is working well in the mathematics lessons, but she is not feeling very competent in mathematics. Her behaviour is externally regulated because her main focus is to get a good grade in mathematics.

Introjected regulation

Another type of extrinsic motivation is introjected regulation. The regulation is partially internalised by the individual but not accepted as one's own. These behaviours are performed with a sense of pressure to avoid guilt and shame and to attain a feeling of pride or worth. According to SDT, the behaviours are considered quite controlling, and they have an external perceived locus of causality (Deci & Ryan, 2000; Ryan & Deci, 2000a, 2002). A classical form of introjection is ego-involvement (Ryan & Deci, 2000a). A student is regulated by introjects if he gets to mathematics class on time to avoid feeling like a bad person.

Identified regulation

Identified regulation is a more autonomous or self-regulated type of extrinsic motivation. If a regulation or goal is personally valued by the individual, and is consciously accepted as one's own goal, the regulation is identified. According to SDT, identified regulation has an internal perceived locus of causality (Deci & Ryan, 2000; Ryan & Deci, 2000a, 2002). Nicole, in my study is working hard with mathematics because she believes it is important for continuing to succeed at mathematics, rather because it is interesting. She believes that mathematics is important for future studies. The regulation for her behaviour is identified because she is doing it for personal reasons.

Integrated regulation

Integrated regulation is the most autonomous type of extrinsic motivation. It does not only involve identifying with the importance of the behaviour, but the regulation is evaluated and brought into harmony with the individuals own personally values, goals, and needs that are already a part of the self (Ryan & Deci, 2002, p. 18). The regulation is fully accepted by the individual. The external regulation is completely internalised to self-regulating, and the result is self-determined extrinsic motivation (Deci & Ryan, 2000; Ryan & Deci, 2002). According to SDT, integrated regulation has many of the same qualities as intrinsic motivation, but there is one essential difference. When people's behaviour is governed by integrated regulations, they are performed volitionally to attain personally important outcomes, rather than because the activity is inherently interesting or enjoyable. The behaviours are performed to attain a separate outcome where the value of the outcome is well integrated with the self. According to SDT, integrated regulation has an internal perceived locus of causality (Deci & Ryan, 2000; Ryan & Deci, 2000a, 2002). Jennifer, in my study, is very clever in mathematics. Doing well in mathematics is personally important to her. She is working hard with mathematics and focuses on conceptual understanding. The

regulation is integrated and her behaviour is self-regulated, but she is not intrinsically motivated for mathematics. She does not think mathematics is enjoyable or interesting.

The continuum illustrated in figure 1 is purely descriptive. According to SDT, the individual does not need to progress through each stage of internalisation. Internalisation of a new regulation of behaviour may happen at any point along this continuum (Deci & Ryan, 2000; Ryan & Deci, 2002).

ACTIVITY THEORY AND RATIONALITY FOR LEARNING

In the book “The politics of mathematics education”, Mellin-Olsen developed his Activity theory, which is a social theory of learning mathematics. Mellin Olsen argues that if students are going to learn mathematics, they must approach their activity with a rationale for learning. He identifies two rationales for learning mathematics in school; an S-rationale (Social rationale) and an I-rationale (Instrumental rationale). In this section I first present the concepts of Generalised others and Ideology. These concepts are important in understanding Mellin-Olsen’s definition of the rationales for learning mathematics. Second I present the I- and S-rationale for learning Mathematics.

Activity Theory

Within Activity Theory the individual and society are considered a unity. The individual is acting on his society at the same time as he is being socialised to it (Mellin-Olsen, 1987, p. 33). The individual is considered a political individual of the society, and that means that the individual is permitted responsibility for his own life situation and for the society. Activity belongs to the individual and is a way of describing the complete life of an individual:

In the broad sense Activity is the way Man acts in his world, transforms it, and is being transformed himself in a variety of ways. Such transformation takes place in environments which are primarily social. (Mellin-Olsen, 1987, p. 38)

Within Activity Theory, the individual experiences himself through others. Mellin-Olsen (1987) operates with the concept “*the Generalised Other, (GO)*” about all the social groups in the environment that have influence on one owns life. It includes the common attitudes, expectations and reactions as experienced by the individual and which function as a referent for his actions. Usually the individual has a system of generalised others, for example, friends, family, neighbours and school. According to Mellin-Olsen the individual may be exposed to various GOs at the same time, and the GOs may communicate different views. Mellin-Olsen further introduces the concept of *ideology*:

In my use of ideology I relate this construct to the individual, in particular the pupil, as a carrier of ideas developed by him in his social relationships, i.e. the attitudes he has adopted from his GOs. (Mellin-Olsen, 1987, p. 155)

A student's ideology is the ideas the student has developed in his social relationships. Ideology is a dialectical concept. It is a result of the influence by his GOs and his own Activities.

Rationality for learning

To understand why students act like they do, Mellin-Olsen introduces the concept of *rationality for learning*:

The individual's rationale belongs to the individual. It is the way he "chooses" to act in his world under the material and social conditions under which he lives. (Mellin-Olsen, 1987, p. 156)

The rationale of behaviour is the result of the individual's ideology. According to the definition above, the individual's rationale is a dialectical concept. The rationale belongs to the individual, but it is a product of the individual's relations to his system of GOs. Mellin-Olsen claims that several rationales may be present for behaviour. He further emphasizes that the individual is not always acting out of a set of rationales, because the individual is considered to be able to evaluate the effects of his behaviour. Mellin-Olsen identified two important rationales for learning mathematics in school. One of the rationales is called an *instrumental rationale (I-rationale)*, because it works as an instrument for the students. This rationale is related to the school's influence of the students' future, by the contribution of formal qualifications to further studies or professions.

In its purest form the I-rationale will tell the pupil that he has to learn, because it will pay out in terms of marks, exams, certificates and so forth (Mellin-Olsen, 1987, p. 157)

The second rationale, which is called *social-rationale (S-rationale)* is saying that knowledge has a value beyond exams and grades. This rationale includes everything that makes the knowledge so important and interesting for the students that they want to acquire it. The S indicates that the student evaluates knowledge through a reference to his GOs which go beyond the I-rationale. The I- and S-rationale work together, and the student's rationale for learning is considered a result of both rationales. The two rationales mutually influence each other, and the student's evaluation of I- and S-knowledge may change over time.

DISCUSSION

As a starting point for my discussion I will present an example presented by Mellin-Olsen :

Jarle is a son of a doctor. From aunts, uncles, neighbours and teachers Jarle understands at an early age that he most likely is going to study, a thought which he also makes his own. For this reason the S-rationale nevertheless works for Jarle, when the teacher presents equations and the class does not understand at all how this can be useful for them. He knows that some time he will meet equations again, maybe as a necessity to become a doctor. For this reason, Jarle has no difficulty with accepting equations and

how to solve them as a meaningful activity in school. (Mellin-Olsen, 1984, p. 39, my translation)

This example illustrates that the student S-rationale can work even though the student does not experience the activity as enjoyable, interesting or challenging. Jarle considers equations to be important in relation to future studies. A student who is acting from the S-rationale has considered the knowledge, through a reference to his generalised others, to be important and interesting. According to Self Determination Theory, a student will internalise the external regulation of a learning activity if significant other or salient reference groups encourage the student to do an uninteresting activity. Within SDT Jarle's type of motivation will be described as identified regulation. Jarle has recognised and accepted the underlying value of learning equations. He has identified with the value of the learning activity, and accepted it as his own. The behaviour is still extrinsically motivated, because he is not doing the activity for its own sake. Another type of extrinsic motivation that also falls under the S-rationale is integrated regulation, which is the most autonomous type of extrinsic motivation. When a student has integrated a regulation, the behaviour is performed to attain goals that are brought into harmony with the student's values, goals or needs. Both types of regulation have an internal perceived locus of causality. The two least autonomous types of motivation in SDT's model are external regulation and introjected regulation. I consider these two types of extrinsic motivation to be similar to the I-rationale, because in the former case the behaviours are regulated by rewards as grades and exams, and in the latter case to avoid guilt and shame and to attain a feeling of pride or worth. Both types of regulation have an external perceived locus of causality.

My aim with this paper is to be informative, clarifying, and critical. Self Determination Theory model of the different types of motivation makes it possible to place the rationales in relation to intrinsic and extrinsic motivation, and that is what I have done in this paper. The model also makes it possible to be critical to the direct translation from S-rationale and I-rationale to intrinsic and extrinsic motivation as for example Holden does (Holden, 2003). Further, the SDT model made it possible to discuss the relation between the frameworks of intrinsic and extrinsic motivation versus social and instrumental rationale for learning.

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POTENTIAL FOR CHANGE OF VIEWS IN THE MATHEMATICS CLASSROOM?

Tine Wedege and Jeppe Skott

Malmö University, Sweden and the Danish University of Education, Denmark

This paper reports on a study which addressed a question of the potential and perceived influence of the Nordic KappAbel competition on the mathematical views and practices of the participating teachers and students. On the basis of an understanding of “views” and “practices” in the mathematics classroom, the term “didactical contract” is presented and used as a metaphor for structuring the analysis of the data from one of the participating teachers and his students. The problem in the study is closely related to the more general one of the role of external sources of influence on teaching/learning processes in the mathematics classroom.

INTRODUCTION

KappAbel is a Nordic mathematics competition for students in lower secondary school. It is based on collaborative work in whole classes: the class counts as *one* participant. The competition begins with two web-based qualifying rounds of joint problem solving activity. In Norway, one class from each county continues to the semi-final. Before meeting for the semi-final, these classes do a project on a given theme (in 2004-05 “Mathematics and the human body”). The classes that progress to the semi finals are represented by four students (two boys and two girls), who are to present their project work at an exhibition and to solve and explain a number of non-routine, investigative tasks. KappAbel, then, focuses on investigations and project work and signals that mathematics does not consist merely of closed lists of concepts and procedures with which to address routine tasks. Also, the emphasis on collaboration in whole classes suggests that there is more to mathematical activity than individuals engaging the development or use of such concepts and procedures. This reflects the aims of KappAbel that are (1) to influence the students’ affective relationships with mathematics (beliefs and attitudes) and (2) to influence the development of school mathematics in line with international reform efforts. The study was conducted in Norway in 2004-05 and sought to contribute to an understanding of the extent to which these aims are met. Hence, the research question we addressed was whether participation in the KappAbel competition has the potential to influence students’ and teachers’ views by influencing the modes of participation in the practices of mathematics classrooms (Wedege and Skott, 2006).

The study includes five types of empirical data, quantitative as well as qualitative: a questionnaire (TS1) administered to the teachers of 2856 grade 9 mathematics classes in Norway, 2004-2005; a questionnaire (TS2) administered to 15 of the teachers whose classes took part in the two introductory rounds of KappAbel and intended to continue with the project work; interviews conducted with eight teachers and six

groups of students, which is the empirical base of this paper; reports and process log books of five classes on the project work of ‘‘Mathematics and the body’’; and finally observations of 10-15 lessons in 3-4 classes.

Elsewhere we have discussed how we have dealt with some of the conceptual and methodological problems of belief research, for instance the ones of using conceptual frameworks that are not well grounded empirically, of over-emphasising teachers’ views of mathematics for their educational decision making, and that no terminology carries unequivocal meanings (Skott & Wedege, 2005; Wedege & Skott, 2006, p. 48 ff.). In this paper we shall briefly outline our understanding of one of the two key concepts in the study: *views*. This notion is linked to changes in school mathematical practices by means of a metaphorical use of *didactical contract*. This conceptual framework presents – together with an inspiration from social practice theory (e.g. Lave, 1988; Wenger, 1998) – a rationale for the Kappabel study.

VIEWS

Belief research has developed into a significant field of study in mathematics education over the last 20 years (e.g. Leder, Pehkonen and Törner (eds.), 2001). One of the recurrent discussions of the field concerns the lack of terminological clarity. There have, then, been many attempts to distinguish between beliefs, conceptions, attitudes, world views and other phrases all of which are meant to capture significant aspects of students’ and teachers’ meta-mathematical orientations, including those related to mathematics teaching and learning.

According to McLeod’s review (1992) of research on affect in mathematics education, ‘‘beliefs’’, ‘‘attitudes’’, and ‘‘emotions’’ were used to describe a wide range of affective responses to mathematics. The terms are not easily distinguishable, but the underlying concepts vary along three dimensions. First they differ in stability, beliefs and attitudes being generally stable, while emotions may change rapidly. Second, they vary in intensity, from ‘‘cold’’ beliefs to ‘‘cool’’ attitudes related to liking or disliking mathematics to ‘‘hot’’ emotional reactions to the frustrations of solving non-routine problems. And third, McLeod distinguishes between beliefs, attitudes, and emotions according to the degree to which cognition plays a role, and the time they take to develop. In figure 1, affect in a broad sense in mathematics education is positioned along a spectrum that runs from *stability* and ‘‘cool’’ on the left (the cognitive end of the spectrum), to fluidity and *intensity* on the right (the affective end of the spectrum).

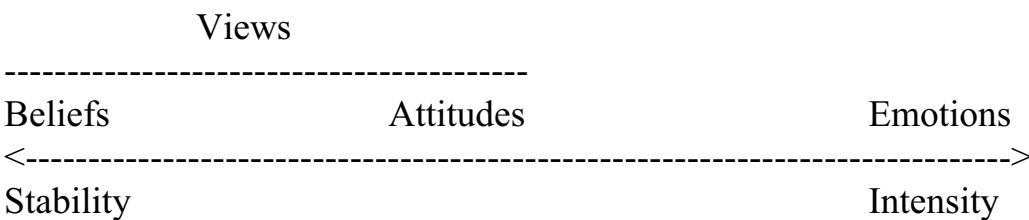


Figure 1: Spectrum of types of affect

This is not the only analytical description in different dimensions of the affective area in mathematics education research (cf. Evans, 2000:43-45), but in terms of terminological clarification, we find these three aspects and the interrelated characteristics both operational and meaningful. In our terminology *beliefs* include also self-perception (e.g. “Mathematics - that’s what I can’t do” (Wedegge, 2002)), aspects of identity (e.g. “In my life, I will never need any mathematics”), and confidence. *Attitudes* (e.g. “The importance of mathematics is increasing with technological development in society” or “Mathematics is the most terrifying school subject”) are more stable than *emotions* (e.g. panic or joy). Students’ emotions are seen in connection to personal goals, however some emotions are related to the social coordination, for example when students share similar ideas (Hannula, 2005). Figure 1 illustrates a continuum from cognitive to affective aspects of people’s relationships with mathematics. Using McCleod’s terminology, we study students’ and teachers’ beliefs of and attitudes towards mathematics at the cognitive end of spectrum, and – for pragmatic reasons – we name these phenomena peoples’ *views of mathematics*.¹

“DIDACTICAL CONTRACT”

The interplay – or the social and mathematical interaction – between teacher and students within the frame of the mathematical instruction is crucial in this study, where the topic is the potential change of practices of mathematics classrooms and in the teacher’s and students’ views of mathematics. We found it relevant to involve the metaphor of “didactical contract” in the design of the study and in subsequent analysis because it might combine the emerging school mathematical practices with views of mathematics and of the learning of mathematics. If you want to infer whether views and practices have changed, you need to look beyond the immediately observable actions and organisations of classroom activity. You have to study if and how the mutual expectations of the participants in those practices have changed, i.e. if and how the contract regulating their interactively developed contributions is evolving.

Brousseau’s concept of didactical contract is well known, or at least the term is frequently used. It originates in the framework of the French school of “Didactique des Mathematiques”, and for Brousseau it is inextricably linked to the theory of didactical situations (Brousseau, 1986). Adopting a somewhat different perspective, Balacheff links the didactical contract to the norms for social interaction in a broader sense. To some extent, then, he removes the concept of the didactical contract from the theoretical framework and the empirical studies on which it is originally based, and he defines the didactical contract as follows:

¹ A broader concept of an individual’s view of mathematics, which is developed on the basis of McCleod’s work, is presented and discussed in Pehkonen and Pietilä (2003), for example. In this paper views is defined as “a compound of knowledge, beliefs, conceptions, attitudes and feelings” (p. 4).

The rules of social interaction in the mathematics classroom include such issues as the legitimacy of the problem, its connection with the current classroom activity, and the responsibilities of both the teacher and pupils with respect to what constitutes a solution or to what is true. We call this set of rules a *didactical contract*. A rule belongs to the set, if it plays a role in the pupils' understanding of the related problem and thus in the constitution of the knowledge they construct. (Balacheff, 1990:260).

In mathematics education literature outside France, the notion of didactical contract is more often used in this latter, broader sense than in the one closely connected to the theory of didactical situations. This is the use we shall make of the term in the following as well. Thus, our use of the didactical contract does not imply that we import the general theoretical framework of the theory of didactical situations. Rather, we use the term *didactical contract* as a metaphor for the set of implicit and explicit rules of social and mathematical interaction in a particular classroom. The didactical contract, then, in our terminology constitutes the rules of the game in that classroom, rules that on the one hand frame the practices that emerge and on the other are regenerated and transformed by those very same practices.

Among the rules of a didactical contract, three central issues may be addressed: (1) What is mathematics and mathematics education? (2) How do you learn mathematics? (3) Why do you learn mathematics?

The problems in the qualifying rounds of KappAbel are not consistent with a didactical contract in which every student who has read and understood the theory, gone over the examples in the textbook and solved the exercises is expected to be able to solve the problem. The tasks seem to require the students to engage in systematically creative investigation, not supported by the contract just described (see www.kappabel.com for examples).

In the research report, we structured the analysis of the qualitative data by the two first issues addressed by the rules of the didactical contract mentioned above. The results were presented in a “first meeting with the teachers” based on the questionnaire (TS2) and in a “second meeting with seven of the teachers and their students” based on interviews and students' reports from the competition. In the following section, we present Steinar at the “second meeting” (Wedegé and Skott, 2006, pp. 156-161).

STEINAR AND HIS STUDENTS

Steinar is a man in his 30s with five years of teaching experience in mathematics. He has a strong background in mathematics and considers himself a mathematician. Through Steinar's answers to the questionnaire TS2 we get the impression that he emphasises the students' learning of mathematics that they will need in their everyday life. When he is asked to characterise good mathematics teaching, he allocates almost the same points to discussion and co-operation, to routines and to project work – yet the least to the last activity. Through the data from two separate interviews, we

meet Steinar again together with four of his students, Anders, Stian, May og Toril and the 9th grade project “Mathematics and the Human Body”.

Mathematics is a tool made of rules

The interviewer asks Steinar to describe a normal mathematics lesson. First he explains that they follow the chapters of the textbook and their content rather slavishly: “We go through the constructions on the blackboard, they work with exercises, and I go round supervising them and ... that sort of thing.” Then he gets more specific:

Steinar: Yes. ... We have three lessons a week. And about fifteen minutes of each lesson I go through what is to be done during the lesson, and what they should finish by the next lesson. ... It’s on a Monday so ... I give them the entire week’s work schedule that they can work on independently. So in a sense it is optional whether they want to listen to me going through the topics or work on their own. I think most of them listen to me. And then, what I go through, they write down in their rule book. ... And then they work on exercises, they sit in pairs and work together. ... [...] ...Next lesson, I also go through a bit of what they ought to get through during that lesson, unless they are already at that point. And I do questions on the blackboard, if someone’s wondering about some question or exercise, if many of them are wondering about the same exercise. But apart from that they do work very independently, they’re a very independent group. They ask each other a lot, and are very easy to deal with (Laughter) (l. 45-60)

During the exposition, Steinar writes one or two rules on the black board; he goes through one or two examples and asks the students to write down the rule in their “rule book”. The tasks are from the book, but when the interviewer asks Steinar, whether he has any kind of dialogue with the student while going through the topics on the blackboard, he says:

S: Oh yes, mm. I try to retrieve information they know from before, and ... much of it was dealt with last year and the year previous to that, after all. Much of it is repetition so ... it’s a matter of getting a dialogue about what they remember from last year up to now, and what they can try to extract from ... new things and try to see a connection ... yes. Then we often have, or we sometimes, sometimes we have, well it’s not in every lesson, but then we do have a bit of those kind of mathematical brainteasers, where we talk a bit about maths. A bit of that problem solving sort of thing, we have a few of those spicy tasks now and again. ... They’re similar to the ones in that Advent Calendar, if you’re familiar with that, with ... “matematikk.org” ... do you know what I mean?

I: Yes

S: Yes, those types of tasks, and having a bit of talk about them. But forty-five minutes, they pass very quickly so ... it’s not always that we have time for all that much of that sort of thing. (l. 74-86)

Steinar uses “brainteasers” and other problem solving tasks to spice up his teaching – something extra, not as part of what he perceives of as the normal teaching of mathematics. “It gets to be a bit too much a matter of routine, I’m starting to think,

but I suppose I'll learn a few new tricks as I go along" (l. 177-178). It sounds as if he considers development and change as coming from outside in the form of tips and tricks. Actually, Steinar calls the KappAbel problems from the qualifying rounds and the class activity around them "a nice interruption of the teaching".

When the interviewer asked the students, how they experience the similarities between the way they have been working during KappAbel and what they normally do during mathematics lessons, they react as follows:

May: It is maths after all

Toril: Equations and that sort of stuff

Anders: We do have tasks sort of, some places. After all, yes, they're all tasks, they're sort of solved in a slightly different way, for example. So in a sense each project is, after all, an exercise. You sort of solve it in a slightly different way. Well, you may have to work a bit more (laughs).

I: Have you ever thought: But, this has nothing to do with mathematics?

Toril: No, I haven't at least.

Anders: No, not really. At least not about our [...], to put it that way. But some of the others were a bit more difficult in terms of discovering the mathematical element. (l. 276-292)

Steinar describes the class as quite homogeneous, and normally the students engage with standard tasks from the textbook listed in their working plan. Steinar encourages a couple of the students to do more challenging tasks explicitly made for the students who do well. Most of them, however, do not take up the challenge.

From the Process Log in the students report, we get the impression that the students do not see mathematical competence and problem solving as the same: it is quite possible to be good in maths without being able to deal with problem solving and vice versa. The class agrees that the four members of the team going to the final "had to be good at maths and able to do problem solving tasks" (Report, p. 5). The character of the KappAbel problems is such that there is not just a single mathematical rule to be followed in order to solve them. According to what seems to be the didactical contract developed between Steinar and his students in this 9th grade, mathematics is a tool made of rules. Problem solving and project work do not appear to be included in the conception of mathematics in this classroom. As Steinar puts it: "... it has been very busy, it's been a somewhat unfamiliar ... way of thinking about maths, bringing it in, integrating it into a topic." (l. 239-240)

You learn mathematics using the rules to solve tasks

Explaining the teacher's role in relation to the students' learning process, Steinar says :

Steinar: Yes, no, well, there are always questions popping up. So I'm going round supervising and listening to how they're doing and: This is going well. And: You've grasped this point and that. And that sort of thing. Of course, I don't do the problems (Laughter). ... Well yes, I suppose I do have a dialogue going with them too ... to get a few impressions of what they've understood. ... Yes, I do have that.

I: But when you're saying that they work very independently, does that mean that they try on their own first, before they contact you?

Steinar: Yes, they seem to try on their own first and then they mostly ask the person next to them, and then ... if they don't get an answer, then they ask me.

I: Well, exciting, because now ...

Steinar: Yes, because I try to be around as little as possible, I want them to try to figure this out a bit by themselves. [inaudible] and then they'll ask me for an answer and then I'll say, "have you looked in your rule book?" - No, he hasn't. "Do that." Then they'll look up their rule book, and for the most part they'll find an answer. So I try to make them even more ... independent, too. (l. 121-139)"

And then they look in the rule book "where they find an answer almost at once", Steinar says. When the interviewer asks him where he gets the inspiration from for his teaching, Steinar says:

Steinar: ... That may well be ... what I think is the smartest thing to do, is to go through the topic and then for them to work with exercises. Because it's ... it's the practise with exercises that makes, that I believe makes, them good at it after all... together with a bit of dialogue. ... In terms of performance – and tests – you do, after all, depend a lot on a rule book, so I put great emphasis on good work with the rule book and that they're able to find the right solution themselves by using it. ... [...] ... I've come to understand that the rule book is ... terribly important. ... which is why I use the approach of going through topics in my lessons, and have them write things down. ... Apart from that, I think it's very important to differentiate the tasks they are given. ... (Laughter). (l. 187-198)

From his experience, Steinar knows that it is routine in solving tasks and use of rule books that make the students better in mathematics.

KappAbel

Steinar explains that it was extremely difficult to get the whole class interested in KappAbel. In the end only half of the students (10) participated in this part of the competition, while the rest had ordinary mathematics lessons.

In the students' Report, it is explicit that project work was a new experience – both to the teacher and the students. However, when the students got down to work they found it fun. They had chosen an open problem on mathematical relations in the human body. In their work, they have only been looking for linear relations (direct proportionality) and they did not find any. However, they also regard this as a result.

Initially, the students did not perceive of the project work as a learning process. In their Process Log they write:

In order for the project to be as efficient as possible, we decided that only half the class would be working on it. If the entire class were to work on the same thing, it would only result in chaos, according to many. In addition, not everyone was equally motivated for working on the project. (Report, p. 5)

This may be understood as if the students consider the product, the exhibition, the competition as the primary purpose of the project. However, from the interview and their report we also get the impression that they have both learnt from the project and enjoyed doing it. In the report they say: "All in all, we think projects in mathematics are something we ought to do more often. We think our chosen problem is original and slightly amusing." They elaborate on this in the interview:

I: What has taking part meant to you?

Toril: Meant?

Anders: I think maybe I find maths more fun now (laughs)

Toril: Yes

May: Same here

Stian: Learnt a bit more

Anders: Thinking a bit, being allowed to think a bit differently about mathematics not just being about sitting there looking at your book and writing, sort of /

I: Yes

Anders: They're slightly different things and sort ofworking together and doing difficult things and trying to manage and cope with it all it's sort of completely different from sitting there reading from the book.

May: Yes (l. 206-226)

In this 9th grade, teacher and students want the mathematical classroom practices to be different. Steinar wants the teaching to be a little more "spicy" and he often thinks of using computers. It seems like the four students have broadened their ideas of what could be done in the mathematics lessons, but they are not sure if you really learn mathematics doing these kinds of activities (problem solving and project work). Their views of mathematics may not have changed much – problem solving still differs from mathematics, because it requires fantasy, creativity and energy. But they did have fun and in Anders' words, the students have been challenged to "think a bit differently about mathematics" when they did the project.

CHANGING VIEWS

We expect neither teachers' nor students' views to change easily. Also, following some of the criticism of mainstream belief research, we do not expect teachers' views

to be immediately and uni-directionally related to his or her contributions to the classroom interactions, or to the practices of mathematics classrooms more generally (for a discussion see chapter 2 in Wedege & Skott, 2006). However, views and practices do change and so do didactical contracts in tandem with changing practices and views of mathematics. The question we were addressing in the KappAbel study is if and how this specific mathematics competition may facilitate such change, or if problem solving and project work is to influence only the islands of instruction specifically directed towards KappAbel. In the case of Steinar and his students, the latter seems to be the case, as the dominant views and practices did not change, although there are shifts in attitudes (“we want to do something else”) and emotions caused by the new kind of mathematical activities.

One of the differences between the KappAbel competition and most other initiatives towards reforming mathematics education is the character and sequence of the steps expected to bring about the envisaged changes. Many attempts to bring about change in teachers’ views on mathematics and its teaching and learning go via a pre- or in-service teacher education course. Teachers are expected subsequently to carry their newly established, reformist educational priorities into the schools. In KappAbel the intention is to immediately restructure the teaching-learning practices of mathematics classrooms, by inserting new types of tasks and novel ways collaborating directly into mathematics classrooms.

Classroom practices may change. As we see in the case of Steinar and his students, change, however, is not merely a question of implementing a few new ideas, for instance in the form of tasks that are meant to insert collaborative and investigative elements in mathematics instruction. Change is a matter of teachers and students engaging differently in the activities that mutually constrain and support each other so as to constantly regenerate and further develop the practices of the classroom. As a consequence of their involvement in these changing practices and of their renegotiation of the didactical contract, students and teachers may develop new ways of conceiving of mathematics, of mathematics in schools, and of the teaching and learning of mathematics, i.e. they may develop new views.

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